ESSAYS ON TAX COMPLIANCE

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ESSAYS ON TAX COMPLIANCE

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INTRODUCTION

INTRODUCTION

The existence of a tax system forces individuals to give part of their income to the government as tax payments. Since individuals dislike paying taxes, different strategies geared towards reducing the tax burden emerge associated to the strategic behavior of taxpayers. Therefore, the tax compliance problem is a phenomenon inherent to the existing tax systems and thus affects to all the developed countries at some extent.

There are different ways to reduce the fiscal burden of an individual relative to that of other taxpayers with the same economic characteristics. Tax evasion is most common alternative that has been analyzed in the tax compliance literature. A large body of the economic analysis on the tax evasion phenomenon has been based on the formalization proposed by Allingham and Sandmo (1972), which is based in turn on the economic analysis of crime initially developed by Becker (1962). According to this approach, individuals voluntarily decide which part of their income will declare, taking into account both the saving associated with paying only the taxes associated with declared income and the potential cost appearing if this illegal behavior is discovered and thus punished[1]. Of course, it is crucial to introduce in this economic setting a legal mechanism of both inspection of declared income and punishment of fraud.

The increasing complexity of tax systems allows some individuals to reduce the tax payments by taking advantage of the incompleteness of the tax codes. This is the case of the so called tax avoidance. While tax evasion is illegal, tax avoidance can not be punished since the latter behavior is not outside the law.[2] Moreover, the progressivity of most of the current tax systems heavily penalizes individuals enjoying high income levels. Therefore, those individuals have strong incentives to avoid taxes by changing their economic activity or their legal residence to countries viewed as "tax heavens". In such countries the tax liability borne by those individuals is either nul or quite light.

The existence of fraudulent behavior, whether it is punishable or not, has important effects on the performance of an economy. First, let us consider the distortions on the properties of the tax code concerning equity. Usually, rich individuals have at their disposal more ways to reduce their tax burden since most of the activities aimed to tax avoidance require the payment of a fixed cost which usually can only be afforded by individuals enjoying a high income level. This fact generates a substantial distortion in terms of equity, which can be even more important if the richer individuals evade systematically a fraction of their income larger than that of the poorer ones. In this case, the tax function ends up being less equitable than the one originally designed by the corresponding government. This also means that the effective tax function becomes more regressive than the nominal one. Second, the problem of tax compliance also affects the incidence of the tax code since different sources of income are associated with different opportunities for both evasion and avoidance. Therefore, some individuals end up paying a share of taxes smaller than the one corresponding to them. Finally, the existence of tax evasion makes more difficult the analysis of the distorting effects of taxation since, given a fixed revenue requirement by the government, more distortionary taxes on declared income may be needed to outweigh the amount of evaded taxes.

In the last decades, the literature on tax compliance has blossomed [see for example Andreoni et al. (1998)]. Tax compliance problem has been studied from many complementary different perspectives. Now, I just want to mention some of the more general approaches. First, it can be

1 Notice that this problem is similar to that of optimal selection of a portfolio consisting of a risky asset and a riskless one.

2 For more details see Stiglitz (1985).

viewed as a tax enforcement problem and this leads to analyze, for instance, the design of optimal auditing policies for a given objective function of the government. Second, we could focus on the taxpayer problem and analyze the tax compliance problem by considering the different alternatives faced by a taxpayer in order to ease his tax burden. More precisely, the analysis of both tax evasion and tax avoidance when the taxpayers take as given the fiscal parameters controlled by the government and maximize their expected utility with respect their declared income, could be an illustrative example of this kind of approach. It is also possible to generalize the implications of the previous models to a macroeconomic context by allowing the possibility of evading behavior in a setting in which the aggregate variables, like consumption or output, are taken into account. Finally, tax compliance can also be linked to the labor market performance since the possibility of fraud may depend on the job type of the taxpayer. This fact may bring about a change in the decisions concerning either the labor supply or the investment in human capital made by individuals.

However, as a complex phenomenon, tax compliance is still a very stimulating area for research. Three questions related to tax compliance are analyzed in this thesis. First, we want to analyze how the evading behavior of consumers and the tax enforcement policy of the government have an influence on economic growth in a dynamic model of general equilibrium. Second, we analyze whether the Ricardian proposition holds in a partial equilibrium context where there is uncertainty associated with the evading behavior. Finally, we study the relationship between the tax rate and the government revenues when individuals may ease their tax burden by resorting either to tax evasion or to tax avoidance.

The analysis of tax evasion has been traditionally made in partial equilibrium contexts where consumers and fiscal authorities interact and the possible implications on the overall economy of tax evasion are left aside. In fact, there are very few studies aimed to generalize the results obtained in partial equilibrium models to macroeconomic models. This scarcity is mainly due to the difficulty associated with introducing uncertainty in dynamic general equilibrium models. Because of this, the existing papers in this literature consider variations of the old Keynesian IS-LM model, where there is no explicit maximizing behavior by the agents and where the dynamic considerations are absent.[3]

In Chapter 1 we try to make a new contribution to the analysis of the implication of tax evasion in a macroeconomic context.[4] In order to achieve this goal we analyze how the tax compliance policy affects the rate of economic growth. It is well known that proportional taxation matters for economic growth since taxes distort the accumulation of capital and, at the same time, affect the supply of productive inputs provided by the government [see, for instance, Barro (1990)]. Therefore, we consider a dynamic general equilibrium model in which the paths of aggregate output, wages, interest rates, saving and consumption are all endogenously determined and this allows us to make an analysis a little more ambitious than the ones that have preceded us. The two complementary instruments available to the tax collecting agency in order to enforce the tax legislation are the inspections and the fines imposed on tax evaders. We thus analyze the effects of changes in the parameters characterizing these two policy instruments on the rate of economic growth.

We consider an overlapping generations model with production à la Diamond (1965) for which we parametrize both preferences and technologies. In such an economy young individuals obtain an income from the labor services they supply to the firms. There is a proportional tax on declared labor income and collected taxes will finance productive inputs supplied by the government, like roads, education, etc. We assume that under reporting of income is a risky and illegal activity. Agents are

3 See, among others, the papers of Peacock and Shaw (1982), Ricketts (1984), Lai and Chang (1988), Lai, Chang and Chang (1995) and Chang and Lai (1996).

4 Chapter 1 is joint with Jordi Caballé.

investigated with positive probability and, if a taxpayer is caught evading, he must pay a proportional penalty on the amount of evaded taxes, as in Yitzhaki (1974). Therefore, the combination of the penalty fee, the audit probability, and the tax rate determines not only the amount of declared income, but also the amount of labor income saved which will be used for consumption in the next period. Such a saving determines the private capital installed in the economy which, together with public capital, determines in turn the evolution of all the remaining macroeconomic variables. The economy ends up displaying a balanced growth path which is characterized by an endogenous growth rate.

In this context we have analyzed the effects of the different auditing policies that the government has at its disposal. These policies are characterized by two parameters, namely, the probability of inspection and the fee to be paid by a taxpayer who is caught evading. Our main findings include the comparative statics of the two tax compliance instruments on the aforementioned endogenous rate of economic growth. Such a comparative statics is generally ambiguous and depends on the importance of publicly provided inputs in the production process. For example, let us restrict ourselves to those combinations of the probability of inspection and of the penalty fee for which evasion disappears. In this scenario a decrease in the fee implies, in the one hand, less revenues, less public spending, and thus less growth. On the other hand, the individuals will enjoy more disposable income so that the private capital of the economy will rise and the growth rate will increase accordingly. We see then that there are two effects exhibiting opposite sign and it will dominate one or the other depending on whether the production function is intensive either in private capital or in public spending. If we consider instead the case where evasion is total, that is, when the consumers decide to hidden all their income, the rate of economic growth is locally increasing in the probability of inspection provided public spending is an input with a sufficiently high productivity.

We also provide two other results also found in partial equilibrium analyses of the tax evasion problem. The first one refers to the non-optimality from the growth viewpoint of an inspection policy inducing truthful revelation of income for exogenously given levels of both the penalty and the tax rates. This is so because, if there is truthful revelation, then a slight reduction in the costly inspection effort reduces negligibly the amount of collected taxes whereas the resources liberated by the tax collection agency can be devoted to the provision of growth enhancing public services. The second result refers to the growth maximizing combination of the two instruments. Such a combination depends on how productive is public capital relative to private capital. In particular, if inducing complete tax compliance is optimal, then a policy with an arbitrarily high penalty fee and a low inspection probability can implement a growth rate arbitrarily close to that of an economy without tax evasion.

Finally, in an example at the end of Chapter 1 we emphasize the fact that policies aimed to maximize the growth rate of the economy might not be the ones that maximize social welfare. For instance, if both the social planner and the taxpayers heavily discount the future, then the planner will tend to choose low values of the penalty fee so as to increase the first period consumption of the individuals. This policy might be associated with low growth rates if the contribution of public capital to output is sufficiently low.

The aim of Chapter 2 is twofold. First , we concentrate our analysis on the circumstances under which Ricardian equivalence holds in a scenario where uncertainty arises as a consequence of tax evasion. Second, we also analyze the relationship between tax evasion and the tax rate when the there is no crowding out.

The Ricardian equivalence proposition says that, if a change in current taxes is completely offset by a change in future taxes, the consumption path of individuals remains unchanged when the government spending path is not modified (Barro, 1974). Some authors as Barsky et al. (1986), Feldstein (1988) and Strawczynski (1995) have analyzed this problem in contexts where uncertainty appeared as a result of exogenous random income shocks hitting the economy. However, in the analysis presented in Chapter 2 we consider that the uncertainty in income is not completely exogenous since taxpayers can control its future distribution by means of the amount of income that they decide to declare.

Furthermore, the sign of the relationship between tax rates and declared income is one of the questions that still is not resolved nowadays. Allingham and Sandmo (1972) showed that under decreasing absolute risk aversion, the relation between declared income and tax rates is ambiguous when the fines are proportional to the unreported income. However, Yitzhaki (1974) found that a rise in the tax rate increases the declared income under decreasing absolute risk aversion and when the fine paid by an audited evader is proportional to the amount of evaded taxes. This modification of the Allingham and Sandmo model generates an unambiguous result which has not been supported by the empirical evidence since several studies have documented that higher tax rates tend to stimulate tax evasion.[5] Many authors such that Gordon (1989), and Klepper, Nagin and Spurr (1991) among others, have searched for other alternative models aimed at explaining this evident contradiction between the empirical findings and the theoretical ones.[6] Our contribution to this literature is to characterize the behavior of declared income as a function of the tax rate in a Ricardian framework that allows us to isolate the tax evasion implications of an increase in the tax rate by disregarding the crowding out effect associated with the higher levels of public spending.

We consider a model where individuals live for two periods. The young individuals must decide both the amount of income they want to report to the tax authorities and the amount they want to save. There is uncertainty in the second period since, if an agent is inspected by the tax authorities, his saving will be reduced by the fine that he has to pay. In this context, we will investigate the effects both on savings and on declared income (and, as a by-product, on consumption) of a variation in the tax rate that leaves unchanged the level of government spending. This means that, if an individual suffers an increase in the tax rate, he will be compensated by a reduction in his future lump-sum taxes.

The results that we have obtained concerning the Ricardian equivalence proposition are totally different depending on the assumption made about the fine paid by taxpayers if an inspection occurs. When the penalty is imposed on undeclared income, the Ricardian equivalence proposition fails to hold, whereas it holds when the fine paid by the taxpayers is a constant fraction of evaded taxes. Moreover, the sign of the relationship between tax rates and declared income also depends on the type of penalties. On the one hand, if the penalty is imposed on unreported income, a rise in the tax rate does not modify the amount of income that an agent has to pay in the case of being caught for a given amount of reported income. This provides an incentive for tax evasion on the margin. Moreover, the lump-sum transfer received by the individuals offsets the income effect accruing from the increase in the tax rate. In this case, the substitution effect becomes crucial and declared income turns to be decreasing in the tax rate. Therefore, individuals end up facing more uncertainty in the second period of life since the variance of their old income is raised. As the incentives for precautionary saving are modified, the consumption profile changes accordingly. On the other hand, if the fine is proportional to the amount of evaded taxes, an increase in the tax rate implies a rise in the penalty and this makes individuals to increase their declared income since the substitution effect has been eliminated and the lump-sum transfers do not completely offset the income effect. However, the amount of evaded taxes remains unchanged now and, hence, neither precautionary savings nor the consumption distribution in the second period are affected.

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5 Clotfelter (1983) and Poterba (1987) report a positive relation between tax rate and undeclared income using a real income data base.

6 Slemrod (1985) and Feinstein (1991) cast some doubts on these results since they argue that it is not possible to distinguish between the effect of the tax rate on evaded income and the overall effect of other variables that are also relevant for the problem under consideration.

Finally, in Chapter 3 we analyze the impact on the tax revenue of a change in the tax rate when the individuals may use two alternative methods for reducing their tax liabilities: tax evasion and tax avoidance. The difference between this two activities lies on the fact that tax evasion is penalized with fees if it is discovered (and this introduces some degree of uncertainty in the individuals decision problem), whereas under the term tax avoidance we include a set of activities that are not strictly illegal and that bring about a reduction of the tax burden without generating any kind of uncertainty. In particular, tax avoidance takes the form of a "tax heaven" in our analysis. A tax heaven is a country or a region where income is tax at very low (or even nul) tax rates.[7] Because of this, in our model a taxpayer that becomes avoider decides to avoid all his income since the fixed costs associated with a change of residence are the same regardless of whether avoidance is partial or total. Some authors like Cowell (1990), Cross and Shaw (1982) and Alm (1988) analyze the behavior of both individuals and government when avoidance and evasion are jointly selected. All those authors use the assumption of a representative consumer and, thus, they disregard the analysis of the behavior of the government revenue as a function of the tax rate. In fact, the comparative statics is immediate in the previous models when fines are proportional to evaded income: revenues increase with the tax rate since there is less evaded income. In our case, individuals have different incomes that are taxed proportionally. This means that not all the individuals select the same method for paying less taxes. We will assume that the income distribution is uniform although we also explore the robustness of our results in an example with a discrete income distribution. Such a discrete distribution will allow us to discuss the implications of income polarization concerning the characterization of the revenue-tax rate relationship.

Our results show that the possibility of choosing between avoiding and evading brings about a tax revenue function exhibiting the shape of a Laffer curve in all the scenarios considered. That is, the relationship between the tax rate and the government revenue is non-monotonic. The intuition of this result lies on the effects that an increase in the tax rate triggers. On the one hand, an increase in the tax rate generates more incentives for avoidance among the richer individuals but, on the other hand, it disincentives fiscal evasion. Therefore, there will be less individuals paying taxes, but those that pay taxes will end up paying more. It is possible to find cases where the latter effect is completely dominated by the former when the cost of avoidance is sufficiently low so that almost all individuals decide to become avoiders. The relative robustness of this result indicates that in our context a policy of high marginal tax rates might not be the appropriate strategy when the objective of the government is to maximize its revenue. Finally, we will like to emphasize that the previous result has some normative implications since, if the costs associated with tax avoidance are low, to set high tax rates and to implement a strong anti-evasion policy is not only ineffective but also regressive because all the rich individuals will avoid their incomes and will pay no taxes.

To conclude I would like to summarize some open lines of research aimed to study different aspects of tax compliance. Some authors analyze the consequences of considering that firms can also adopt some kind of evading behavior. Following this line of research Yaniv (1995, 1996) develops a general model of tax evasion applicable to several types of evasion that might be practiced by the firm. Lee (1998) analyzes the influence of tax evasion on the output decision of a monopolist when the audit probability or the penalty rate depends on the reports made by such monopolist. Finally, Joulfaian and Rider (1998) examine the compliance pattern of small businesses by using the Taxpayer Compliance Measurement Program (TCMP) data. In fact, it could be interesting to extend this analysis to a general equilibrium context where firms and individuals can exhibit some kind of fraudulent behavior.

On the other hand, some papers as the ones of Bishop et al. (1999) and Trandel and Snow (1999) try to analyze the implications of the existence of some fraud on the equity of the tax

7 Some examples of tax heavens are Andorra, Monaco and Panamá. To obtain a more detailed list of countries that are viewed as tax heavens see Monroy and Coronado (1998).

system.[8] In particular, Bishop et al. (1999) use the TCMP micro data to study the equity effects of non-compliance, while Trandel and Snow (1999) analyze how the presence of an underground economy influences the relationship between tax rates, progressivity, and non-payment. Obviously, the existence of some alternatives to reduce the tax liabilities of a taxpayer modifies the initial progressivity of the tax function. Therefore, to study how the progressivity of the tax function changes when the individuals try to diminish their tax burden (by means of avoidance and/or evasion) becomes a natural question to be posed in this framework.

As another interesting line of research, I just want to mention the fact that some authors pinpoint the problem of the corruption of tax authorities. Corruption is also a tax compliance problem since, if individuals are caught evading but the tax inspectors do not punish them, the government perceives the evading behavior as if it were honest. Obviously, the previous problem will induce some reform of the optimal inspection policy by the government so as to offset the negative effects on revenues generated by the existence of corruption. As an example we could mention the paper of Hindriks et al. (1999) that examines the implications of corruptibility and the potential abuse of authority for the effects and optimal design of tax collection schemes. In a related work, Sanyal et al. (1998) show that in some circumstances a higher tax results in smaller net revenue for the government in a context where the tax administration is corrupt. Following this line, we could try to characterize some anti-evasion policies that are less susceptible to corruption. For example, the policy of levying a lump-sum tax that is independent of the income of taxpayers is quite immune to corruption . However, this tax policy has the disadvantage of being quite regressive. Therefore, it seems evident that the question as how to characterize anti-evasion policies being simultaneously immune to corruption and progressive at some extent, is still an open question.

8 There are already several theoretical and empirical papers that analyze the implications in terms of equity of implementing a progressive tax function. As examples, see the works of Ok (1995), Panadés (1999) and Young (1987, 1990).

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CHAPTER 1

TAX EVASION AND ECONOMIC GROWTH

1.1. INTRODUCTION

The aim of this chapter is to analyze how the tax compliance policy affects the rate of economic growth. It is well known that proportional taxation matters for economic growth since taxes distort the accumulation of capital and, at the same time, affect the supply of productive inputs provided by the government [see, for instance, Barro (1990)]. However, an effective tax system must be enforceable, that is, it must provide incentives to the taxpayers for tax compliance. The two complementary instruments available to the tax collecting agency in order to enforce the tax legislation are the inspections and the fines imposed on tax evaders. We thus analyze the effects of changes in the parameters characterizing these two policy instruments on the rate of economic growth. To this end, we have to consider a dynamic general equilibrium model in which the paths of aggregate output, wages, interest rates, saving and consumption are all endogenously determined. We should mention at this point that little attention has been paid to the macroeconomic implications of tax evasion.[1]

Our general equilibrium approach forces us to an extreme stylization of the economy under study. Thus, we consider an overlapping generations model with production á la Diamond (1965) for which we parametrize both preferences and technologies. In such an economy young individuals obtain an income accruing from the labor services they supply to the firms. There is a proportional tax on declared labor income and collected taxes will finance productive inputs supplied by the government, like roads, education, etc. We assume that under reporting of income is a risky and illegal activity. Agents are investigated with positive probability and, if a taxpayer is caught evading, he must pay a proportional penalty on the amount of evaded taxes, as in Yitzhaki (1974). Therefore, the combination of the penalty fee, the audit probability, and the tax rate determines not only the amount of declared income, but also the amount of labor income saved which will be used for consumption in the next period. Such a saving determines the private capital installed in the economy which, together with public capital, determines in turn the evolution of all the remaining macroeconomic variables. The economy ends up displaying a balanced growth path which is characterized by an endogenous growth rate.

Our main findings include the comparative statics of the two tax compliance instruments on the aforementioned endogenous rate of economic growth. Such a comparative statics is generally ambiguous and depends on the importance of publicly provided inputs in the production process. We also provide two other results also found in partial equilibrium analyses of the tax evasion problem. The first one refers to the non optimality from the growth viewpoint of an inspection policy inducing truthful revelation of income for exogenously given levels of both the penalty and the tax rates. This is so because, if there is truthful revelation, then a slight reduction in the costly inspection effort reduces negligibly the amount of collected taxes whereas the resources liberated by the tax collection agency can be devoted to the provision of growth enhancing public services. The second result refers to the growth maximizing combination of the two instruments. Such a combination depends on how productive is public capital relative to private capital. In particular, if inducing complete tax compliance is optimal, then a policy with an arbitrarily high penalty fee and a low inspection probability allows the implementation of a growth rate arbitrarily close to that of an economy without tax evasion.

The chapter is organized as follows. Section 1.2 presents the taxpayer optimization problem. Section 1.3 determines the equilibrium of the economy from the interaction among consumers,

1 The papers of Peacock and Shaw (1982), Ricketts (1984), Lai and Chang (1988), Lai, Chang and Chang (1995) and Chang and Lai (1996) are among the few exceptions that introduce macroeconomic considerations in the analysis of tax evasion. However, all these papers rely on variants of the traditional Keynesian (IS-LM) model in which neither the dynamics nor the maximizing behaviour of consumers are made explicit. Moreover, they concentrate the analysis on the relationship between tax evasion and total tax collection.

firms and the government. Section 1.4 analyzes the consequences of the tax enforcement policy on economic growth. As a warning of the limitations of our positive approach, Section 1.5 develops an example of how the objective of growth maximization may conflict with the objective of a social planner trying to maximize a social welfare function. Section 1.6 concludes the chapter.

1.2. THE TAX EVASION PROBLEM

Let us consider an overlapping generations (OLG) economy populated by a continuum of identical individuals living for two periods. A new generation is born in each period and there is no population growth. Generations are indexed by the period in which they are born. Individuals own a unit of labor when they are young (the first period of their lives) and such a unit of labor is supplied inelastically to the firms in exchange of a wage. Labor income is subjected to a proportional tax and the tax rate is $\tau \in (0,1)$. An individual of generation t declares a level x_t of labor income during the first period of life. Therefore, the amount of taxes paid voluntarily will be τx_t. Since tax evasion is possible, x_t might be less than the real wage w_t. With probability $p \in (0,1)$ individuals are subjected to investigation by the tax authority and, if such an investigation takes place, the tax collecting agency detects the true labor income earned by the taxpayer. In such a case, the taxpayer will have to pay a proportional penalty rate $\pi > 1$ on the amount of evaded taxes $\tau(w_t - x_t)$.

Our specification of the tax evasion problem is the same as that of Yitzhaki (1974) since the penalty is imposed on evaded taxes while Allingham and Sandmo (1972) assume instead that the penalty is on undeclared income. Note however that all our analysis can be adapted to the setup of Allingham and Sandmo by replacing the penalty rate π by $\hat{\pi}/\tau$, where $\hat{\pi}$ would be the penalty rate on unreported income.

Consumption in the first period of life takes place after taxes on declared income have been paid but before the potential inspection occurs. Let s_t denote the income disposable after an individual has consumed and paid the taxes on declared income. Young individuals may save part of their income and earn a gross rate of return R_{t+1} on the amount saved. The saving of an agent which has not been audited is s_t while the saving of an audited agent will be $s_t - \pi\tau(w_t - x_t)$.

We now introduce two doses of realism in the tax system in order to prevent counterfactual behavior by the taxpayers. If an individual declares more than his true labor income and such an individual is audited, then the excess tax contribution is just returned. In other words, the penalty rate applying to "negative" tax evasion is 1. Under such an assumption, no individual will declare more than his true wage because excess tax contribution is in fact a risky investment having a negative risk premium.[2]

Moreover, the tax legislation does not feature a "loss offset", which means that the tax rate applying to negative income is zero. Hence, the tax law establishes that only agents declaring

2 Recall that risk averse agents take risky positions if and only if the associated risk premium is strictly positive [see Arrow (1970)].

positive income must fill the tax form and pay the corresponding taxes on declared income. Note that individuals not filling the tax form are in fact "declaring" a zero labor income.

Capital income will be consumed when individuals are old (i.e., in the second period of life). An old individual does not have any other source of income and thus her consumption will be $R_{t+1}(s_t - \pi\tau(w_t - x_t))$ if she has been audited, or $R_{t+1}\, s_t$ if she has not. Since the inspection occurs after consumption has taken place, taxing the income of old agents is not enforceable and, therefore, capital income is tax exempt.

The following table summarizes the sequence of events within each period of life:

First period of life
Individuals work
Wages are paid
Individuals declare their labor income and pay the corresponding taxes
Young consumption takes place
Tax inspection occurs with probability p and the corresponding penalty is paid
Capital market opens and effective saving takes place

Second period of life
Return on saving is paid
Old consumption takes place

The preferences of an agent of generation t are represented by the time-additive Von Neumann-Morgenstern utility function

$$u\left(C_t^1\right) + \delta E\left(u\left(\tilde{C}_{t+1}^2\right)\right), \tag{1.1}$$

where C_t^1 denotes consumption in the individual's first period of life (young consumption) and $\tilde{C}_{t+1}^2$ is the random consumption in the second period of life (old consumption). The random variable $\tilde{C}_{t+1}^2$ takes two values, C_{t+1}^{2A} and C_{t+1}^{2N}, which correspond to old consumption if the individual has been audited, and old consumption if she has not been audited, respectively. The parameter $\delta > 0$ is the discount factor.

Therefore, an individual of generation t chooses both the income declared $x_t \in [0, w_t]$ and the intended saving s_t in order to solve the following program:

$$\text{Max}\ \left\{u\left(C_t^1\right) + (1-p)\delta u\left(C_{t+1}^{2N}\right) + p\delta u\left(C_{t+1}^{2A}\right)\right\}, \tag{1.2}$$

subject to

$$C_t^1 = w_t - \tau x_t - s_t\ ,$$
$$C_{t+1}^{2N} = R_{t+1}\, s_t\ ,$$

and

$$C_{t+1}^{2A} = R_{t+1}\left(s_t - \pi\tau(w_t - x_t)\right).$$

For tractability we will assume that the expected utility representation u is logarithmic, i.e., $u(C) = \ln C$. The analysis can be generalized to an isoelastic utility, $u(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$, $\sigma > 0$. However, the cost of such a generalization will be to have a saving function which is not independent of the interest rate. This will complicate the analysis without providing new insights. Moreover, the relationship between saving and the interest rate is an unsolved empirical question and to build a model abstracting from such a relationship is thus a reasonable, defensive position.[3] Clearly, our results will be qualitatively similar if we assume instead isoelastic utilities having a parameter σ sufficiently close to 1.

Lemma 1. *The solution to the individual's optimization program* (1.2) *is given by*

$$x_t = Xw_t \text{ , and } s_t = Sw_t \text{ ,}$$

where

$$X = \begin{cases} 0 & \text{if} \quad p\pi \le \frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p}, \\ \frac{(1-p)\,\tau(1+\delta p\pi)-(1-p\pi)\,(p\delta+\tau)}{p\tau(\pi-1)\,(1+\delta)} & \text{if} \quad \frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p} < p\pi < 1, \\ 1 & \text{if} \quad p\pi \ge 1, \end{cases} \tag{1.3}$$

and

$$S = \begin{cases} S_1 & \text{if} \quad p\pi \le \frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p}, \\ \frac{\pi\delta(1-p)\,(1-\tau)}{(\pi-1)\,(1+\delta)} & \text{if} \quad \frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p} < p\pi < 1, \\ \frac{\delta(1-\tau)}{(1+\delta)} & \text{if} \quad p\pi \ge 1, \end{cases} \tag{1.4}$$

with

$$S_1 = \frac{(\delta+(1+\delta(1-p))\,\pi\tau)+\sqrt{(\delta+(1+\delta(1-p))\,\pi\tau)^2-4\delta\pi\tau(1-p)\,(1+\delta)}}{2(1+\delta)}. \tag{1.5}$$

Proff. See the appendix.

Notice that intended saving before tax inspection s_t and reported income x_t are both linear in actual wages. Note also that the conditions for obtaining an interior solution for the declared income, $x_t \in (0, w_t)$, are

$$\frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p} < p\pi < 1. \tag{1.6}$$

3 In fact, the Von Neumann-Morgenstern utlity function (1.1) is time-additive, homothetic, and exhibits a saving function which is independent of the interest rate if and only if u is logarithmic.

The first inequality is achieved by setting the penalty rate at a sufficiently high level for a given value of p. Finally, it should be pointed out that (1.6) is satisfied by a plausible parameter configuration like

$$\tau = 0.25\ ,\ \ \pi = 3\ ,\ \ p = 0.05\ ,\ \ \delta = 0.75\ . \qquad (1.7)$$

For a given value of $p \in (0,1)$, it is clear from (1.6) that the set of values of the penalty rate π for which $x_t \in (0, w_t)$ constitutes an open interval $(\underline{\pi}, \overline{\pi})$ with

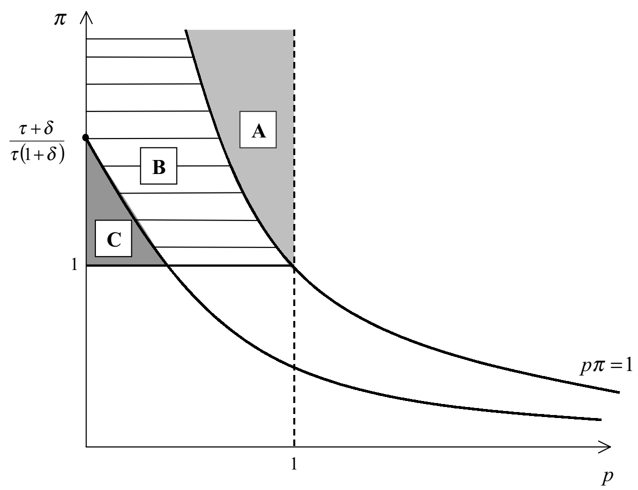
$$1 < \underline{\pi} = \frac{(\tau + \delta)}{\tau(1+\delta) + \delta(1-\tau)p} < \frac{1}{p}$$

and $\overline{\pi} = \frac{1}{p}$. Moreover, for a given value of the penalty rate $\pi > 1$, the set of values of the audit probability p for which $x_t \in (0, w_t)$ constitutes also an open interval $(\underline{p}, \overline{p})$ with $\overline{p} = \frac{1}{\pi}$. It can be proved that the infimum $\underline{p}$ of such an interval is equal to zero when the mild condition $(\pi - 1)\,\tau + \delta(\pi\tau - 1) \geq 0$ is imposed and $0 < \underline{p} < \frac{1}{\pi}$ whenever $(\pi - 1)\tau + \delta(\pi\tau - 1) < 0$.

Figure 1.1 shows the regions of the parameters p and $p\pi$ (the expected penalty rate) for which we obtain either interior or boundary solutions for the declared income x_t. In the interior of region B, the optimal solution satisfies $x_t \in (0, w_t)$ whereas $x_t = w_t$ in region A and $x_t = 0$ in region C. The function of p defining the frontier between regions B and C is given by the first expression in (1.6). Note that such an expression is an increasing and concave function of p which is equal to zero whenever $p = 0$ and is equal to one at $p = 1$.

FIGURE 1.1

INTERIOR AND BOUNDARY SOLUTIONS FOR THE TAXPAYER PROBLEM



The following partial derivatives concerning the behavior of both the propensity to declare X and the propensity to save S for an interior solution are obtained from (1.3) and (1.4):

$$\frac{\partial X}{\partial p} = \frac{\pi\delta(1-\tau)}{\tau(\pi-1)(1+\delta)} > 0\ ,$$

$$\frac{\partial X}{\partial \pi} = \frac{\delta(1-p)(1-\tau)}{\tau(\pi-1)^2\ (1+\delta)} > 0\ ,$$

$$\frac{\partial S}{\partial p} = \frac{-\delta\pi(1-\tau)}{(\pi-1)(1+\delta)} < 0\ ,$$

$$\frac{\partial S}{\partial \pi} = \frac{-\delta(1-p)(1-\tau)}{(\pi-1)^2\ (1+\delta)} < 0\ .$$

As expected, reported income is increasing in both the probability of investigation and the penalty rate π. Since individuals increase the income reported with p and π, this translates into a decrease of intended saving.

The effects of marginal changes in the policy parameters on the propensity to save S when $p\pi < \frac{(\tau+\delta)p}{\tau(1+\delta)+\delta(1-\tau)p}$ can be obtained directly from implicitly differentiating the fist order condition of problem (1.2). Since $X = 0$ in such a parameter region, the first order condition with respect to s_t is

$$-u'(w_t - s_t) + (1-p)R_{t+1}\,u'(R_{t+1}\,s_t) + pR_{t+1}\,u'(R_{t+1}(s_t - \pi\tau w_t)) = 0 \qquad (1.8)$$

Implicitly differentiating (1.8) we get

$$\frac{\partial s_t}{\partial \pi} = \frac{p(R_{t+1})^2\ \tau w_t u''(R_{t+1}(s_t - \pi\tau w_t))}{u''(w_t - s_t) + (1-p)(R_{t+1})^2\,u''(R_{t+1}\,s_t) + p(R_{t+1})^2\,u''(R_{t+1}(s_t - \pi\tau w_t))} > 0\ ,$$

and

$$\frac{\partial s_t}{\partial p} = \frac{R_{t+1}[u'(R_{t+1}\,s_t) + u'(R_{t+1}(s_t - \pi\tau w_t))]}{u''(w_t - s_t) + (1-p)(R_{t+1})^2\ u''(R_{t+1}\,s_t) + p(R_{t+1})^2\ u''(R_{t+1}(s_t - \pi\tau w_t))} > 0\ ,$$

where the latter inequality comes from the fact that $u'(R_{t+1}(s_t - \pi\tau w_t)) > u'(R_{t+1}\,s_t)$. Therefore, we can conclude that $\frac{\partial S}{\partial \pi} > 0$ and $\frac{\partial S}{\partial p} > 0$ in the interior of region C of Figure 1.1.

On the other hand, it is obvious from (1.4) that marginal changes in the tax compliance policy have no effects on the propensity to save S when $p\pi > 1$, i.e., when truthful revelation of income is already achieved.

1.3. EQUILIBRIUM

There are competitive firms in the economy that produce a single good according to the following Cobb-Douglas technology:

$$Y_t = BK_t^{\alpha} \hat{L}_t^{1-\alpha}, \quad \text{with} \quad B > 0, \quad \alpha \in (0,1), \tag{1.9}$$

where Y_t is the output, K_t is the private capital used by each firm and $\hat{L}_t$ denotes the efficiency units of labor hired by each firm. Note that K_t might be interpreted as a composite capital embodying both physical and human capital. Efficiency units of labor are proportional to both the number L_t of physical units of labor and the level g_t of capital supplied by the government (public capital) per worker, i.e.,

$$\hat{L}_t = D\, L_t\, g_t, \quad \text{with} \quad D > 0,$$

Therefore, we are assuming that public capital increases proportionally the productivity of each worker as in Barro (1990). The services provided by public capital are assumed completely rival for the users so that is the amount of public capital per capita and not the total amount that enters in the production function. Moreover, we assume that there are neither user charges nor congestion effects associated with public services. Examples of such public services include public education, transportation systems, maintenance of law and order, etc.[4] Public capital is thus a productive externality from the firms viewpoint. Hence, the production function (1.9) can be written as

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha} g_t^{1-\alpha},$$

where $A = BD^{1-\alpha}$. We assume that both private and public capital fully depreciate after one period.

Taking g_t as given, the optimal demands for private capital and workers by firms must satisfy the first order conditions for profit maximization

$$w_t = A(1-\alpha)K_t^{\alpha} L_t^{-\alpha} g_t^{1-\alpha}, \tag{1.10}$$

and

$$R_t = A\alpha K_t^{\alpha-1} L_t^{1-\alpha} g_t^{1-\alpha}. \tag{1.11}$$

Given the constant returns to scale assumption, competitive firms will make zero profits and its number remains thus indeterminate. We normalize the number of firms to one per worker. Hence, equilibrium in the labor market implies that $L_t = 1$ for all t. Therefore, (1.10) and (1.11) become in equilibrium

$$w_t = A(1-\alpha)K_t^{\alpha} g_t^{1-\alpha}, \tag{1.12}$$

4 Barro and Sala-i-Martín (1992) discuss the growth implications of alternative assumptions on the nature of publicly provided services.

and

$$R_t = A\alpha K_t^{\alpha-1} g_t^{1-\alpha}. \tag{1.13}$$

On the other hand, equilibrium in the capital market implies that effective saving must be equal to the private capital installed in the next period,

$$K_{t+1} = (1-p)\, s_t + p\big(s_t - \pi\tau(w_t - x_t)\big). \tag{1.14}$$

Since in this large economy a fraction p of individuals is subjected to tax investigation, the first term on the RHS of (1.14) is the effective saving of the non audited population whereas the second term is the effective saving of the audited population. Substituting s_t and x_t by their optimal values, (1.14) becomes

$$K_{t+1} = Mw_t\ , \tag{1.15}$$

where

$$M = S - p\pi\tau(1-X). \tag{1.16}$$

Note that $M > 0$ since effective saving after inspection is strictly positive. Using Lemma 1, we get the following explicit expression for M:

$$M = \begin{cases} S_1 - p\pi\tau & \text{if} \quad p\pi \le \dfrac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p}\ , \\ \dfrac{\pi\delta(1-\tau)\left(1-2p+p^2\pi\right)}{(\pi-1)(1+\delta)} & \text{if} \quad \dfrac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p} < p\pi < 1\ , \\ \dfrac{\delta(1-\tau)}{1+\delta} & \text{if} \quad p\pi \ge 1 \end{cases} \tag{1.17}$$

where S_1 is given in (1.5).

The government finances the stock of public capital by means of both the proportional taxes on declared income and the penalty fees collected from the audited taxpayers in the previous period. We assume that the government faces a proportional inspection cost c per unit of audited income. Therefore, the budget constraint of the government is

$$g_{t+1} = (1-p)\,\tau x_t + p\big(\tau x_t + \pi\tau(w_t - x_t)\big) - cpw_t\ , \tag{1.18}$$

where the first term on the RHS of (1.18) are the taxes paid by the non audited taxpayers, the second term are the taxes plus the penalty fees paid by audited taxpayers, and the last term is the cost associated with tax inspection. Substituting the equilibrium value of x_t we get

$$g_{t+1} = Gw_t\ , \tag{1.19}$$

where

$$G = (1-p\pi)\tau X + p\pi\tau - cp \, . \tag{1.20}$$

We can use again Lemma 1 to get an explicit expression for G in terms of the exogenous parameters,

$$G = \begin{cases} p(\pi\tau - c) & \text{if} \quad p\pi \le \dfrac{(\tau+\delta)p}{\tau(1+\delta)+\delta(1-\tau)p}, \\ \dfrac{(1-p\pi)(\tau(\pi-1)+\delta\pi\tau(1-p)+\delta(1-p\pi))}{(\pi-1)(1+\delta)} + p\pi\tau - cp & \text{if} \quad \dfrac{(\tau+\delta)p}{\tau(1+\delta)+\delta(1-\tau)p} < p\pi < 1, \\ \tau - cp & \text{if} \quad p\pi \ge 1. \end{cases} \tag{1.21}$$

We will assume that $\tau > c$ since this assumption, together with the fact that $X \in [0,1]$, ensures that G is strictly positive. That is, if the unitary cost of inspection c is lower than the tax rate, the tax system generates resources for positive public spending. Plugging (1.12) into (1.19) we obtain

$$g_{t+1} = GA(1-\alpha)K_t^{\alpha}\, g_t^{1-\alpha} \, ,$$

which can be rewritten as

$$\frac{g_{t+1}}{g_t} = GA(1-\alpha)\left(\frac{K_t}{g_t}\right)^{\alpha}. \tag{1.22}$$

Furthermore, from dividing (1.15) by (1.19), we get

$$\frac{K_t}{g_t} = \frac{M}{G}, \tag{1.23}$$

and thus (1.22) becomes

$$\gamma \equiv \frac{g_{t+1}}{g_t} = GA(1-\alpha)\left(\frac{M}{G}\right)^{\alpha} = A(1-\alpha)M^{\alpha}\, G^{1-\alpha} \, . \tag{1.24}$$

Therefore, the growth rate γ of public spending is constant for all t along an equilibrium path. Hence, combining (1.12) and (1.23) we get

$$w_t = A(1-\alpha)\left(\frac{M}{G}\right)^{\alpha} g_t \, , \tag{1.25}$$

and, thus, wages also grow at the same rate γ. Since the reported income x_t and the intended savings s_t are proportional to wages, and the same occurs with the several consumptions, as dictated by the constraints of problem (1.2), all these variables also grow at the rate γ. Finally, from (1.13) and (1.23), the equilibrium interest rate is constant and equal to

$$R_t = A\alpha\left(\frac{G}{M}\right)^{1-\alpha}. \tag{1.26}$$

Note that this economy is always in a balanced growth path and thus displays no transition. This should not be surprising since the constant returns to scale assumption, together with the fact that public spending is proportional to installed capital [see (1.23)], implies that the model becomes of the Ak type. Recall that the infinite horizon versions of the Ak models of Barro (1990) and Rebelo (1991) did not display transition either.

1.4. GROWTH EFFECTS OF THE TAX COMPLIANCE POLICY

The effects of changes in the tax enforcement parameters on the rate γ of economic growth are exclusively determined by the induced changes in M and G as it can be seen from (1.24). The following partial derivatives for interior solutions can be obtained from (1.17) and (1.21) after some tedious algebra:

$$\frac{\partial M}{\partial \pi} = -\frac{\delta(1-p\pi)(1-\tau)(p(\pi-2)+1)}{(\pi-1)^2(1+\delta)} < 0\,, \tag{1.27}$$

$$\frac{\partial M}{\partial p} = \frac{-2\delta\pi(1-\tau)(1-p\pi)}{(\pi-1)(1+\delta)} < 0\,, \tag{1.28}$$

$$\frac{\partial G}{\partial \pi} = \frac{\delta(1-p\pi)(1-\tau)(p(\pi-2)+1)}{(\pi-1)^2(1+\delta)} > 0\,, \tag{1.29}$$

$$\frac{\partial G}{\partial p} = \frac{2\delta\pi(1-\tau)(1-p\pi)-c(1+\delta)(\pi-1)}{(\pi-1)(1+\delta)}. \tag{1.30}$$

The sign of the last partial derivative is ambiguous. However, $\frac{\partial G}{\partial p} > 0$ if and only if

$$2\delta\pi(1-\tau)(1-p\pi) > c(1+\delta)(\pi-1). \tag{1.31}$$

This condition is satisfied whenever the unitary cost of inspection is sufficiently low for given values of p and π. For instance, the parameter configuration in (1.7) exhibits a positive derivative of G with respect to p if and only if $c < 0.819$. This is a mild restriction since a reasonable calibration of the model would place the value of c not above of 0.03. Under condition (1.31), the qualitative effects of p and π on M are the opposite of those on G. Therefore, in such a case, the private to public capital ratio $\frac{K_t}{g_t}$ is decreasing in both the probability of inspection p and the penalty rate π, as follows from (1.23). Moreover, from (1.26), we see that the equilibrium interest rate is then increasing in both parameters of the tax compliance policy.

At the interior of the parameter region for which $X = 0$, the effects of changes in p and π on M are ambiguous since the derivatives $\frac{\partial M}{\partial p}$ and $\frac{\partial M}{\partial \pi}$ might be either positive or negative depending on a quite complicated and non-intuitive relation involving p, π, δ and τ. Inspection of (1.21) in such a region reveals clearly that G is locally increasing in both the penalty rate π and the audit probability p.

Finally, if $p\pi > 1$, which means that there is complete tax compliance, then $\frac{\partial M}{\partial \pi} = \frac{\partial M}{\partial p} = \frac{\partial G}{\partial \pi} = 0$ and $\frac{\partial G}{\partial p} < 0$.

An interesting, unambiguous comparative statics result refers to the effects of the parameter c on the rate of growth. Since G is decreasing in the inspection cost c [see (1.20)], and such a cost does not affect M, improvements in the inspection technology directly translate into higher growth rates.

On the other hand, the tax compliance policy might have ambiguous effects on the rate of growth depending on the technological parameter α, as it can be seen by logarithmically differentiating (1.24),

$$\frac{\partial(\ln\gamma)}{\partial p} = \frac{\alpha}{M}\frac{\partial M}{\partial p} + \frac{(1-\alpha)}{G}\frac{\partial G}{\partial p}, \tag{1.32}$$

$$\frac{\partial(\ln\gamma)}{\partial \pi} = \frac{\alpha}{M}\frac{\partial M}{\partial \pi} + \frac{(1-\alpha)}{G}\frac{\partial G}{\partial \pi}. \tag{1.33}$$

Therefore, as an immediate consequence of the previous discussion and from inspection of (1.17), (1.21), (1.27), (1.29) and (1.33), we can state the following proposition referred to marginal changes in the inspection cost c, and the penalty rate π on evaded taxes:

Proposition 2. *(a) The rate of growth γ is decreasing in the unitary inspection cost c.*

(b) The rate of growth γ is not affected by marginal changes in the penalty rate π when $p\pi > 1$.

(c) Consider a tax compliance policy pair (p, π) *such that there is under reporting of income, i.e.,* $p\pi < 1$*. If* α *is sufficiently close to zero, then the rate of growth* γ *is locally increasing in the penalty rate* π*.*

(d) Consider a tax compliance policy pair (p, π) *such that* $X \in (0,1)$*. If* α *is sufficiently close to one, then the rate of growth* γ *is locally decreasing in the penalty rate* π*.*

Clearly, the parameter α measures the importance of private capital in the production process. If α is close to one, then the contribution of government spending to aggregate output is small so that, at an interior solution, a decrease in the penalty rate will reduce the resources devoted to government spending while it will increase private capital accumulation (M will increase). The latter effect will dominate the reduction in public resources due to the decrease in G. The converse argument applies when α is close to zero.

We should point out that the nature of the two instruments available to the tax authority is quite different. Usually, the tax legislation establishes both the tax rate τ and the penalty rate π on evaded income while the probability p of inspection depends on the effort made by the tax collection agency. Such an effort is not verifiable and, therefore, an specific value of p cannot be enforced by law. Moreover, the probability p of inspection can be almost instantaneously adjusted by the tax authority whereas the modification of the penalty rate should undergo a rather lengthy parliamentary process. Therefore, let us assume now that the penalty rate is fixed at a finite level, and consider a tax authority trying to maximize the rate of economic growth for a given tax rate. The following proposition establishes the non-desirability from the growth viewpoint of auditing policies inducing taxpayers to be honest.

Proposition 3. *For every given finite value of* π*, the rate* γ *of economic growth is never maximized by selecting an audit probability* p *which induces taxpayers to declare their true labor income.*

Proof. See the appendix.

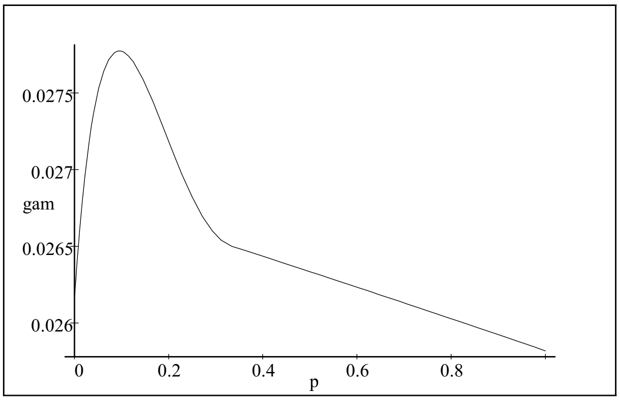
The intuition behind the previous proposition is quite obvious. First, note that condition (1.31) does not hold when the expected penalty rate $p\pi$ takes a value around 1 and, according to (1.30), G is locally decreasing in p in such a case. Then, given a finite penalty rate and an initial position of complete tax compliance, the tax collection agency may increase G (the government revenues) by reducing, say, the number of tax inspectors whereas the induced reduction in M (the private capital accumulation) is almost zero around $p = \frac{1}{\pi}$ [see (1.28)].

As an illustration of Proposition 3, the two panels of Figure 1.2 show examples of growth rates as functions of the probability p on the domain $(0,1)$. In the case considered in Panel 1.2A, we obtain a kind of Laffer curve, and growth is maximized when the audit probability is $p = 0.095 \in (\underline{p}, \overline{p})$. However, Panel 1.2B shows a situation in which public spending is so unproductive $(\alpha = 0.9)$ that the growth rate is strictly decreasing in p on the whole domain. Note that the two plotted functions are strictly decreasing in $\left(\frac{1}{\pi} - \varepsilon, 1\right)$ for some $\varepsilon > 0$.

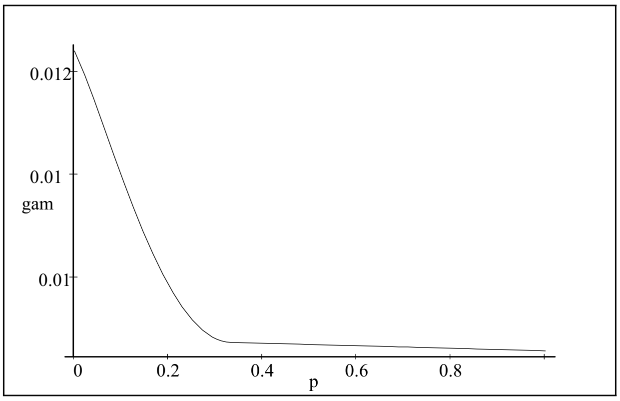
FIGURA 1.2

THE RELATION BETWEEN γ AND p

$\delta = 0.75$, $\pi = 3$, $\tau = 0.25$, $c = 0.03$, $A = 0.3\left(\underline{p} = 0\ ,\ \overline{p} = 0.33\right)$



Panel 1.2A: $\alpha = 0.7$



Panel 1.2B: $\alpha = 0.9$

The argument of Proposition 3 would not apply if we fixed the probability of inspection and modified instead the penalty rate. In such a case, the growth rate might be monotonically increasing, monotonically decreasing or non monotonic in the penalty rate π, as the different panels of Figure 1.3 show. In particular, for the case analyzed in Panel 1.3A, which corresponds to an economy with highly productive public capital ($\alpha = 0.2$), to induce complete tax compliance through high penalties is a growth maximizing policy. However, recall that it is impossible to obtain an uniformly increasing function when the variable p is in the horizontal axis, as follows from Proposition 3. The Panels 1.3B and 1.3C correspond to situations in which the growth maximizing penalty rates are $\underline{\pi}$ and 1, respectively, so that declared income is zero in both cases.[5] Panel 1.3D corresponds to a situation in which maximum growth is achieved at an interior solution of the taxpayer optimization problem since the growth maximizing penalty rate is $\pi = 2.77 \in \left(\underline{\pi}, \overline{\pi}\right)$. Finally, notice that the rate of growth remains always unaf-

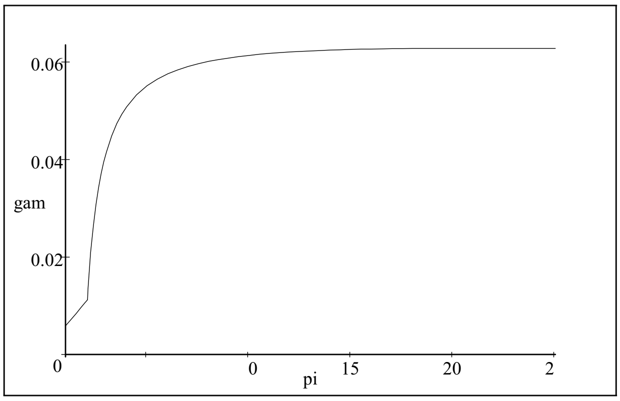
5 Panel 3C is obtained under an unrealistically high audit probability (p = 0.9).

fected by changes in the penalty rate whenever $\pi \geq \frac{1}{p}$ since then truthful revelation of income is already achieved and no additional revenues accrue from tax inspections.

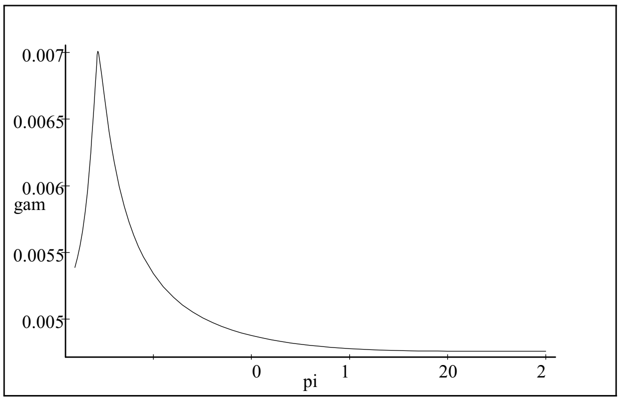
FIGURA 1.3

THE RELATION BETWEEN γ AND p

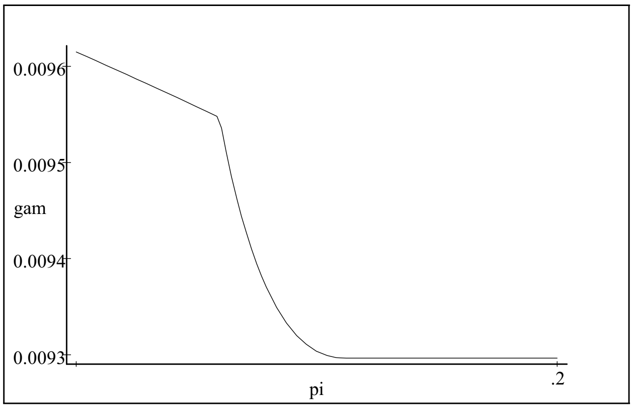
$\delta = 0.75$, $\tau = 0.25$, $c = 0.03$, $A = 0.3$



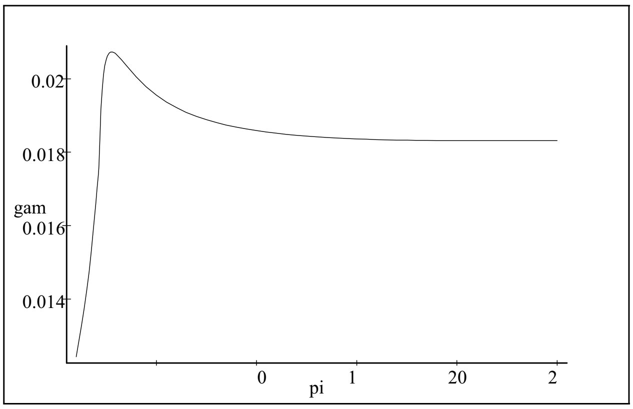
Panel 1.3A: $\alpha = 0.2$, $p = 0.05$ ($\underline{\pi} = 2.14$ and $\overline{\pi} = 20$)



Panel 1.3B: $\alpha = 0.95$, $p = 0.05$ ($\underline{\pi} = 2.14$ and $\overline{\pi} = 20$)



Panel 1.3C: $\alpha = 0.9$, $p = 0.9$ ($\underline{\pi} = 1.06$ and $\overline{\pi} = 1.11$)



Panel 1.3D: $\alpha = 0.8$, $p = 0.05$ ($\underline{\pi} = 2.14$ and $\overline{\pi} = 20$)

To end this discussion about the effects of changes in a single parameter of the tax enforcement policy, we state a couple of additional comparative statics results concerning the audit probability p. Both results are provided for the sake of completeness and they follow directly from performing straightforward derivatives.

Proposition 4. *(a) The rate of growth* γ *is locally decreasing in the audit probability* p *when* $p\pi > 1$.

(b) Consider a tax compliance policy pair (p, π) *such that* $X = 0$*. If* α *is sufficiently close to zero, then the rate of growth* γ *is locally increasing in the audit probability* p*.*

It should be pointed that the comparative statics analysis of changes in p is even less clear than that of π. This is so because the sign of the relation between p and G when $X \in (0,1)$ is ambiguous depending on whether condition (1.31) holds or not.

Our previous discussion is dramatically modified if the tax authority can control simultaneously both instruments of the tax enforcement policy. Observe that agents report their true wages when $p\pi \geq 1$. Then, the rate of growth under such a policy is given by

$$\gamma = A(1-\alpha)\left(\frac{\delta(1-\tau)}{1+\delta}\right)^{\alpha}(\tau - cp)^{1-\alpha} ,$$

as follows from (1.17), (1.21) and (1.24). Such a growth rate is strictly decreasing in p and it is clear that a growth rate arbitrarily close to

$$\gamma^{*} = A(1-\alpha)\left(\frac{\delta(1-\tau)}{1+\delta}\right)^{\alpha}\tau^{1-\alpha} , \quad (1.34)$$

can be implemented by means of a complete tax compliance policy displaying a probability p of inspection arbitrarily low and a penalty rate π arbitrarily high with $p\pi \geq 1$ (see Figure 1.1). Such a policy consisting on "hanging evaders with probability zero" has received attention in the theoretical literature on tax evasion when the government seeks to maximize its revenues [see, among many others, Kolm (1973)].[6] It is easy to check algebraically from (1.17) and (1.21) that, if $X \in (0,1)$, then $M > \frac{\delta(1-\tau)}{1+\delta}$ and $G < \tau$. Hence, the desirability from the growth viewpoint of complete tax compliance will depend on whether the technological parameter α is high or low, as it can bee seen from comparing (1.24) with (1.34). In particular, to induce honest behavior by the taxpayers is desirable whenever public capital is very productive (i.e., when α is sufficiently close to zero). The following proposition summarizes more precisely the results:

Proposition 5. *(a) Consider the set of tax compliance policies inducing true reports of labor income, that is, policies satisfying* $p\pi \geq 1$*. Then, the supremum of the set of rates of growth associated with such policies is* γ^**. Moreover, for all* $\varepsilon > 0$*, there exists a policy pair* $(p(\varepsilon), \pi(\varepsilon))$*, with* $p(\varepsilon)\cdot\pi(\varepsilon) = 1$*, such that the rates of growth associated with the policies* $(p(\varepsilon), \pi)$*, with* $\pi \geq \pi(\varepsilon)$*, are equal to* $\gamma^* - \varepsilon$*. Furthermore, the function* $p(\varepsilon)$ *is strictly increasing while* $\pi(\varepsilon)$ *is strictly decreasing, and* $\lim_{\varepsilon\to 0} p(\varepsilon) = 0$ *while* $\lim_{\varepsilon\to 0} \pi(\varepsilon) = \infty$*.*

(b) If α *is sufficiently close to zero, then there exists a policy pair* (p, π) *inducing complete tax compliance which displays faster economic growth than any other policy inducing tax evasion. Conversely, if* α *is sufficiently close to one, then there exists a policy pair* (p, π) *inducing tax evasion which displays faster economic growth than any other policy inducing complete tax compliance.*

Let us point out that Proposition 3 does not contradict the first sentence in part (b) of Proposition 5. In the former we were keeping fixed the penalty rate at a finite level whereas in the latter both p and π were moving simultaneously in opposite directions with π approaching infinity. Obviously, the growth rate given in (1.34) is never achieved by a complete compliance policy but just arbitrarily approximated.

6 Moreover, Friedland, Maital and Rutenberg (1978) have documented the effectiveness of such an extreme policy in their experimental work.

1.5. GROWTH AND WELFARE: AN EXAMPLE

It should be noticed that so far we have conducted just a positive analysis of the growth effects of the tax enforcement parameters. The normative analysis in a OLG model like ours will depend on the objective function of the social planner and, in particular, on the weights assigned to each generation in her objective function.

To see how the preferences of the social planner might conflict with the objective of maximizing economic growth, we consider the following example. We follow Samuelson (1967, 1968) and assume that the objective of a time-consistent social planner is to maximize

$$V_s = (1-\beta)\sum_{t=s}^{\infty}\beta^{t-s}\left[u\left(C_t^1\right)+\delta E\left(u\left(\tilde{C}_{t+1}^2\right)\right)\right], \text{ for all s },$$

where $\beta \in (0,1)$. That is, the planner discounts the utility of future generations at a constant rate and all the members of the same generation will be equally treated.

For the illustrative purposes of this example, let us assume that the values of p and π that the tax authority can select lie in the interior of region B of Figure 1.1. From the constraints of problem (1.2) we obtain the following optimal interior consumptions:

$$C_t^1 = \left(\frac{1-\tau}{1+\delta}\right)w_t \ ,$$

$$C_{t+1}^{2A} = \left(\frac{\delta p\pi(1-\tau)}{1+\delta}\right)w_t \ R_{t+1} \ ,$$

$$C_{t+1}^{2N} = \left(\frac{\delta\pi(1-p)(1-\tau)}{(\pi-1)(1+\delta)}\right)w_t \ R_{t+1} \ .$$

Using (1.25) and (1.26), and since $g_t = g_s \ \gamma^{t-s}$, where g_s is the government spending at time s, we obtain

$$w_t = A(1-\alpha)\left(\frac{M}{G}\right)^{\alpha} g_s \ \gamma^{t-s} \ ,$$

and

$$w_t \ R_{t+1} = A^2\,(1-\alpha)\alpha\left(\frac{M}{G}\right)^{2\alpha-1} g_s \ \gamma^{t-s} \ .$$

Therefore, the equilibrium consumptions are $C_t^1 = \hat{C}^1\gamma^{t-s}$, $C_{t+1}^{2A} = \hat{C}^{2A}\gamma^{t-s}$, and $C_{t+1}^{2N} = \hat{C}^{2N}\gamma^{t-s}$, where

$$\hat{C}^1 = \left(\frac{1-\tau}{1+\delta}\right) A(1-\alpha)\left(\frac{M}{G}\right)^{\alpha} g_s \ , \tag{1.35}$$

$$\hat{C}^{2A} = \left(\frac{\delta p\pi(1-\tau)}{1+\delta}\right) A^2 (1-\alpha)\alpha\left(\frac{M}{G}\right)^{2\alpha-1} g_s \ ,$$

and

$$\hat{C}^{2N} = \left(\frac{\pi\delta(1-p)(1-\tau)}{(\pi-1)(1+\delta)}\right) A^2 (1-\alpha)\alpha\left(\frac{M}{G}\right)^{2\alpha-1} g_s \ .$$

Then, the objective of the social planner becomes

$$V_s = (1-\beta)\sum_{t=s}^{\infty} \beta^{t-s}\left[\ln\left(\hat{C}^1\gamma^{t-s}\right) + (1-p)\delta\ln\left(\hat{C}^{2N}\gamma^{t-s}\right) + p\delta\ln\left(\hat{C}^{2A}\gamma^{t-s}\right)\right] =$$
$$= (1-\beta)\sum_{t=s}^{\infty} \beta^{t-s}\left[\ln\hat{C}^1 + p\delta\ln\hat{C}^{2A} + (1-p)\delta\ln\hat{C}^{2N} + (1+\delta)\ln\gamma^{t-s}\right] =$$
$$(1-\beta)\sum_{t=s}^{\infty} \beta^{t-s}\left[\ln\hat{C}^1 + p\delta\ln\left(\frac{\hat{C}^{2A}}{\hat{C}^{2N}}\right) + \delta\ln\hat{C}^{2N} + (1+\delta)\ln\gamma^{t-s}\right],$$

and since

$$\frac{\hat{C}^{2A}}{\hat{C}^{2N}} = \frac{p(\pi-1)}{1-p},$$

and

$$\sum_{t=s}^{\infty} \beta^{t-s}\ln\gamma^{t-s} = \frac{\beta}{(1-\beta)^2}\ln\gamma \ ,$$

the objective function of the planner is simply

$$V_s = \ln\hat{C}^1 + \delta\ln\hat{C}^{2N} + p\delta\ln\left(\frac{p(\pi-1)}{1-p}\right) + \frac{(1+\delta)\beta}{1-\beta}\ln\gamma \ .$$

With such an objective function it is quite easy to construct examples for which the planner might prefer a policy generating a path with slow capital accumulation. For instance, assume that the taxpayers and the planner discount heavily the future, that is, both δ and β are close to zero. In such a case the planner would tend to select a high value for $\hat{C}^1$. Note that the ratio $\frac{M}{G}$ is decreasing in π so that, regardless of the value of α, the planner will choose low values of the penalty rate as dictated by (1.27), (1.29) and (1.35). However, this policy with low penalties may cause low rates of growth if α is close to zero. Indeed, if α is low, fast growth is attained when the government sets severe penalties on tax evaders as follows from part (c) of Proposition 2 (see also the example considered in Panel 1.3A).

1.6. EXTENSIONS

The simple model we have just considered has allowed the analysis of the growth implications of different tax compliance policies. Due to its simplicity, many extensions are possible. We just mention three of them.

The first one refers to the explicit recognition of involuntary mistakes in the process of filling the tax form, as in Rubinstein (1979). In this case, the penalty fee on detected tax evaders should be set at a moderate level since both the inefficiency and the inequality generated by severe penalties applied infrequently could be politically unbearable. The analysis could give rise then to an endogenous penalty rate, and it will thus provide further support for the non optimality of complete tax compliance in the spirit of our Proposition 3. The relevance of such a proposition relies indeed on the fact that legislators do not set very severe penalties on tax evaders since they are perhaps aware that many taxpayers commit (unverifiable) mistakes by accident when they fill their tax forms. Therefore, fines cannot tend to infinity, which conforms with the assumption of Proposition 3. However, this extension would require agents working for more than one period since the repeated interaction between taxpayers and the tax collecting agency would be now a key element of the model.

The second extension would be to consider inspection policies for which the probability of an audit depends on the income declared, as in Reinganum and Wilde (1985). Those authors show that net fiscal revenues increase with such policies. The growth implications of such inspection policies remain thus unexplored.

Finally, in our model growth is achieved by means of the accumulation of both private and public capital. However, there are other ways in which sustained growth can be achieved, like for instance through human capital accumulation [see, among many others, Caballé and Santos (1993)]. The analysis of changes in the tax compliance policy on such richer models could provide insights on both the short-run and the long-run effects. This is so because those models typically display some transitional dynamics while such a dynamics is absent in the model considered in the present chapter.

1.7. APPENDIX

Proof of Lemma 1. To obtain the solution to the individual's problem (1.2) we must first assume that the solution is interior, that is, $x_t \in (0, w_t)$. In this case, the first order conditions with respect to x_t and s_t yield, after some tedious algebra, the following solution:

$$x_t = \left(\frac{(1-p)\,\tau(1+\delta p\pi) - (1-p\pi)\,(p\delta+\tau)}{p\tau(\pi-1)\,(1+\delta)} \right) w_t \,, \tag{1.36}$$

and,

$$s_t = \left(\frac{\pi\delta(1-p)\,(1-\tau)}{(\pi-1)\,(1+\delta)} \right) w_t \,.$$

It can be checked from (1.36) that such a conjectured solution satisfies in fact $x_t > 0$ if and only if

$$p\pi > \frac{(\tau+\delta)\,p}{\tau(1+\delta)+\delta(1-\tau)\,p}. \tag{1.37}$$

On the other hand, it can be seen from manipulating (1.36) that a necessary and sufficient condition for $x_t < w_t$ is

$$p\pi < 1. \tag{1.38}$$

Since $x_t \in [0, w_t]$ as a consequence of the aforementioned tax law, we have that $x_t = 0$ when (1.36) is nonpositive, which means that the agent will not fill the tax form in such a circumstance. Hence, to obtain the solution for problem (1.2) when condition (1.37) does not hold, we impose $x_t = 0$ and solve the maximization problem for s_t. The corresponding optimal propensity to save is then given in (1.5). Furthermore, $x_t = w_t$ when $p\pi \geq 1$ so that, in such a case, we solve the maximization problem (1.2) for s_t after imposing $x_t = w_t$ in its constraints.

Proof of Proposition 3. Observe that the growth rate γ is obviously a continuous function of the inspection probability p for all $p \in (0, 1)$. Hence, we only have to prove that there exist a number $\varepsilon \in \left(0, \frac{1}{\pi} - \underline{p} \right)$ such that the rate of growth γ is strictly decreasing in p on the interval $\left(\frac{1}{\pi} - \varepsilon, 1 \right)$. We will first prove that the derivative $\frac{\partial \gamma}{\partial p}$ is strictly negative for $p \in \left(\frac{1}{\pi}, 1 \right)$. Notice that $X = 1$ when $p \in \left(\frac{1}{\pi}, 1 \right)$. Thus, from (1.17) and (1.21), it holds that $M = \frac{\delta(1-\tau)}{(1+\delta)}$, $G = \tau - cp$, and

$$\gamma = A(1-\alpha) \left(\frac{\delta(1-\tau)}{1+\delta} \right)^{\alpha} (\tau - cp)^{1-\alpha}, \tag{1.39}$$

as follows from evaluating (1.24) in such a parameter region. Clearly, (1.39) is strictly decreasing in p. Next, since γ has continuous derivatives with respect to p on $\left(\underline{p}, \frac{1}{\pi} \right)$, we must compute the left derivative of γ with respect to p at $\frac{1}{\pi}$. In order to compute $\left.\frac{\partial \gamma}{\partial p}\right|_{p \to \left(\frac{1}{\pi}\right)^-}$ we only have to evaluate the derivative of (1.24) at an interior solution and take the limit as $p \to \frac{1}{\pi}$. From (1.28) and (1.30) we obtain $\left.\frac{\partial M}{\partial p}\right|_{p \to \left(\frac{1}{\pi}\right)^-} = 0$ and $\left.\frac{\partial G}{\partial p}\right|_{p \to \left(\frac{1}{\pi}\right)^-} = -c < 0$. Therefore, from (1.32), we get $\left.\frac{\partial (\ln \gamma)}{\partial p}\right|_{p \to \left(\frac{1}{\pi}\right)^-} < 0$. We have thus proved that for some $\varepsilon > 0$ the rate of growth is strictly decreasing in the interval $\left(\frac{1}{\pi} - \varepsilon, 1\right)$ and, hence, a policy inducing complete tax compliance is not growth maximizing.

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CHAPTER 2

TAX EVASION AND RICARDIAN EQUIVALENCE*

2.1. INTRODUCTION

The aim of this chapter is to analyze when the Ricardian equivalence proposition holds in a framework where uncertainty arises as a consequence of tax evasion. The Ricardian equivalence proposition says that, if a change in current taxes is completely offset by a change in future taxes, the consumption path of individuals remains unchanged when the government spending path is not modified (Barro, 1974). There is a large body of literature analyzing whether this proposition holds under uncertainty. For example, Barsky et al. (1986) introduce individual uncertainty about future income and show how a tax cut coupled with a future income tax increase can stimulate consumer spending through the precautionary motive for saving. Feldstein (1988) considers a model with altruistic individuals where income uncertainty implies that the bequests are also uncertain and that there is a positive probability of leaving zero bequests. The resulting corner solution for equilibrium bequests causes a failure of Ricardian equivalence since individuals cannot offset the change in the calendar of payments to the government by means of intergenerational transfers. Finally, Strawczynski (1995) analyzes the sources of the Ricardian equivalence failure when either current or future income are uncertain and the non-negativity constraint on bequests is binding in some states of nature.

In all the aforementioned models, the nature of income uncertainty is exogenous. In this chapter we will present a model where uncertainty about future income arises because taxpayers can evade taxes and they may be audited by the tax authorities. Thus, whereas uncertainty is completely exogenous in the previous models, in our model the degree of uncertainty can be controlled at some extend by the taxpayers, since the amount of declared income determines the distribution of future income.

I consider a model where individuals live for two periods. The young individuals must decide both the amount of income they want to report to the tax authorities and the amount they want to save. There is uncertainty in the second period since, if an agent is inspected by the tax authorities, his saving will be reduced by the fine that he has to pay. In this context, we will investigate the effects both on savings and on declared income (and, as a by-product, on consumption) of a variation in the tax rate that leaves unchanged the level of government spending. This means that, if the tax rate is increased individuals will be compensated by a reduction in his future lump-sum taxes. Note then that our analysis is the typical Ricardian one: we study the effects of changing the financing policy for a given path of government spending. Of course, all the fiscal unbalances incurred by the government generate the corresponding change in the outstanding public debt.

We find that the Ricardian equivalence proposition fails to hold when the penalties on evaders are proportional to the amount of unreported income while the proposition holds when penalties are proportional to the amount of evaded taxes. When fees are set on evaded income, the result we obtain is similar to the one appearing in the aforementioned papers. However, the mechanism at work is now endogenous. In fact, we show that evasion increases with the tax rate and this generates more uncertainty in the second period income, since there is an increase in the gap between the disposable income of an audited individual and the disposable income of an individual who has not been inspected. Therefore, in this context the income tax is harmful, since it forces taxpayers to misreport their income, and consequently to increase their precautionary savings. Such precautionary savings are indeed costly, since young agents are forced to consume less that they would have chosen if proportional taxes and fees were absent. On the other hand, the Ricardian equivalence proposition holds under penalties that are proportional to the amount of evaded taxes, since now the increase in taxes results in an increase of declared income, which leaves unchanged the amount of evaded taxes. Therefore, the degree of uncertainty about second period income remains unchanged as does the amount of fines.

One of the most puzzling results in the tax evasion literature refers to the relationship between tax rates and declared income. Allingham and Sandmo (1972) showed that, under decreasing absolute risk aversion, the relation between declared income and tax rates is ambiguous when the fines are proportional to the unreported income, while declared income is decreasing in the tax rate under the less realistic assumption of non-decreasing absolute risk aversion. On the other hand, Yitzhaki (1974) assumed that the fine paid by an audited evader is proportional to the amount of evaded taxes and found that an increase in the tax rate increases declared income under decreasing absolute risk aversion. This modification of the Allingham and Sandmo model generates an unambiguous result that has not been supported by the empirical evidence since several studies have documented that higher tax rates tend to stimulate tax evasion.[1]

Many authors have searched for alternative models aimed at explaining this contradiction between the empirical findings and the theoretical predictions.[2] In this line of research, Yaniv (1994), in a reexamination of the model of Allingham and Sandmo, shows that it is possible to find a negative relation between declared income and tax rates when the individual's utility exhibits constant relative risk aversion and some other restrictions on the parameters are imposed. On the other hand, Yitzhaki (1987) shows that, when the inspection probability depends on evaded income, an increase in the tax rate might also result in an increase in evaded income.

I will show that it is possible to find the previous empirically plausible relation under much less stringent assumptions in a setup where the government budget constraint is taken into account and the path of government spending remains unchanged, that is, when taxes are modified as a consequence of the government financing policy and the size of the "crowding out" is kept constant. In a similar line of research, Koskela (1983) analyzes in a static context the implications concerning tax evasion of "compensated" changes in the tax rate. He finds that, under decreasing absolute risk aversion and when the taxpayer is compensated with a lump-sum transfer that keeps the expected government revenue unchanged, an increase in the marginal tax rate either stimulates or discourages tax evasion depending on the nature of the penalty scheme.

The chapter is organized as follows. Section 2.2 presents the individual's decision problem. Section 2.3 introduces the government budget constraint. In Section 2.4, I analyze the effects both on the consumption path and on the amount of evasion of a rise in the proportional tax rate that is compensated by a reduction in the future lump-sum tax. Finally, Section 2.5 contains some concluding remarks.

2.2. THE TAXPAYERS' PROBLEM

Let us consider a large economy populated by a continuum of identical individuals. The mass of individuals is normalized to one. These individuals live for two periods (called periods 1 and 2) and, when they are young, receive an exogenous income y which is the same for all. This income is subjected to a proportional tax rate $\tau \in (0, 1)$. Each individual declares an amount of income equal to x

1 Clotfelter (1983) and Poterba (1987) report a positive relation between tax rate and undeclared income using a real income data base.

2 See Cowell and Gordon (1988), Gordon (1989), and Klepper, Nagin and Spurr (1991), among others.

and, therefore, the amount τx denotes the taxes that are voluntarily paid. Each agent will be audited by the tax authorities with probability p. The inspection allows the tax authorities to find out the true income of an audited individual. Note that, even if the income of an individual were known by the tax authorities, no penalties could be imposed without an inspection that legally established the existence of tax fraud. Individuals have to pay a fine $F(\tau)$ on unreported income if they are caught. The potential dependence of the fine on the tax rate will allow us to cope with the cases of proportional fines both on unreported income and on evaded taxes. Consumption in the first period of life takes place after taxes on declared income have been paid but before the potential inspection occurs. Let S denote the saving of each agent before the potential inspection occurs. Therefore, the effective saving of an agent who has not been audited is also equal to S, while the effective saving of an inspected agent will be $S - F(\tau)(y - x)$. Finally, individuals in the first period of life may have to pay a lump-sum tax T_1.

In their second period of life, individuals only receive the capital income accruing from their effective saving. The gross rate of return on saving is exogenously given and is equal to a constant R. Capital income is devoted to purchase second period consumption and to pay the lump-sum tax T_2. We allow both T_1 and T_2 to take negative values whenever individuals receive lump-sum transfers from the government. We assume that capital income is tax exempt. Notice that consumption in the second period is a random variable that takes the value $R[S - F(\tau)(y - x)] - T_2$ with probability p and the value $RS - T_2$ with probability $1 - p$.

The temporal sequence of events in each period is summarized in the following table:

First period
Individuals receive their exogenous income
Individuals declare their income and pay the corresponding proportional taxes
Individuals pay the first period lump-sum tax
First period consumption takes place
Tax inspection occurs with probability p and the corresponding penalty is paid
Effective saving takes place

Second period
Return on effective saving is received
Individuals pay the second period lump-sum tax
Second period consumption takes place

The preferences of an individual are defined by a time-additive Von Neumann-Morgenstern utility function

$$u(C_1) + \delta E\left(u\left(\tilde{C}_2\right)\right) + \delta V(G),$$

where C_1 is the first period consumption of an individual, $\tilde{C}_2$ is the random consumption in the second period, and G is the level of some public good provided by the government in the second period of individuals' life. The random variable $\tilde{C}_2$ takes two values, C_2^Y and C_2^N, which correspond to second period consumption if the consumer is inspected and if he is not, respectively. The parameter $\delta > 0$ is the discount factor applying on future utility. The utility function u is twice continuously differentiable with $u'>0$ and $u''<0$, and satisfies the Inada conditions $\lim_{C\to 0} u'(C)=\infty$ and $\lim_{C\to\infty} u'(C)=0$ in order to guarantee interior solutions for consumption.

Therefore, taking as given the level of public spending G, a consumer chooses the amount of saving S and the declared income x in order to solve the following program:

$$\text{Max}\left\{u(C_1)+(1-p)\,\delta u\left(C_2^N\right)+p\delta u\left(C_2^Y\right)+\delta V(G)\right\},$$

subject to

$$C_1 = y-\tau x - S - T_1\,, \tag{2.1}$$

$$C_2^N = RS - T_2\,, \tag{2.2}$$

$$C_2^Y = R\big(S-F(\tau)\,(y-x)\big)-T_2\,. \tag{2.3}$$

An interior solution to the previous maximization problem must satisfy the following first order conditions:

$$u'(C_1)\,\tau = p\delta RF(\tau)\,u'\left(C_2^Y\right), \tag{2.4}$$

$$u'(C_1) = (1-p)\,\delta Ru'\left(C_2^N\right)+p\delta Ru'\left(C_2^Y\right). \tag{2.5}$$

According to equation (2.4), the consumer equates the marginal utility obtained from an unit of declared income with the marginal utility when he has been audited. Equation (2.5) tells us that he equates the utility of an extra unit of first period consumption with the expected utility obtained from a marginal increase in second period consumption. Finally, substituting equation (2.4) into (2.5) in order to eliminate C_2^Y, we obtain[3]

$$u'(C_1) = \left(\frac{F(\tau)-\tau}{F(\tau)}\right) = (1-p)\,\delta Ru'\left(C_2^N\right), \tag{2.6}$$

2.3. THE BUDGET CONSTRAINT OF THE GOVERNMENT

The government finances an arbitrary level of public spending G in the second period with the proportional taxes on declared income, the lump-sum taxes and the penalty fees collected

3 Alternatively, combining (2.4) and (2.5) in order to eliminate C_1, we obtain the typical first order condition appearing in models of tax evasion with a single period, that is,

$$(1-p)\,\tau u'\left(C_2^N\right)=p\big(F(\tau)-\tau\big)\,u'\left(C_2^Y\right)$$

In this equation the marginal utility obtained from an extra unit of consumption when the inspection does not occur is equated with the loss that takes place when the individual is caught and, thus, punished. It is obvious from the previous equation that positive evasion $(x<y)$ occurs if and only if $(1-p)\tau > p\big(F(\tau)-\tau\big)$, which is the usual condition found in the tax evasion literature.

from the audited taxpayers. Since the law of large numbers applies in this large economy, a proportion p of consumers are inspected. We assume that there is no cost associated with tax inspection. Therefore, the intertemporal budget constraint of the government is[4]

$$G = RT_1 + T_2 + R\left[(1-p)\,\tau x + p\left(\tau x + F(\tau)\,(y-x)\right)\right]. \qquad (2.7)$$

Note that if $G = 0$ and $T_1 = 0$, the budget constraint (2.7) is identical to that of a fully funded social security system with proportional contributions and lump-sum benefits, and where individuals could misreport their labor income. Totally differentiating (2.7) with respect to, T_1, T_2, τ and x, we obtain

$$RdT_1 + dT_2 + R\left[x + pF'(\tau)\,(y-x)\right]d\tau + R\left(\tau - pF(\tau)\right)dx = 0\,. \qquad (2.8)$$

Our next step is to analyze the effect on the individual's consumption of changes in the fiscal policy keeping constant the level G of public spending. In particular, we will consider changes in the proportional tax rate τ accompanied by changes in the second period lump-sum taxes T_2 such that (2.8) is satisfied.

2.4. THE EFFECTS OF CHANGES IN TAXES

Barsky et al. (1986) have analyzed whether the Ricardian equivalence proposition applies when the profile of individual endowments is exogenously uncertain. When tax evasion is present, the uncertainty is not so exogenous since an individual chooses the level of uncertainty he wants to bear when filling his income report. For example, if agents decide to declare all their true income, then the variance of their second period consumption vanishes and, in consequence, the individual will face no uncertainty.

In order to evaluate the effect of the change in the tax rate on consumption, first observe from (2.1), (2.2) and (2.3), that

$$dC_1 = -xd\tau - \tau dx - dS - dT_1\,, \qquad (2.9)$$

$$dC_2^N = RdS - dT_2\,, \qquad (2.10)$$

$$dC_2^Y = RdS + RF(\tau)\,dx - RF'(\tau)\,(y-x)\,d\tau - dT_2\,. \qquad (2.11)$$

Substituting the government budget constraint (2.8) into (2.10) and (2.10), we obtain

$$dC_2^N = RdS + RdT_1 + R\left[x + pF'(\tau)\,(y-x)\right]d\tau + R\left(\tau - pF(\tau)\right)dx\,, \qquad (2.12)$$

4 The variable G can also be viewed as the final value of government spending, i.e., $G = RG_1 + G_2$, where G_i is the government spending in period i. Such a final value would also enter additively in the utility function of individuals.

$$dC_2^Y = RdS + RdT_1 + R\left[(1-p)F(\tau) + \tau\right]dx + R\left(x - (1-p)F'(\tau)(y-x)\right)d\tau . \qquad (2.13)$$

Define the index of absolute risk aversion $\Phi(C) = -\frac{u''(C)}{u'(C)} > 0$. Taking the first order conditions (2.4) and (2.6), and logarithmically differentiating both sides of these equations, we obtain

$$\Phi(C_1)\,dC_1 - \frac{1}{\tau}d\tau = \Phi\left(C_2^Y\right)dC_2^Y - \frac{F'(\tau)}{F(\tau)}d\tau , \qquad (2.14)$$

$$\Phi(C_1)\,dC_1 - \left(\frac{F(\tau)\left(F'(\tau)-1\right) - F'(\tau)\left(F(\tau)-\tau\right)}{F(\tau)\left(F(\tau)-\tau\right)}\right)d\tau = \Phi\left(C_2^N\right)dC_2^N . \qquad (2.15)$$

Finally, using the expressions (2.9), (2.12) and (2.13) to substitute into (2.14) and (2.15), and assuming $dT_1 = 0$, we obtain the following equations:

$$\begin{aligned}\left[-\Phi(C_1) - R\Phi\left(C_2^Y\right)\right]\frac{dS}{d\tau} &= \frac{1}{\tau} - \frac{F'(\tau)}{F(\tau)} + \Phi(C_1)\,x + \Phi\left(C_2^Y\right)Rx \\ &- \Phi\left(C_2^Y\right)(1-p)RF'(\tau)(y-x) + \left[\Phi(C_1)\,\tau + \Phi\left(C_2^Y\right)\left(R(1-p)F(\tau) + R\tau\right)\right]\frac{dx}{d\tau} ,\end{aligned} \qquad (2.16)$$

$$\begin{aligned}\left[-\Phi(C_1) - R\Phi\left(C_2^N\right)\right]\frac{dS}{d\tau} &= \left[\frac{F(\tau)\left(F'(\tau)-1\right) - F'(\tau)\left(F(\tau)-\tau\right)}{F(\tau)\left(F(\tau)-\tau\right)}\right] \\ &+ \Phi(C_1)\,x + \Phi\left(C_2^N\right)Rx + \Phi\left(C_2^N\right)RpF'(\tau)(y-x) + \left[\Phi(C_1)\,\tau + \Phi\left(C_2^N\right)R\left(\tau - pF(\tau)\right)\right]\frac{dx}{d\tau} .\end{aligned} \qquad (2.17)$$

Observe that we have a system of two equations and two unknowns, $\frac{dS}{d\tau}$ and $\frac{dx}{d\tau}$. Solving this system, we will obtain the sign of the previous derivatives.

It is important to remark that our results will crucially depend on the assumptions made about the fine that an individual must pay if he is inspected. We consider two alternative assumptions. The first one consists of imposing a penalty proportional to undeclared income and independent of the tax rate. In this case, we have $F(\tau) = \pi < 1$ as in Allingham and Sandmo (1972). For every unreported unit of income the taxpayer must pay a constant proportion π. This specification also requires that $\pi > \tau$ since, otherwise, tax evasion would not be punished. The second specification is based on imposing the penalty on evaded taxes as in Yitzhaki (1974). In this case we have that $F(\tau) = \pi\tau$ with $\pi > 1$, where the inequality restriction is necessary to guarantee that a tax evader pays a penalty greater than the taxes paid by a honest taxpayer.

First, we will examine the case $F(\tau) = \pi$. The following proposition summarizes the results.

Proposition 6. *Let* $F(\tau) = \pi < 1$. *Assume that the variation in the proportional tax rate* τ *is compensated with a variation of the second period lump-sum tax* T_2*, such that it leaves unchanged the government spending level* G*. Then,*

(a) the declared income x *is decreasing in the tax rate;*

(b) the amount of evaded taxes $\tau(y - x)$ *is increasing in the tax rate;*

(c) if the utility function exhibits decreasing absolute risk aversion, $p < \frac{1}{2}$*, and* $\pi < 2\tau$*, then first period consumption is decreasing in the tax rate;*

(d) second period consumption when no inspection takes place is increasing in the tax rate;

(e) second period consumption when the agent is audited is decreasing in the tax rate.

Proof. See the appendix.

Our specification allows to characterize unambiguously the tax evasion decision in a Ricardian framework. The most important finding in this respect is that declared income is decreasing in the tax rate. In other words, when the tax rate increases, individuals likewise increase the amount of unreported income, which is a result that is in accord with the empirical evidence. Recall that the theoretical literature in this area obtains either ambiguous results or empirically supported results under very restrictive assumptions on the utility function when the probability of being detected is constant. In particular, when the fines are on unreported income, Allingham and Sandmo (1972) are only able to guarantee the positive relationship between tax rates and unreported income under the not very appealing assumption of non-decreasing absolute risk aversion. Note that in our framework no additional assumptions on utility functions besides concavity are needed to obtain the positive relation between tax rates and unreported income.[5]

The intuition behind items (a) and (b) of the previous proposition lies in the absence of the income effect when the tax rate increases, since this effect is completely offset by the lump-sum transfer. On the other hand, an increase in the tax rate makes honest behavior more expensive when compared with cheating, and this generates a substitution effect that stimulates tax evasion.[6]

The effect of a tax rate increase on intended savings S is ambiguous in general. While the aforementioned substitution effect induces more precautionary saving since more tax evasion amounts to more risk associated with second period income, the lump-sum transfers goes in the opposite direction since the increase in second period income due to the transfer implies a reduction of savings.

Items (c), (d) and (e) of Proposition 6 tell us that Ricardian Equivalence fails since consumption is affected by a change in the timing of taxes.[7] This should not be surprising since Barsky et al. (1986) and Strawczynski (1995) have already obtained this result in different contexts with uncertainty. However, in our context income uncertainty is endogenous whereas these authors assumed a completely exogenous mechanism generating such uncertainty. When the tax rate increases, individuals evade more income, which implies in turn that they will bear more risk in the second period since the amount of fines to be paid in case of inspection depends on the level of evasion. Therefore, income taxes and evasion fees are the cause of income uncertainty and are thus the driving force of precautionary savings. In fact, higher tax rates (coupled with an equivalent increase of lump-sum

5 As we have already mentioned in the introduction, such an empirical positive relation can also be obtained if the probability of inspection were not constant. This is the case in the model of Yitzhaki (1987) where the inspection probability depends positively on evaded income.

6 It is important to remark that, when the penalty does not depend on the tax rate, a rise in tax rate does not alter the marginal utility in the case of being audited for a given x.

7 In this paper, we are usually referring to a Ricardian equivalence exercise consisting of a tax increase today coupled with a cut in lump-sum taxes (or an increase in lump-sum transfers) tomorrow. Of course, our results would be symmetric if we had considered instead the more standard instrumentation of a tax reduction today coupled with an increase in taxes tomorrow.

taxes) induce more cheating, more income risk and, consequently, more inefficient precautionary savings. Note that the inefficiency of precautionary savings hinges on the fact that individuals end up saving more than they would in absence of both taxes and fees.

The sign of the effects on the consumption path are not ambiguous under the empirically plausible parametric restrictions $p < \frac{1}{2}$ and $\pi < 2\tau$, and under the assumption of decreasing absolute risk aversion.[8] In particular, first period consumption decreases when the tax rate is raised. From (2.9), we can observe that the variation in first period consumption depends on three different effects when $dT_1 = 0$. The first order effect implies that first period consumption goes down when the tax rate increases since the taxpayer has to pay more taxes and, in consequence, has less disposable income. On the other hand, when the tax rate increases, we know that declared income diminishes, and then the effective tax payment goes down and this has a positive effect on first period consumption. Finally, the third effect is given by the impact on savings of a change in the tax rate. In general this effect is ambiguous so that it is not possible to know in which direction the variation in savings modifies first period consumption. However, in the present context the first order effect outweighs the indirect effects induced by the changes in both declared income and saving.

We can also observe that an increase in the tax rate implies a reduction in second period consumption when the individual is audited and an increase in consumption when he is not. In fact, declared income decreases with the tax rate since evasion would become more attractive. This results immediately in an increase in second period consumption if the individual is not detected. However, the higher level of evasion translates into a higher amount of fees paid in case of inspection, which will imply in turn a lower consumption in such a case.

Finally, we can compute the effect of a rise in the tax rate on aggregate second period consumption, and we obtain the following corollary:

Corollary 7. *Let* $\overline{C}_2 = (1-p)C_2^N + pC_2^Y$. *If the utility function exhibits decreasing absolute risk aversion,* $p < \frac{1}{2}$, *and* $\pi < 2\tau$, *then* $\overline{C}_2$ *is decreasing in the tax rate.*

Proof. The proof follows from a direct computation.

This result tells us that, despite the probability of being audited being smaller than that of not being audited, the negative effect on C_2^Y outweighs the positive effect on C_2^N.

Let us analyze now the consequence of assuming that $F(\tau) = \pi\tau$, that is, proportional fines are imposed on evaded taxes. The following proposition summarizes the results:

Proposition 8. *Let* $F(\tau) = \pi\tau > 1$. *Assume that the variation in the proportional tax rate* τ *is compensated with a variation of second period lump-sum tax* T_2, *such that it leaves unchanged the government spending* G. *Then,*

(a) the declared income x *is increasing in the tax rate;*

8 The restriction $\pi < 2\tau$ implies that the individual must pay the taxes he has evaded plus a fine which amounts to less than 100 % of evaded taxes.

(b) the amount of evaded taxes $\tau(y - x)$ *is not affected by changes in the tax rate;*

(c) the intended saving S *is decreasing in the tax rate;*

(d) consumption C_1, C_2^N *and* C_2^Y *is not affected by changes in the tax rate.*

Proof. See the appendix.

We can see in this case that Ricardian Equivalence proposition holds. This result clearly differs from that obtained by Barsky et al. (1986) in a model with exogenous uncertainty and flat rate taxes. This is so because in our model a tax cut modifies not only the saving decision but also the amount of declared income. In particular, these two effects have the opposite sign. When the penalty is imposed on evaded taxes, the penalty rate $F(\tau)$ increases proportionally with τ. Therefore, the substitution effect is eliminated. Observe also that, for given levels of declared income x and saving S, an increase in the tax rate τ reduces both first period consumption and second period consumption in case of inspection as a consequence of the proportional increase in the penalty rate. Therefore, the lump-sum compensation would not completely offset the effect on the distribution of second period consumption and, as a consequence, the individual must raise his declared income.[9] Moreover, the amount of evaded taxes $\tau(y - x)$ is not modified since the decrease in tax rates is exactly offset by the reduction in the amount of evaded income and, therefore, the amount of fees paid if the individual is inspected does not change.

In this case, the effect of raising the tax rate τ on intended saving S is not ambiguous. Note that the intended saving of an individual decreases since declared income increases and, at the same time, there is a lump-sum compensation in the second period of life.

Observe that old consumption in both states of the nature (C_2^N and C_2^Y) remains unaltered because of two effects. First, an increase in the tax rate generates a decrease in capital income $\left(R\frac{dS}{d\tau}\right)$ which is equal to the corresponding decrease in second period lump-sum taxes $\left(\frac{dT_2}{d\tau}\right)$. Second, as we said, the amount of evaded taxes does not change. Consequently, the increase in the tax rate does not modify the risk that individuals bear in their second period of life so that neither precautionary saving nor the consumption profile is affected by the tax rate.

A positive relation between declared income and the tax rate was also found by Yitzhaki (1974) under decreasing absolute risk aversion, while we obtain this relation under just concavity of the utility function. It should be stressed that in Yitzhaki's model, life cycle considerations are absent and the government absorbs the revenues from proportional taxes instead of channelling these revenues to the private sector through either lump-sum transfers or tax rebates. Note that, with fines proportional to the taxes evaded and constant probability of inspection, the result concerning the relationship between declared income and tax rates is now at odds with empirical evidence.

To conclude our discussion, we should mention that the effects of a modification in the tax rate on both the declared income and the consumption path are the same if the tax compensation were made through a modification of the first period lump-sum tax T_1. Trivially, the change in the calendar of lump-sum taxes should not have any real effect on the consumption path regardless of whether $F(\tau) = \pi\tau$ or $F(\tau) = \pi$.

9 Balassone and Jones (1998) provide a detailed discussion about the sign of the income effect in a similar context.

2.5. CONCLUSION

I have considered a very simple Ricardian model that allows us to analyze the implications of tax evasion on the equilibrium consumption path. The results differ depending on the assumption made about the fine paid by taxpayers if an inspection occurs. When the penalty is imposed on undeclared income, the Ricardian equivalence proposition fails to hold, whereas it holds when the fine paid by the taxpayers is a constant fraction of evaded taxes. Moreover, the sign of the relationship between tax rates and declared income also depends on the type of penalties. Our Ricardian framework also allows us to isolate the tax evasion implications of an increase in the tax rate by disregarding the crowding out effect accruing from the higher levels of public spending.

The reason behind the previous contradictory results lies in the effects triggered by tax evasion. There are two basic effects: the substitution effect and the income effect. On the one hand, if the penalty is imposed on unreported income, a rise in the tax rate does not modify the amount of income that an agent has to pay if caught for a given amount of reported income. This provides an incentive for tax evasion at the margin. Moreover, the lump-sum transfer received by the individuals offsets the income effect accruing from the increase in the tax rate. In this case, the substitution effect becomes crucial and declared income turns to be decreasing in the tax rate. Therefore, individuals end up facing more uncertainty in the second period of life since the variance of their old income is raised. As the incentives for precautionary saving are modified, the consumption profile changes accordingly. On the other hand, if the fine is proportional to the amount of evaded taxes, an increase in the tax rate implies an increase in the penalty and this leads individuals to increase their declared income since the substitution effect has been eliminated and the lump-sum transfers do not completely offset the effect on the structure of uncertainty. However, the amount of evaded taxes remains unchanged now and, hence, neither precautionary savings nor the consumption distribution in the second period are affected.

It should be noticed that the results of this chapter concerning Ricardian equivalence are immediately extended to an economy with production in which firms hire both the labor supplied by agents and the capital accruing from saving net of outstanding public debt. The rental prices of both labor and capital will be equal to their respective marginal productivities. Whenever Ricardian equivalence holds in our setup, it will hold in such a general equilibrium context. This is so because the change in the financing policy of the government will not affect neither the consumption path nor the capital lent to the firms, and this is consistent with invariant wages and invariant rates of return from accumulated savings. Obviously, the failure of Ricardian equivalence in our model translates into the same failure in the corresponding economy with productive firms.

2.6. APPENDIX

Proof of Proposition 6. Assume that the penalty is imposed on undeclared income, that is, $F(\tau) = \pi$. In this case, it follows that $F'(\tau) = 0$. Thus, equations (2.16) and (2.17) can be simplified to

$$\left[-\Phi(C_1) - R\Phi\left(C_2^Y\right)\right]\frac{dS}{d\tau} = \frac{1}{\tau} + \Phi(C_1)\,x + \Phi\left(C_2^Y\right)Rx + \left[\Phi(C_1)\,\tau + \Phi\left(C_2^Y\right)R(\tau + (1-p)\,\pi)\right]\frac{dx}{d\tau},$$

$$\left[-\Phi(C_1)-R\Phi\left(C_2^N\right)\right]\frac{dS}{d\tau}=-\frac{1}{\pi-\tau}+\Phi(C_1)\,x+\Phi\left(C_2^N\right)Rx+\left[\Phi(C_1)\,\tau+\Phi\left(C_2^N\right)R(\tau-p\pi)\right]\frac{dx}{d\tau}.$$

Solving this system, we obtain the following explicit solutions for $\frac{dS}{d\tau}$ and $\frac{dx}{d\tau}$:

$$\frac{dS}{d\tau}=\frac{\alpha D-\beta A}{\alpha C-\beta B}, \tag{2.18}$$

$$\frac{dx}{d\tau}=\frac{BD-AC}{\alpha C-\beta B}, \tag{2.19}$$

where

$$\alpha=\left[\Phi(C_1)\,\tau+\Phi\left(C_2^Y\right)R(\tau+(1-p)\,\pi)\right],$$

$$\beta=\left[\Phi(C_1)\,\tau+\Phi\left(C_2^N\right)R(\tau-p\pi)\right],$$

$$A=\left[\frac{1}{\tau}+\Phi(C_1)\,x+\Phi\left(C_2^Y\right)Rx\right],$$

$$B=\left[-\Phi(C_1)-R\Phi\left(C_2^Y\right)\right],$$

$$C=\left[-\Phi(C_1)-R\Phi\left(C_2^N\right)\right],$$

$$D=\left[-\frac{1}{\pi-\tau}+\Phi(C_1)\,x+\Phi\left(C_2^N\right)Rx\right].$$

Simplifying and collecting terms, it is easy to see that $\alpha C-\beta B<0$ and $BD-AC>0$. We obtain thus a negative relation between tax rates and reported income. The positive relation between evaded taxes $\tau(y-x)$ and the tax rate τ follows immediately.

To compute the effect of a change in the tax rate on consumptions, first rewrite (2.9), (2.10) and (2.11) as

$$\frac{dC_1}{d\tau}=-x-\tau\frac{dx}{d\tau}-\frac{dS}{d\tau}, \tag{2.20}$$

$$\frac{dC_2^N}{d\tau}=R\frac{dS}{d\tau}-\frac{dT_2}{d\tau}, \tag{2.21}$$

$$\frac{dC_2^Y}{d\tau}=R\frac{dS}{d\tau}+R\pi\frac{dx}{d\tau}-\frac{dT_2}{d\tau}. \tag{2.22}$$

Likewise, assume $dT_1 = 0$ and rewrite (2.8) as

$$\frac{dT_2}{d\tau} = -Rx - R(\tau - p\pi)\frac{dx}{d\tau}. \tag{2.23}$$

Next, we substitute the solutions of $\frac{dS}{d\tau}$, $\frac{dT_2}{d\tau}$ and $\frac{dx}{d\tau}$ given in (2.18), (2.19) and (2.23) into the expressions (2.20), (2.21) and (2.22). We obtain

$$\frac{dC_1}{d\tau} = \frac{\beta(Bx + A) - \alpha(Cx + D) - \tau(BD - AC)}{\alpha C - \beta B}, \tag{2.24}$$

$$\frac{dC_2^N}{d\tau} = \frac{R[\alpha(D + xC) - \beta(A + xB) + (\tau - p\pi)(BD - AC)]}{\alpha C - \beta B}, \tag{2.25}$$

$$\frac{dC_2^Y}{d\tau} = \frac{R[\alpha(D + xC) - \beta(A + xB) + (\tau + (1-p)\pi)(BD - AC)]}{\alpha C - \beta B}. \tag{2.26}$$

Since we know that $\alpha C - \beta B < 0$, the sign of these expressions is determined by the sign of the numerator. Making use of the index of absolute risk aversion $\Phi(C) = -\frac{u''(C)}{u'(C)}$ and rearranging the expressions (2.24), (2.25) and (2.26), we obtain

$$\frac{dC_1}{d\tau} = \frac{\frac{1}{\pi - \tau}\Phi\left(C_2^Y\right)R(1-p)\,\pi - \frac{1}{\tau}\Phi\left(C_2^N\right)Rp\pi}{\alpha C - \beta B}, \tag{2.27}$$

$$\frac{dC_2^N}{d\tau} = \frac{-\Phi(C_1)\,p\pi\frac{\pi}{(\pi - \tau)\,\tau} - \frac{1}{\pi - \tau}\Phi\left(C_2^Y\right)R\pi}{\alpha C - \beta B}, \tag{2.28}$$

$$\frac{dC_2^Y}{d\tau} = \frac{\frac{\pi}{\tau}\left[\Phi(C_1)\,(1-p) + \Phi\left(C_2^N\right)R\right] + \frac{1}{\pi - \tau}\Phi(C_1)\,(1-p)\pi}{\alpha C - \beta B}. \tag{2.29}$$

It is immediate to see that under decreasing absolute risk aversion, $p < 1/2$ and $\pi < 2\tau$, the numerator of expression (2.27) is positive, which implies that $\frac{dC_1}{d\tau} < 0$. Moreover, the numerator of (2.28) is unambiguously negative since the index of absolute risk aversion is positive and $\pi > \tau$, and this implies that $\frac{dC_2^N}{d\tau} > 0$. Likewise, it can be checked that the numerator of (2.29) is unambiguously positive so that $\frac{dC_2^Y}{d\tau} < 0$.

Proof of Proposition 8. If we assume that the fine takes the form $F(\tau)=\pi\tau$, we have that $F'(\tau)=\pi$ and equations (2.16) and (2.17) become

$$\left[-\Phi(C_1)-R\Phi\left(C_2^Y\right)\right]\frac{dS}{d\tau}=\Phi(C_1)\,x+\Phi\left(C_2^Y\right)R\left[x-(1-p)\,\pi(y-x)\right]+\left[\Phi(C_1)\,\tau+\Phi\left(C_2^Y\right)R\tau\left(1+(1-p)\right)\pi\right]\frac{dx}{d\tau},$$

$$\left[-\Phi(C_1)-R\Phi\left(C_2^N\right)\right]\frac{dS}{d\tau}=\Phi(C_1)\,x+\Phi\left(C_2^N\right)R\left[x+p\pi(y-x)\right]+\left[\Phi(C_1)\,\tau+\Phi\left(C_2^N\right)R\tau(1-p\pi)\right]\frac{dx}{d\tau}.$$

The solution to this system yields

$$\frac{dx}{d\tau}=\frac{y-x}{\tau}>0, \tag{2.30}$$

$$\frac{dS}{d\tau}=-y<0. \tag{2.31}$$

Concerning the amount of evaded taxes $\tau(y-x)$, let us differentiate with respect to τ to obtain

$$\frac{d[\tau(y-x)]}{d\tau}=y-x-\tau\frac{dx}{d\tau}.$$

From (2.30), it immediately follows that $\frac{d[\tau(y-x)]}{d\tau}=0$.

In order to compute the effect of a change in τ on the consumption path, rewrite the equations (2.9), (2.10) and (2.11) as

$$\frac{dC_1}{d\tau}=-x-\tau\frac{dx}{d\tau}-\frac{dS}{d\tau}, \tag{2.32}$$

$$\frac{dC_2^N}{d\tau}=R\frac{dS}{d\tau}-\frac{dT_2}{d\tau}, \tag{2.33}$$

$$\frac{dC_2^Y}{d\tau}=R\frac{dS}{d\tau}+R\tau\pi\frac{dx}{d\tau}-R\pi(y-x)-\frac{dT_2}{d\tau}. \tag{2.34}$$

Substituting (2.30) into (2.8), we obtain

$$\frac{dT_2}{d\tau}=-Ry. \tag{2.35}$$

Finally substituting (2.30), (2.31) and (2.35) into the equations (2.32), (2.33) and (2.34), we find that

$$\frac{dC_1}{d\tau}=0,\quad \frac{dC_2^N}{d\tau}=0,\quad \text{and}\quad \frac{dC_2^Y}{d\tau}=0.$$

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CHAPTER 3

TAX AVOIDANCE AND THE LAFFER CURVE

3.1. INTRODUCTION

To pay taxes is viewed as an unpleasant obligation and, consequently, many taxpayers try to reduce their tax liabilities. Most of the literature has concentrated on tax evasion as a way of achieving such a reduction. However, this method implies that the taxpayers bear some risk since, if they are discovered underreporting their true income, they are punished with a penalty.

The growing complexity of the tax code has given rise to tax avoidance as an alternative method of reducing the payment of taxes. We mean by tax avoidance the set of actions which allow taxpayers to enjoy a legal reduction in their tax liabilities. In general these actions are riskless but costly.[1] Usual practices of taking full advantage of the tax code include, for instance, income splitting, postponement of taxes, and tax arbitrage across income facing different tax treatments.[2] On the other hand, the emergence of certain countries, called *tax heavens*, where the income is lower-taxed or non-taxed at all, has encouraged to a great extent the tax avoidance activity. For example, it is well known that some taxpayers have their fiscal residence located in a tax heaven in order to pay less taxes while their productive activities are located in a country exhibiting much higher tax rates.[3] In this chapter, we will concentrate on the analysis of tax avoidance associated with the existence of tax heavens. Therefore, we will assume that tax avoidance is linked to the source of income so that, when an individual becomes avoider, he shelters all his income.

The behavior of the taxpayer geared towards minimizing their tax bill has evident implications for the government revenues. Even if the exact measurement of the tax evasion and tax avoidance activities is, by definition, difficult to achieve, some estimations have been performed. For instance, the US Internal Revenue Service has estimated in 1994 an overall compliance rate of 87%, which may seem quite high in relative terms but, if we translated this rate into dollar terms, we would see that each 1% increase in compliance translates to USD 7-10 billion in terms of government revenues (see Lyons, 1996).

The aim of this chapter is to analyze the relationship between the tax rate and the tax revenue raised by the government when agents choose between evading and avoiding as two alternatives ways of reducing their tax liabilities. If the tax rate affects the attitude of individuals towards tax compliance, then any change in the tax rate affects the tax revenue in two ways. On the one hand, for a given level of tax compliance, collected taxes increase with the tax rate. On the other hand, an increase in the tax rate may increase the set of taxpayers who avoid their taxes, and this diminishes the tax revenue obtained by the government. When the former effect is stronger for low tax rates and weaker for high tax rates, we will be facing the typical Laffer curve relating government revenues with tax rates, that is, this relationship will exhibit an inverted U shape.

The government can achieve a larger tax revenue by increasing the tax rate until its largest feasible value, when the revenue-tax rate relationship is increasing. However, when this relationship has the shape of a Laffer curve, as a consequence of tax evasion and tax avoidance activities, the maximum tax revenue is achieved by selecting a tax rate strictly lower than 100%. Therefore, governments seeking to maximize its revenue will never find optimal to extract all the income from the

1 In some cases these actions encompass some uncertainty. This happens when the avoidance strategies depend on uncertain events or when the hoopholes in the tax law are exploited until the margin.

2 For more details see Stiglitz (1985).

3 This is a very common practice for firms in Netherlands, since the Netherlands Antilles is considered a tax heaven jurisdiction. Other tax heavens, located in Europe, are for example Andorra and Monaco, which usually shelter a few rich individuals.

private sector through proportional taxes even if the income of each individual is unaffected by the tax rate.

Some authors have analyzed taxpayer behavior using a representative agent approach when both avoidance and evasion are jointly chosen. These authors study the implications that this choice has on some traditional results of the tax evasion literature but disregard the analysis of the behavior of the government revenue as a function of the tax rate. In fact, the comparative statics is immediate in the previous analysis when the fines are proportional to evaded income: revenues increase with the tax rate since there is less evaded income.[4] It should be noticed that these authors emphasize the aspects of complementariety between these two activities. We consider instead a framework where the individuals only choose one of the two alternatives. Therefore, we are treating tax evasion and tax avoidance as substitutive strategies and, thus, we are somewhat following the idea of polarization between evaders and shelters proposed by Cowell (1990). According to this author, by choosing to shelter income, an individual draws attention to himself and therefore becomes a prime target for investigation and, thus, it is inadvisable to try evading and avoiding at the same time. The presence of legal tax shelters induces thus a polarization between evaders and avoiders.

In our work, we dispense with the assumption of a representative agent, since we consider that all the individuals have a different exogenous income although their preferences are represented by a common utility function. This means that not all the individuals select the same method for paying less taxes. We will assume that the income distribution is uniform although we will also explore the robustness of our results in an example with a discrete income distribution. Such a discrete distribution will allow us to discuss the implications of income polarization concerning the characterization of the revenue-tax rate relationship.[5] Notice that in our model individuals are indexed by different levels of income but both the aggregate income and the distribution of income among individuals will remain unaltered when the tax rate changes. Thus, our analysis focuses exclusively on the contribution of the tax code and the tax enforcement policy to the revenue-tax rate relationship and then it disregards the effects of tax compliance on labor supply and other macroeconomic variables.

We show in this chapter that the Laffer curve could emerge in all the scenarios we consider. In order to get such an existence result we need to assume that the cost of avoidance is sufficiently low so as to allow almost all the individuals to avoid their income for high values of the tax rate. The robustness of this result indicates that in our context a policy of high marginal tax rates might not be the appropriate strategy when the objective of the government is to maximize its revenue.

The chapter is organized as follows. Section 3.2 presents a description of the individual behavior both when the taxpayer is an evader and when he is an avoider. The tax revenue function of the government is presented in Section 3.3. Section 3.4 provides the analysis of the tax rate-tax revenue relationship both when the non-avoiders are honest and when they evade. In Section 3.5 we carry out a more detailed analysis on the existence of the Laffer curve when a isoelastic utility function is assumed. Section 3.6 characterizes the Laffer curve when the income distribution function is not uniform but discrete. Finally, the main conclusions and extensions of the chapter are presented in Section 3.7. Most of the formal proofs are relegated to the Appendix.

4 See, for example Cross and Shaw (1982) and Alm (1988) who analyze both the individual and government behavior when avoidance and evasion are jointly selected, under the assumption that avoidance is not a risky activity. See also Alm and Mccallin (1990) who carry out a mean-variance analysis when both evasion and avoidance are considered as risky assets.

5 Waud (1988) has studied the existence of a Laffer curve for the government revenue in a context where both tax evasion and tax avoidance take place. This author establishes the existence a Laffer curve for the expected tax revenue in a partial equilibrium context with a representative agent. See also Waud (1986) who studied the appearance of the Laffer curve of the tax revenue when tax evasion and tax avoidance appear simultaneously in a classical supply-side model.

3.2. INDIVIDUAL BEHAVIOR

Let us consider an economy with a continuum of agents who are indexed by their income y. Each individual receives an idiosyncratic exogenous income y drawn from an uniform distribution on $[0, \overline{Y}]$. The preferences of agents are represented by a common Von Neumann-Morgenstein utility function $U(\cdot)$ defined on after-tax income I. The first and second derivatives U' and U'' exist and are continuous with $U' > 0$ and $U'' < 0$. Therefore, agents are risk averse. Moreover, we assume that the utility function $U(\cdot)$ satisfies the Inada condition: $\lim_{I \to 0} U'(I) = \infty$.

In this economy agents are supposed to pay proportional taxes with a flat-rate tax $\tau \in [0, 1]$. However, agents have two alternative ways of reducing their tax bill. On the one hand, they can misreport part of their income (tax evasion) and, on the other hand, they can avoid the tax payments using legal methods (tax avoidance). Tax evasion is a risky activity because if the individual is inspected, he has to pay the evaded taxes plus a penalty. Although we will assume that tax avoidance is a riskless action, there are positive costs associated with it. In our model, taxpayers can be classified on one of two categories: evaders or avoiders. The case of honest taxpayers also arises as a corner solution of the evaders problem and will also be studied.

3.3. TAX EVADERS

Let us consider the standard Allingham and Sandmo (1972) model of tax evasion using expected utility analysis. In what follows, we will adopt the variation of such a model presented by Yitzhaki (1974). The individual declares an amount of income equal to x and he will be audited by the tax authorities with probability p. The true income y is always discovered by the inspection. Therefore, agents reduce their tax payments by $\tau(y - x)$ whenever inspection does not occur. If an individual is caught evading, he has to pay a proportional fine $s > 0$ on evaded taxes. We assume that overreporting is never rewarded. Therefore, we can restrict the declared income x to be no greater than y.[6] On the other hand, x is no lower than zero since the tax code does not feature a loss-offset, that is, negative declared income is viewed by the tax authorities as equivalent to zero declared income. We define the evaded income as $e = y - x$ where $e \in [0, y]$. Then, the random net income of an evader with true income y and evaded income e is equal to[7]

$$y - \tau y + \tau e \text{, if the individual is not inspected,}$$

and

$$y - \tau y - s\tau e \text{, if the individual is inspected.}$$

6 When an individual declares more than his true income, he will only receive the excess tax contribution if he is audited. Therefore, $s\tau = 1$ whenever $x > y$.

7 Note that $y - \tau y + \tau e = y - \tau x$.

We assume that the parameters defining the tax inspection policy, p and s, are given exogenously throughout the chapter.

Each taxpayer chooses the amount e of evaded income in order to maximize his expected utility

$$E[U(I)] = (1-p)\,U(y-\tau y+\tau e) + pU(y-\tau y-s\tau e) \equiv u(e). \tag{3.1}$$

The first order condition for an interior solution of the maximization of (3.1) is

$$(1-p)\,U'(y-\tau y+\tau e)\,\tau - pU'(y-\tau y-s\tau e)\,s\tau = 0. \tag{3.2}$$

The corresponding second-order condition is automatically satisfied because of the assumption of concavity of the utility function.

The following lemma shows the solutions for the maximization problem (3.1):

Lemma 9. *The optimal evaded income* $e(\tau, y)$ *as a function of the tax rate* τ *and the individual income* y *is:*

(a) if $\tau(1+s) < 1$,

$$e(\tau, y) = \begin{cases} 0 & \text{for } s \geq \dfrac{1-p}{p} \\ \hat{e}(\tau, y) & \text{for } \left(\dfrac{1-p}{p}\right)\dfrac{U'(y)}{U'(y-\tau y-s\tau y)} < s < \dfrac{1-p}{p} \\ y & \text{for } s \leq \left(\dfrac{1-p}{p}\right)\dfrac{U'(y)}{U'(y-\tau y-s\tau y)}, \end{cases}$$

(b) if $\tau(1+s) \geq 1$,

$$e(\tau, y) = \begin{cases} 0 & \text{for } s \geq \dfrac{1-p}{p} \\ \hat{e}(\tau, y) & \text{for } s < \dfrac{1-p}{p}, \end{cases}$$

where $\hat{e}(\tau, y)$ *is the solution for the evaded income* e *to equation* (3.2).

Proof. See the Appendix.

Lemma 9 states the intuitive result that the evader will declare less than his actual income if the penalty s is small enough, whereas the amount declared is positive whenever s is large enough. Note that when $s \geq \dfrac{1-p}{p}$ taxpayers become honest.

Finally, substituting $e(\tau, y)$ into the expected utility we have

$$V(\tau, y) = (1-p)\,U(y - \tau y + \tau e(\tau, y)) + pU(y - \tau y - s\tau e(\tau, y)), \tag{3.3}$$

which is the maximum value achieved by the expected utility, that is, the indirect utility function.

3.2.2. Tax Avoiders

The structure of the tax system may offer some ways of reducing the tax bill of the agents. This implies that they will pay less taxes since the avoided income will be taxed at a lower tax rate. To simplify the analysis, let us assume that the avoided income is taxed at a zero tax rate.[8] Moreover, we consider that tax avoidance takes the form of sheltering a source of income in a tax heaven. Since we assume that individuals have only a source of income (typically associated with a single economic activity), when an individual decides to become an avoider, he shelters his total income.

There exist positive costs associated with the tax avoidance activity given by the function $C(y)$. These costs include, for example, the cost of obtaining the relevant information about the tax clauses, or the payment to a tax advisor, or the necessary amount to create a firm in order to declare the personal income as corporate income, which in some countries is lower-taxed. We assume that $C(y) = k + cy$, where k is a positive fixed cost and $c \in (0, 1)$ is a proportional cost per unit of avoided income. Note that the existence of a fixed cost will prevent the poorest individuals from avoiding since they can not afford it. The utility achieved by an avoider is thus $U(y - cy - k)$. Observe that if there is an individual who wants to be an avoider then the following two conditions must hold simultaneously: (i) $\tau > c$ and (ii) $k \le (1-c)\overline{Y}$. Condition (i) eliminates the values of the tax rate for which tax avoidance never takes place, since for $\tau \le c$ an individual obtains more utility being honest than being an avoider.[9] Condition (ii) ensures that at least the richest individual will be an avoider for $\tau > c$. In order to simplify the analysis, we will assume that condition (ii) holds in what follows.

3.3. GOVERNMENT

The government obtains resources from the taxes paid by the individuals who are not avoiders and from the fines that individuals must pay if they are caught evading. Since the government inspects each individual with probability p, a fraction p of individuals is inspected in this large econ-

8 This analysis can easily be modified to accommodate a positive tax rate for avoided income.

9 The utlity achieved by a honest tax payer is $U(y - \tau y)$. Since the utility function is monotonically increasing and $\tau \le c$, we have

$$U(y - \tau y) \ge U(y - cy) > U(y - cy - k).$$

omy. For the sake of simplicity we assume that there are no costs associated with tax inspection.[10] Note that the density function $f(y)$ of a uniform distribution of income on $[0, \overline{Y}]$ is

$$f(y) = \begin{cases} \dfrac{1}{\overline{Y}} & \text{for} \quad 0 \leq y \leq \overline{Y} \\ 0 & \text{elsewhere.} \end{cases}$$

Therefore, the total government tax revenue per capita is given by the following Lebesgue integral

$$G(\tau) = \int_{A(\tau)} [(1-p)\,\tau(y - e(\tau, y)) + p\tau(y + se(\tau, y))] \frac{1}{\overline{Y}} dy \,, \tag{3.4}$$

where $A(\tau)$ is the Lebesgue measurable subset of incomes in $[0, \overline{Y}]$ for which the individuals are non-avoiders when the tax rate is τ. Obviously, $G(\tau)$ is a continuous function on $[0, 1]$. Note that the avoiders do not pay taxes so that the evaders are the only individuals who pay taxes although possibly less than what they should. Note also that the tax rate τ affects $G(\tau)$ through its effects both on $e(\tau, y)$ and on the set $A(\tau)$. Then, our objective will be to analyze the behavior of the government tax revenue $G(\tau)$ when the tax rate changes. The next section provides the corresponding analysis.

3.4. THE LAFFER CURVE

In this section we will analyze whether the relationship between the tax rate and the government revenue can be described by a Laffer curve. There exist several definitions of the Laffer curve. In particular we will consider only two of them. The weakest one involves a non-monotone relationship between $G(\tau)$ and τ. A stronger definition of the Laffer curve requires the existence of a single maximum for $G(\tau)$ in the open interval $(0, 1)$. In this case the Laffer curve displays the typical inverted U shape. Laffer curves conforming the different previous two definitions will be found throughout the chapter.

To carry out the analysis we need to separate the case where non-avoiders are honest from the case where they are strict evaders. In the former case uncertainty vanishes since the individuals declare their true income, while in the latter case the individuals hide part of their true income and they face up to a potential inspection which will reduce their net income. The analysis will differ in these two cases.

10 Note that the assumption of a zero inspection cost does not affect the present analysis whenever the government were unable to identify ex-ante who is an evader and who is an avoider. In this case, under a constant probability p of inspection, the total inspection cost will be unaffected by the proportion of evaders and avoiders and, thus, it will be independent of the tax rate.

3.4.1. Honest Behavior

In this case the individuals who are not avoiders report their true income so that $e(\tau, y) = 0$. Following Lemma 9 this occurs when $s \geq \frac{1-p}{p}$. To study how the total tax revenue is modified when τ increases, we need to know which individuals become avoiders. This decision depends on the difference between the cost of being honest and the cost of being an avoider. For example, if either the costs associated with tax avoidance are high or the tax rate is low, poor individuals will prefer paying their corresponding taxes. Thus, for an individual with income y, avoiding is optimal whenever:[11]

$$U(y - \tau y) \leq U(y - cy - k). \tag{3.5}$$

Solving (3.5) for τ, we obtain that

$$\tau(y) \geq c + \frac{k}{y}.$$

Therefore, for $\tau < c + \frac{k}{y}$ the individual with income y will prefer being honest rather than avoider. In particular, we can find the tax rate that leaves the richest individual indifferent between avoiding and being honest, that is,

$$\bar{\tau}(\bar{Y}) = c + \frac{k}{\bar{Y}}. \tag{3.6}$$

Observe that $\bar{\tau}(\bar{Y})$ is greater than the proportional cost of avoiding c if the fixed cost k is strictly positive. Hence, for $\tau < \bar{\tau}(\bar{Y})$ all the individuals will pay their corresponding taxes. Obviously, $\tau(\bar{Y}) < 1$ for $k < (1-c)\bar{Y}$.

Now we need to know how many individuals will be honest for each possible value of the tax rate above $\bar{\tau}(\bar{Y})$. It turns out that, for $\tau \in [\bar{\tau}(\bar{Y}), 1]$ there exists a threshold level of income $y_H^*(\tau) \in (0, \bar{Y})$ making an individual indifferent between being honest and avoiding. In particular, $y_H^*(\tau)$ is the value of y satisfying (3.5) with equality. Formally,

$$y_H^*(\tau) = \frac{k}{\tau - c}. \tag{3.7}$$

Therefore, the individuals having incomes lower than $y_H^*(\tau)$ will tell the truth, while the individuals with higher incomes will be avoiders. Note that when $k = 0$, $y_H^*(\tau) = 0$. Hence, all the individuals become

11 We adopt the innocuous convention that when an individual is indifferent between being honest and avoiding, he is honest.

either honest or avoiders. Note also that $y_H^*(\tau)$ decreases with τ so that, if the tax rate increases, the lower incomes will also take advantage from avoidance.[12] Hence, for $\bar{\tau}(\overline{Y}) < \tau \le 1$ only the individuals who can afford the fixed cost will be avoiders. In this case the tax revenue is given by the total taxes paid for the individuals who are honest.

The following Lemma gives us the expression for the total tax revenue function:

Lemma 10. *Let* $s \ge \frac{1-p}{p}$. *Then, the total tax revenue will be given by the following two-part function:*

$$G(\tau) = \begin{cases} \dfrac{\tau\overline{Y}}{2} & \text{if } \quad 0 \le \tau \le \bar{\tau}(\overline{Y}) \\[2ex] \dfrac{\tau}{2\overline{Y}}\left(\dfrac{k}{\tau - c}\right)^2 & \text{if } \quad \bar{\tau}(\overline{Y}) < \tau \le 1. \end{cases}$$

Proof. See the Appendix.

The next proposition characterizes the behavior of $G(\tau)$ with respect to the tax rate.

Proposition 11. *Assume that* $s \ge \frac{1-p}{p}$. *Then, the total tax revenue* $G(\tau)$ *is a continuous function on* $[0,1]$ *that achieves its unique maximum value when* $\tau = \bar{\tau}(\overline{Y})$.

Proof. See the Appendix.

The intuition of this result is quite obvious. When the tax rate of the economy is lower than $\bar{\tau}(\overline{Y})$, all the individuals pay their corresponding taxes and the tax revenue is increasing in the tax rate. On the other hand, when the tax rate is higher than $\bar{\tau}(\overline{Y})$, two different effects take place. The first one is the same as before since higher tax rates have a positive effect on the tax revenue. However, there exists now an additional effect since an increase in the tax rate implies that some individuals (the richest ones) prefer now becoming avoiders and, in consequence, they do not pay taxes any longer. This effect has a clear negative impact on the tax revenue. As the Proposition 11 shows, the negative effect outweighs the positive one, and for $\tau > \bar{\tau}(\overline{Y})$ the tax revenue is decreasing in the tax rate. Note that the Laffer curve obtained in this subsection conforms with the stronger definition of such a curve, that is, the curve has a unique maximum on $(0,1)$.

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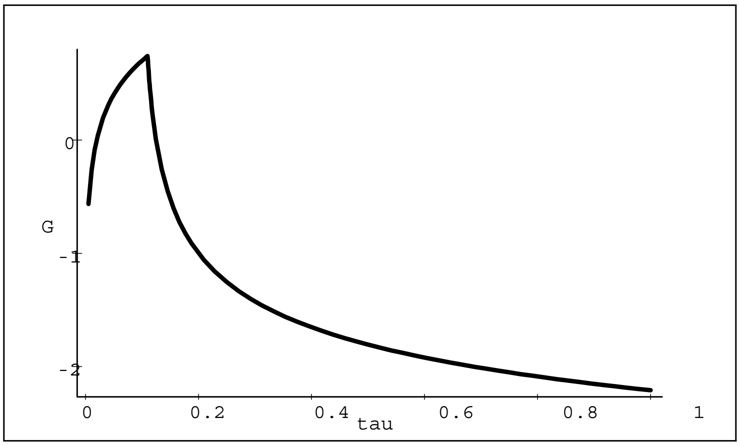
12 Observe that $y_H^*(\bar{\tau}(\overline{Y})) = \overline{Y}$ and, thus, the threshold level $y_H^*(\tau)$ will be lower than $\overline{Y}$ for $\tau \in [\bar{\tau}(\overline{Y}), 1]$ since $y_H^*(\tau)$ decreases with τ.

As an illustration of Proposition 11, Figure 3.1 displays the total tax revenue as a function of the tax rate when $c = 0.1$, $\overline{Y} = 100$, $k = 1$. In this case we obtain a standard Laffer curve with a maximum at $\overline{\tau}(\overline{Y}) = 0.11$.

FIGURE 3.1

THE FUNCTION WHEN THE INDIVIDUALS ARE HONEST, USING LOGARITHMIC SCALE

($c = 0.1, \overline{Y} = 100, k = 1$)



3.4.2. Tax Evasion

When $s < \frac{1-p}{p}$ program (3.1) yields a strictly positive amount of evaded income, $e(\tau, y) > 0$. The decision of being either an avoider or an evader will depend on the individual's income and the values of the parameters of the model. For $\tau \le c$ all the individuals will prefer evading since the avoidance is only attractive for tax rates strictly greater than c. Therefore, for $0 < \tau \le c$ the government obtains some positive revenue at least from the fines collected by the inspection.[13]

For $\tau > c$, tax avoidance can appear. Thus, an individual prefers avoiding to evading when $V(\tau, y) < U(y - cy - k)$, and vice versa. Hence, given p, s, c and k, we can define $y^*(\tau)$ as the income that leaves an individual indifferent between evading and avoiding when the tax rate is τ, that is,

$$V(\tau, y^*(\tau)) = U(y^*(\tau) - cy^*(\tau) - k), \qquad (3.8)$$

where the function $V(\tau, y)$ is defined in (3.3). The following lemma establishes the existence of such a threshold income level $y^*(\tau)$:

13 The government does not know if the individuals will prefer evading all their income or evading only a part of it. Note that the fines collected include the amount of evaded taxes plus the penalty paid as a punishment to evasion.

Lemma 12. *Assume that* $s < \frac{1-p}{p}$*, and* $(1+s)\tau > 1+sc$*. Then there exists an income* $y^*(\tau) > 0$ *such that all the individuals having an income lower than* $y^*(\tau)$ *are evaders while all those with an income higher than* $y^*(\tau)$ *are avoiders.*

Proof. See the Appendix.

As we expected, the previous results tell us that the richest individuals avoid the payment of their taxes while the poorest ones choose the evasion as a way of reducing their tax liabilities. This is so because the poorest individuals can not face up to the fixed cost k of avoiding. Notice that assuming that $(1+s)\tau > 1+sc$, implies that $(1+s)\tau > 1$, which ensures that the problem (3.1) has an interior solution.[14] In other words, Lemma 12 only applies for $0 < e(\tau, y) < y$. This fact does not constitute a very restrictive assumption since the available data about tax evasion shows that tax evasion takes place in a partial way.

The next proposition summarizes the main result concerning the existence of the Laffer curve in this context:

Proposition 13. Assume $s < \frac{(1-p)}{p}$. If the fixed cost k is sufficiently small, then the total tax revenue $G(\tau)$ is non-monotonic in τ, for $\tau \in [0, 1]$.

Proof. See the Appendix.

Note that, when tax evasion occurs and the fixed cost of avoidance is low, we are only able to make a characterization of the Laffer curve in the weakest sense. To ensure that almost all individuals are avoiders when the tax rate is converging to one, we need to assume that the fixed cost is small enough. Therefore, the tax revenue will take a low value because only the poorest individuals pay taxes. On the contrary, in the absence of tax evasion (see subsection 4.1), we were able to make a stronger characterization which embodied the existence of a single maximum of the revenue function without any additional assumption about the fixed cost.

The importance of the Proposition 13 is evident since a higher tax rate does not mean a higher tax revenue if the individuals can avoid their income taxes. For instance, if the individuals did not have the possibility of avoiding, an increase in the tax rate implies an increase in the tax revenue under decreasing absolute risk aversion.[15] This is so because, when the fine is proportional to the amount of evaded taxes, an increase in the tax rate translates into a rise in the amount that the taxpayer has to pay as a penalty. This induces less evasion since the substitution effect has been eliminated as a consequence of imposing penalties on evaded taxes and not on evaded income. Therefore, the final outcome will be an increase in the tax revenue.[16]

14 See Yitzhaki (1974).

15 A sufficient condition to ensure that nobody wants to avoid is $k > (1-c)\overline{Y}$.

16 If tax avoidance is not allowed, then the tax revenue is given by

$$G(\tau) = \int_0^{\overline{Y}} \left[(1-p)\,\tau(y - e(\tau, y)) + p\tau(y + se(\tau, y))\right] \frac{1}{\overline{Y}}\,dy.$$

It is important to remark the important role that the tax avoidance costs play in the analysis that we have carried out. First, the fixed cost is necessary to guarantee that not everybody takes advantage from avoidance. Second, if the costs associated with the tax avoidance are too high, nobody wants to avoid and the tax revenue function will be increasing in the tax rate.

In the next section, we examine exhaustively an example with an isoelastic utility function which will allow us a more precise characterization of the revenue function $G(\tau)$.

3.5. AN EXAMPLE: THE ISOELASTIC UTILITY FUNCTION

The isoelastic utility function has been widely used in several instances of economic analysis ranging from financial economics to macroeconomics. Let us only consider the interesting case where the evasion takes place (i.e. $s < \frac{1-p}{p}$) since the honest case has been fully characterized in Section 4.1 without using any specific functional form.

The optimal evasion when the utility function is $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, with $\gamma > 0$, becomes the following:

$$e(\tau, y) = \begin{cases} y & \text{if} \quad \tau \le \tau^* \\ \left[\frac{(A-1)(1-\tau)}{\tau(1+As)}\right] y & \text{if} \quad \tau > \tau^*, \end{cases} \tag{3.9}$$

Calculating the derivative of $G(\tau)$ respect to the tax rate we have

$$\frac{dG(\tau)}{d\tau} = \int_0^{\overline{Y}} \left[(1-p)\left(y - e(\tau, y) - \tau\frac{\partial e(\tau, y)}{\partial \tau}\right) + py + ps\left(e(\tau, y) + \tau\frac{\partial e(\tau, y)}{\partial \tau}\right)\right] \frac{1}{\overline{Y}} dy.$$

It is easy to see that $\frac{dG(\tau)}{d\tau} > 0$ since $\frac{\partial e(\tau, y)}{\partial \tau} < 0$ under the assumption of decreasing absolute risk (see Yitzhaki, 1974) and $s < \frac{(1-p)}{p}$.

where $A = \left(\frac{ps}{1-p}\right)^{-\frac{1}{\gamma}}$ and $\tau^* \in (0,1)$ is the tax rate which separates full from partial evasion. Notice that $A > 1$ since $s < \frac{1-p}{p}$. The value of τ^* is

$$\tau^* = \frac{1-\left(\frac{sp}{1-p}\right)^{\frac{1}{\gamma}}}{1+s}, \qquad (3.10)$$

which is obtained rearranging the condition for an interior solution ($e < y$) given by Lemma 9. We can observe that the optimal evasion is a constant proportion $\phi(\tau) \in [0,1]$ of the true income y, where $\phi(\tau)$ can be expressed as

$$\phi(\tau) = \begin{cases} 1 & \text{if} \quad \tau \leq \tau^* \\ \frac{(A-1)(1-\tau)}{\tau(1+As)} & \text{if} \quad \tau > \tau^*. \end{cases} \qquad (3.11)$$

The decision to be a full or a partial evader depends on the level of the tax rate. When the tax rate is small enough, individuals decide to evade all their income because if they are inspected, the penalty that they must pay is not very large. Nevertheless, when the tax rate increases the fine paid becomes also higher and this discourages individuals from evading all their income. Note that $\phi(1) = 0$. This means that the optimal evasion is equal to zero when $\tau = 1$ as a consequence of the assumption $\lim_{I \to 0} U'(I) = \infty$.

To see if the tax revenue displays the shape of a Laffer curve, we need to know how the individuals modify their behavior when the tax rate changes. Obviously, if the individuals become avoiders, they do not pay taxes any longer and this fact has a significative impact on the tax revenue. Basically, we will consider two different values for the tax rate which imply qualitative changes in the individuals' behavior. On the one hand, we have the value τ^* of the tax rate which separates full from partial evasion. This value depends on the policy inspection parameters, s and p, and the parameter γ of the utility function. On the other hand, we have the proportional cost c of avoidance which affects the decision of becoming avoider. In this framework, we consider two possible scenarios: $\tau^* \leq c$ and $\tau^* > c$. The main difference between these two cases is that when $\tau^* \leq c$ the taxpayers are only full evaders for very low values of the tax rate $(\tau \leq \tau^*)$, while they are full evaders for a larger range larger of tax rates whenever $\tau^* > c$.

3.5.1. Case 1: $\tau^* \leq c$

The first step is to calculate the different components of the tax revenue function $G(\tau)$. If $0 \le \tau \le \tau^*$, everybody prefers evading since $\tau^* < c$. Furthermore, as $\tau \le \tau^*$ all the individuals will choose to be full evaders, that is, $e(\tau, y) = y$. Then, the tax revenue will be only composed by the penalties paid by the consumers who are caught evading.

When the tax rate is higher than τ^*, the evaders choose to declare only a part of their true income, that is, $e(\tau, y) < y$. On the other hand, all the individuals prefer being evaders for all $\tau \in [0, c]$. Therefore, we need to know if some individual prefers becoming avoider in the interval $\tau \in (c, 1]$. Observe that for an individual with income y we can calculate which is the value of the tax rate which leaves him indifferent between evading and avoiding. More precisely, this value is such that satisfies the following equality:

$$(1-p)\left[y - \tau y + \tau \frac{(A-1)(1-\tau)}{\tau(1+As)} y\right]^{1-\gamma} + p\left[y - \tau y - s\tau \frac{(A-1)(1-\tau)}{\tau(1+As)} y\right]^{1-\gamma} = (y - cy - k)^{1-\gamma}. \quad (3.12)$$

Solving for τ we get[17]

$$\hat{\tau}(y) = 1 - \left[\frac{(1-c)}{D} - \frac{k}{Dy}\right], \quad (3.13)$$

where

$$D \equiv \left[\left(\frac{1+s}{1+As}\right)\left((1-p)A^{1-\gamma} + p\right)^{\frac{1}{1-\gamma}}\right]. \quad (3.14)$$

Observe that $\hat{\tau}(y)$ is decreasing in y and this means that for high income levels $\hat{\tau}(y)$ will be small because the rich individuals can meet the cost of avoidance more easily. In particular, the first potential avoider is the richest individual, i.e., the one with $y = \overline{Y}$. Therefore, we can find which is the tax rate that leaves the richest individual indifferent between evading or avoiding by substituting $y = \overline{Y}$ into the expression (3.13). Thus, we get

$$\hat{\tau}(\overline{Y}) = 1 - \left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right]. \quad (3.15)$$

The following Lemma ensures that $\hat{\tau}(\overline{Y}) \in (c, 1)$:

Lemma 14. *Assume* $s < \frac{1-p}{p}$*, then*

17 We adopt the innocuous convention that when an individual is indifferent between evading and avoiding, he evades.

(a) $D > 1$

(b) $\hat{\tau}(\overline{Y}) \in (c, 1)$.

Proof. See the Appendix.

Note that the value $\hat{\tau}(\overline{Y})$ is greater than c because the richest individual prefers being evader when $\tau = c$. This implies that, if $\tau^* \le \tau \le \hat{\tau}(\overline{Y})$, all the individuals prefer evading. Thus, as the evasion takes place only partially, the tax revenue comes both from the taxes voluntarily paid and from the fines paid by the inspected individuals.

Finally, when $\hat{\tau}(\overline{Y}) < \tau \le 1$, it is no longer true that the best option for all individuals is to become evaders. While the tax rate is growing the individuals who enjoy higher incomes will tend to prefer avoiding. Therefore, there exists an income $y_P^*(\tau)$ that makes an individual indifferent between being a partial evader and being avoider for a given tax rate. We can calculate the explicit form of this threshold level since $y_P^*(\tau)$ is such that (3.12) holds. The value of $y_P^*(\tau)$ is given by

$$y_P^*(\tau) = \frac{k}{(1-c)-(1-\tau)D}. \tag{3.16}$$

Note that $\tau \in (\bar{\tau}(\overline{Y}), 1]$ ensures that $y_P^*(\tau) < \overline{Y}$. Thus, in this case the tax revenue is only composed by the payments collected from the individuals who do not avoid.

The following Lemma gives us the expression of the function $G(\tau)$:

Lemma 15. *Let* $s < \frac{1-p}{p}$ *and* $\tau^* \le c$. *Then, the total tax revenue will be given by the following three-part function:*

$$G(\tau) = \begin{cases} \frac{1}{2} p\tau(1+s)\overline{Y} & \text{if} \quad 0 \le \tau \le \tau^* \\ \frac{1}{2}\tau(1-\phi(\tau)+p(1+s)\phi(\tau))\overline{Y} & \text{if} \quad \tau^* < \tau \le \hat{\tau}(\overline{Y}) \\ \frac{1}{2\overline{Y}}\tau(1-\phi(\tau)+p(1+s)\phi(\tau))\left(y_P^*(\tau)\right)^2 & \text{if} \quad \hat{\tau}(\overline{Y}) < \tau \le 1. \end{cases}$$

Proof. See the Appendix.

The next proposition describes the Laffer curve.

Proposition 16. Assume that $\tau^* \leq c$ and $s < \frac{1-p}{p}$. Then, the total tax revenue $G(\tau)$ is strictly increasing on $[0, \hat{\tau}, (\overline{Y}))$ and strictly decreasing on $(\hat{\tau}(\overline{Y}), 1]$.

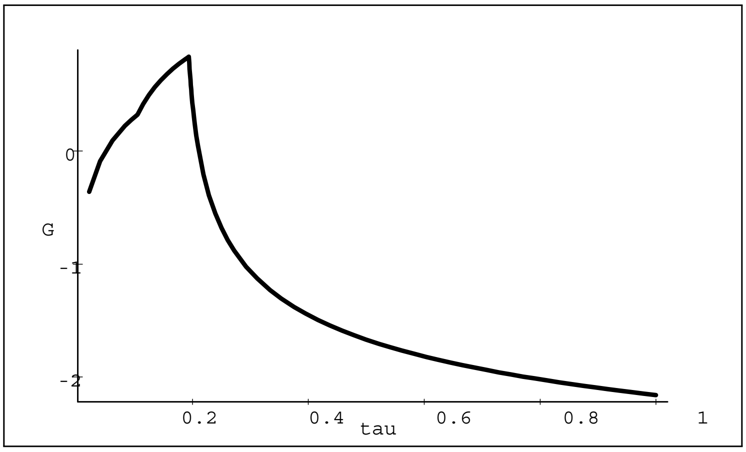
Proof. See the Appendix.

We can illustrate the previous proposition by evaluating the tax rates $\hat{\tau}(\overline{Y})$ given in expression (3.15) and τ^* given in expression (3.10) at $p = 0.1$, $s = 3$, $c = 0.15$, $\overline{Y} = 100$, $\gamma = 2$ and $k = 1$. We get $\tau^* = 0.10566$ and $\hat{\tau}(\overline{Y}) = 0.19376$. Observe that in this case $s < \frac{1-p}{p}$, $\tau^* < c$ and $\hat{\tau}(\overline{Y}) > c$. Figure 3.2 shows the tax revenue for these values.

FIGURE 3.2

THE FUNCTION $G(\tau)$ WHEN THE EVASION TAKES PLACE, USING LOGARITHMIC SCALE AND $\tau^* \leq c$

($p = 0.1$, $s = 3$, $c = 0.15$, $\overline{Y} = 100$, $k = 1$, $\gamma = 2$)



Note that the function $G(\tau)$ is increasing in $\tau \in (0, \hat{\tau}(\overline{Y}))$ since when $\tau < \hat{\tau}(\overline{Y})$ all the individuals prefer being evaders. This means that a higher tax rate implies a higher tax revenue as a consequence of two positive effects. First, the increase in the tax rate causes a raise of the tax revenue accruing from declared income. Second, the individuals declare a higher part of their true income since evaded income is decreasing in τ. When $\tau > \hat{\tau}(\overline{Y})$ not all the individuals want to be evaders. In particular, only the individuals with an income greater than $y_P^*(\tau)$ will evade. Thus, in this case we can distinguish two different effects on tax revenue. On the one hand, there is a positive effect due to the fact that a higher tax rate implies that the individuals who are evaders pay more taxes. On the other hand, we have a negative effect since an increase of the tax rate causes a decrease on $y_P^*(\tau)$ which implies that less people pays taxes. As it has been proved, the second effect offsets the first one and the tax revenue is decreasing in τ for $\tau \in \left(\hat{\tau}\left(\overline{Y}\right),1\right]$. Therefore, the maximum value of the government revenue is reached at the tax rate $\hat{\tau}\left(\overline{Y}\right)$.

Finally, let us point out that for the case $\tau^* < c$ we have been able to obtain a Laffer curve in its strongest form under isoelastic preferences, while this characterization was not available without such a parametrization of the utility function (see Proposition 13). Moreover, we have dispensed with the more vague assumption of "sufficiently" low fixed avoidance cost required in Proposition 13.

3.5.2. Case 2: $\tau^* > c$

Like in case 1 we will calculate the multi-part tax revenue function. We know that if $\tau < \tau^*$, the evader individuals are full evaders, that is, they evade all their true income. Moreover, all the individuals prefer evading rather than avoiding if $\tau \le c$. Therefore, we need to know which will be the behavior of the richest individual when $\tau = \tau^*$. If he prefers being full evader, all the individuals will be full evaders. This situation would be similar as the one analyzed in the case 1. On the other hand, the richest individual could prefer being avoider at $\tau = \tau^*$. The following lemma establishes the condition under which such a circumstance occurs:

Lemma 17. *The richest individual of this economy strictly prefers being an avoider to being a full evader at* $\tau = \tau^*$ *if and only if*

$$k < \overline{Y}\left[(1-c) - \left[(1-p) + pA^{\gamma-1}\right]^{\frac{1}{1-\gamma}}\right]. \qquad (3.17)$$

Proof. See the Appendix.

Observe that the fulfillment of this condition requires small values for both the fixed cost k and the proportional cost c since this makes easy for the richest individual to become avoider at $\tau = \tau^*$.

Assume from now that condition (3.17) holds. In consequence, we can ensure that there exists a tax rate smaller than τ^* leaving the richest individual indifferent between being full evader and avoiding. In particular, this tax rate satisfies:

$$(1-p)\left[\overline{Y}\right]^{1-\gamma} + p\left[\overline{Y} - \tau\overline{Y} - s\tau\overline{Y}\right]^{1-\gamma} = \left(\overline{Y} - c\overline{Y} - k\right)^{1-\gamma}. \qquad (3.18)$$

Solving for τ we obtain

$$\tilde{\tau}\left(\overline{Y}\right)=\frac{1-\left[\frac{1}{p}\left(1-c-\frac{k}{\overline{Y}}\right)^{1-\gamma}-\frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s}. \qquad (3.19)$$

Note that condition (3.17) guarantees that $\tilde{\tau}\left(\overline{Y}\right)\in\left(c,\tau^*\right)$. Then, for $0\le\tau\le\tilde{\tau}\left(\overline{Y}\right)$ we have that all the individuals are full evaders. In consequence, the tax revenue will be equal to the penalties paid by full evaders which have been inspected.

When $\tilde{\tau}\left(\overline{Y}\right)\le\tau\le\tau^*$ it is not longer true that everybody prefers evading to avoiding. We define $y_T^*(\tau)$ as the income for which an individual is indifferent between evading all his true income and avoiding. The value of $y_T^*(\tau)$ is[18]

$$y_T^*(\tau)=\frac{k}{(1-c)-\left((1-p)+p(1-\tau-\tau s)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}.$$

Observe that $k\in\left(0,(1-c)\overline{Y}\right)$ and $\tau\in\left(\tilde{\tau}\left(\overline{Y}\right),\tau^*\right)$ ensure that $y_T^*(\tau)\in 0,\left(\overline{Y}\right)$. Hence, the tax revenue is only composed by the penalties paid by the inspected individuals who prefer being full evaders.

Finally, if $\tau^*<\tau\le 1$, the evaded income is lower than the true income since for $\tau>\tau^*$ the full evaders become partial evaders.[19] In this case the tax revenue collected by the tax authorities comes both from the taxes voluntarily paid and from the penalties paid by the inspected partial evaders.

The following Lemma gives us the expression of the function $G(\tau)$:

Lemma 18. *Let* $s<\frac{1-p}{p}$ *and* $\tau^*>c$. *Then, the total tax revenue will be given by the following three-part function:*

$$G(\tau)=\begin{cases}\frac{1}{2}p\tau(1+s)\,\overline{Y} & \text{if } \quad 0\le\tau\le\tilde{\tau}\left(\overline{Y}\right)\\[2ex] \frac{1}{2\overline{Y}}p\tau(1+s)\left(y_\tau^*(\tau)\right)^2 & \text{if } \quad \tilde{\tau}\left(\overline{Y}\right)<\tau\le\tau^*\\[2ex] \frac{1}{2\overline{Y}}\tau\left(1-\phi(\tau)+p(1+s)\,\phi(\tau)\right)\left(y_P^*(\tau)\right)^2 & \text{if } \quad \tau^*<\tau\le 1.\end{cases}$$

18 The threshold $y_T^*(\tau)$ obtained equating the utility from full evasion to the utility from avoiding.

19 Note that $y_T^*(\tau)=y_P^*(\tau)$ for $\tau=\tau^*$.

Proof. See the Appendix.

The following proposition gives us the results obtained concerning the existence of the Laffer curve:

Proposition 19. Assume $\tau^* > c$ and $s < \frac{1-p}{p}$. The total tax revenue $G(\tau)$ is non monotonic in τ, for $\tau \in [0,1]$ if $k < (1-c)\,\overline{Y}\sqrt{c(1-\phi(c)+p(1+s)\,\phi(c))}$.

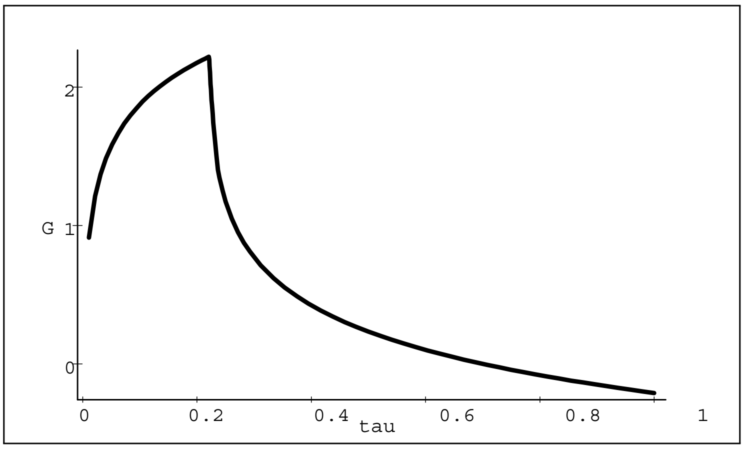
Proof. See the Appendix.

In order to illustrate this result we evaluate the expressions (3.19) and (3.10) at $p = 0.1$, $s = 1.5$, $c = 0.1$, $\overline{Y} = 100$, $\gamma = 2$, $k = 1$. Thus, we have $\tau^* = 0.10566$ and $\hat{\tau}(\overline{Y}) = 0.19376$. Then, the following figure shows the tax revenue function $G(\tau)$ for the previous parameters values:

FIGURE 3.3

THE FUNCTION $G(\tau)$ WHEN THE EVASION TAKES PLACE USING LOGARITHMIC SCALE AND $\tau^* > c$

($p = 0.1$, $s = 1.5$, $c = 0.1$, $\overline{Y} = 100$, $k = 1$, $\gamma = 2$)



When $\tau^* > c$ we only are able to make a characterization of the Laffer curve in the weakest sense like in the general case (see subsection 4.2). However, we do not invoke the assumption of "sufficiently" low fixed avoidance cost to obtain the Laffer curve since in this particular case we have found an explicit formula for the threshold fixed cost of avoidance below which a Laffer curve is obtained.

3.6. A DISCRETE DISTRIBUTION OF INCOME

In the previous setup, we have assumed that the income distribution of the individuals of the economy was uniform. However, we can ask if the previous results about the existence of a Laffer curve relating tax revenues with tax rates would change if the income distribution is modified. Since it is impossible to address such a question with enough generality, in this section we consider a simple discrete income distribution with only three income classes y_1, y_2 and y_3, where $y_1 < y_2 < y_3$. Individuals are distributed into these three classes according to the proportions $\{\alpha_1, \alpha_2, \alpha_3\}$ respectively, where $\alpha_1 + \alpha_2 + \alpha_3 = 1$. In order to simplify the exposition we assume, without loss of generality, that $y_1 = 0$, $y_2 = m$ and $y_3 = 1$, where $0 < m < 1$. The combination of the proportions α_1, α_2 and α_3 as well as the size of m, will allow us to analyze different types of discrete income distributions. Observe that this distribution corresponds to an economy with a high degree of polarization income whereas under a uniform distribution such a polarization was absent. For simplicity we adopt the isoelastic utility function taken in Section 5. The present analysis will be divided into two parts: first we assume that $\tau^* \leq c$ holds and we then will examine the case where $c < \tau^*$.

3.6.1. Case 1: $\tau^* \leq c$

Like in Section 5 we will find the different parts of the tax revenue function. Hence, for $\tau \in [0, \tau^*]$ all the individuals evade all their income, and then the tax revenue is given by the penalties paid by the evaders inspected. In this context the individuals with income $y_1 = 0$ do not contribute to the tax revenue because although they declare all their true income, the taxes paid are zero.[20]

When the current tax rate is greater than τ^*, partial evasion takes place. Moreover, for $\tau \leq c$ all the individuals will want to be evaders. However, when the tax rate is greater than c, the advantage of evasion over avoidance only comes from the fixed cost associated to avoidance. Similarly to the case with an uniform distribution of income, we can find which is the tax rate that leaves the richest individuals indifferent between avoiding and evading. Formally, we obtain this value substituting the value of y_3 into the expression (3.13). This yields

$$\hat{\tau}_{y3} = 1 - \left[\frac{(1-c)-k}{D} \right]. \tag{3.20}$$

For $\tau \in (\tau^*, \hat{\tau}_{y3}]$ everybody is a partial evader. In this case, the tax revenue is composed both by the taxes voluntary paid and by the penalties from inspected individuals. Note that there exists a discontinuity at $\tau = \hat{\tau}_{y3}$ since the richest agents stop paying taxes and the tax revenue then falls. The magnitude of this jump depends on the proportion α_3, since a proportion very large of rich individuals implies that a large fraction of people does not pay their taxes any longer and this fact causes a dramatic decrease of the tax revenue. Hence, for $\tau > \hat{\tau}_{y3}$ only the individuals who have income y_2 continue paying taxes. However, as the tax rate raises the benefit obtained from evading becomes smaller since a high tax rate diminishes the expected utility of an evader. Then, from expression

20 Note from (3.9) that the individuals with $y_1 = 0$ do not evade.

(3.13) the value of the tax rate that will leave the individuals with income y_2 indifferent between evading and avoiding is

$$\hat{\tau}_{y2} = 1 - \left[\frac{(1-c)}{D} - \frac{k}{Dm} \right]. \tag{3.21}$$

Then, $\hat{\tau}_{y2}$ is as the maximum value of the tax rate that has associated a positive value of the tax revenue since for $\tau > \hat{\tau}_{y2}$ the tax revenue becomes zero. Therefore, for $\tau \in (\hat{\tau}_{y3}, \hat{\tau}_{y2}]$ the tax revenue is equal to the taxes voluntarily paid by the individuals with income y_2 and to the fines paid by the inspected ones.

Observe from expression (3.21) that $\hat{\tau}_{y2} < 1$ if $k < (1-c)m$. This is so because when the fixed cost is large enough, the individuals with income y_2 never resort to avoidance and thus, they pay taxes until $\tau = 1$. Obviously, for $\tau > \hat{\tau}_{y2}$ all the individuals except the ones who have income y_1 prefer avoiding and, as a consequence, the tax revenue becomes zero.

The next Lemma gives us the expression of the function $G(\tau)$.

Lemma 20. *Let $s < \frac{1-p}{p}$ and $\tau^* \leq c$. Then, the total tax revenue will be given by the following four-part function:*

$$G(\tau) = \begin{cases} p\tau(1+s)(\alpha_2 m + \alpha_3) & \text{if } \; 0 \leq \tau \leq \tau^* \\ (\alpha_2 m + \alpha_3)[\tau(1-\phi(\tau) + p(1+s)\phi(\tau))] & \text{if } \; \tau^* < \tau \leq \hat{\tau}_{y3} \\ \alpha_2 m[\tau(1-\phi(\tau) + p(1+s)\phi(\tau))] & \text{if } \; \hat{\tau}_{y3} < \tau \leq \hat{\tau}_{y2} \\ 0 & \text{if } \; \hat{\tau}_{y2} < \tau \leq 1. \end{cases}$$

Proof. See the Appendix.

The following proposition summarizes the results concerning the existence of the Laffer curve when different assumptions on the proportions α_i when $i = 1, 2, 3$ are imposed.

Proposition 21. *Assume that $s < \frac{1-p}{p}$ and $k \in (0, m(1-c))$.*

(a) if $\alpha_3 \geq \alpha_2$, then the tax revenue function $G(\tau)$ has a single maximum at $\tau = \hat{\tau}_{y3}$,

(b) if the proportion α_3 of rich individuals is small enough, the tax revenue function $G(\tau)$ has a single maximum at $\tau = \hat{\tau}_{y2}$.

Proof. See the Appendix.

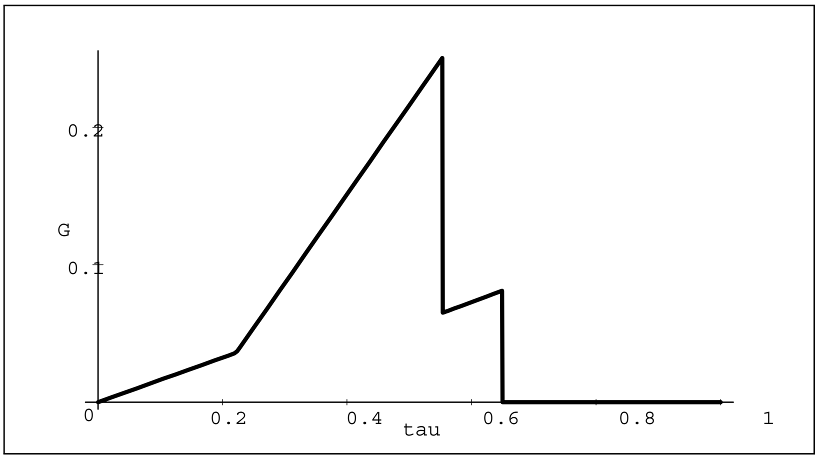
The intuition of result (a) relies on the fact that, if α_3 is large enough, when the tax rate is equal to $\hat{\tau}_{y3}$ many rich individuals stop paying taxes because they prefer avoiding their income. This constitutes a very significant reduction of the collected taxes since the great source of revenues comes from the rich individuals. Although the tax rate rises, the tax revenue can not recover the previous value achieved when $\tau = \hat{\tau}_{y3}$. This is so because the individuals with income y_2, which are the only ones who pay taxes, have not enough income to offset the loss that the avoidance of the rich individuals brings about. Observe that the assumption about the size of the fixed cost k allows the individuals with income y_2 to avoid his income for $\tau > \hat{\tau}_{y2}$, since otherwise when $k \in (m(1-c), (1-c))$ those individuals never take advantage from avoidance for $\tau \in (0,1)$.

Figure 3.4 shows the behavior of the tax revenue described in part (a) of proposition 6.1. The parameters values considered are: $p = 0.1$, $s = 3$, $\gamma = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.4$, $\alpha_3 = 0.4$, $m = 0.7$, $c = 0.25$ and $k = 0.25$.

FIGURE 3.4

THE FUNCTION $G(\tau)$ WHEN THE INCOME DISTRIBUTION IS DISCRETE AND $\alpha_3 > \alpha_2$

($p = 0.1$, $s = 3$, $c = 0.25$, $k = 0.25$, $\gamma = 0.5$, $\alpha_1 = 0.4$, $\alpha_2 = 0.2$, $\alpha_3 = 0.4$, $m = 0.7$**)**



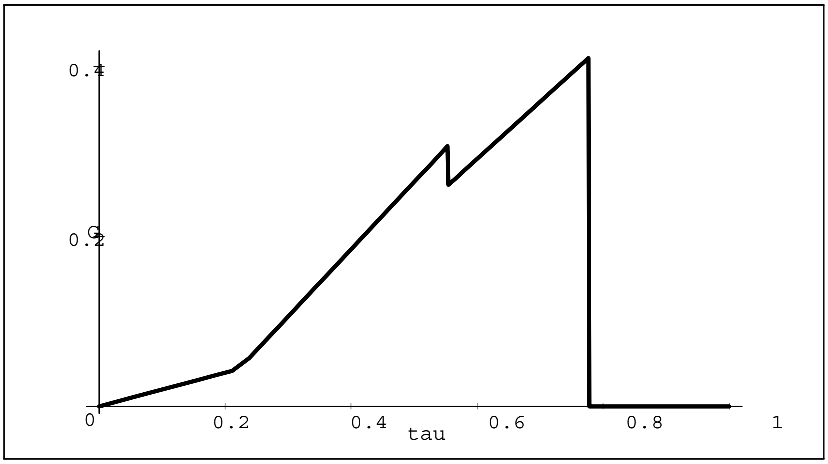
On the other hand, the part (b) of the previous proposition says us that when α_3 is quite small there is almost no difference between $G_2(\tau)$ and $G_3(\tau)$. Although rich individuals are avoiders, their weight on the total income is very small, and the tax revenue quickly offsets the loss caused by the avoidance of the rich individuals. Note that if $\alpha_3 < \alpha_2$ but α_3 and α_2 are close enough, we get the same results as in part (a) of Proposition 21.

Figure 3.5 illustrates, the behavior of the tax revenue described by statement (b) when $p = 0.1$, $s = 3$, $\gamma = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.8$, $\alpha_3 = 0.1$, $m = 0.7$, $c = 0.25$ and $k = 0.25$.

FIGURE 3.5

THE FUNCTION $G(\tau)$ WHEN THE INCOME DISTRIBUTION IS DISCRETE AND $\alpha_3 < \alpha_2$

($p = 0.1$, $s = 3$, $c = 0.25$, $k = 0.25$, $\gamma = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.8$, $\alpha_3 = 0.1$, $m = 0.7$ **)**



Observe that, even if the results stated in Proposition 21 do not depend explicitly on the value of m, this value plays an important role in the analysis of the behavior of the government revenue under alternative income distributions. An increase of m, for given values of α_1, α_2 and α_3, gives rise to two opposite effects on the government revenues. On the one hand, if the value of the parameter m increases, then the individuals having an income equal to y_2 become richer and, therefore, they pay more taxes and contribute more to the revenues raised by the government. On the other hand, from inspection of (3.21) we can check that the value of the tax rate that will leave the individuals with income y_2 indifferent between evading and avoiding decreases as m increases. This implies in turn that the government is not going to get revenues for lower values of the tax rate.

In order to get a more precise intuition on the role played by the parameter m, let us consider an income distribution where α_1 and α_3 are small and α_2 is large. In this context, when m is close enough to zero one obtains an income distribution where a vast majority of individuals is poor. In this case, the government could set very high tax rates since poor individuals will keep paying taxes since they could not afford the avoidance costs. This kind of fiscal policy ends up being very regressive since the individuals that contribute to the increase in government revenues are only the ones having lower income. However, when m is close to 1 and the vast majority of individuals is thus quite rich, the government should set tax rates not very high so as to prevent individuals from avoiding. Obviously, since now the individuals enjoy a higher income, the value of the government revenue is generically higher than in the previous case displaying high tax rates with poor individuals.

Finally, observe that when the income distribution function is discrete, the existence of the Laffer curve in the weak sense is always guaranteed by the non-continuity of the function $G(\tau)$.

3.6.2. Case 2: $\tau^* > c$

The results obtained in this case will depend on the behavior that individuals adopt at $\tau = \tau^*$. More specifically, we will have the following scenarios according to the values of the parameters:

Scenario A

$$0 < c < \tilde{\tau}_{y3} < \tilde{\tau}_{y2} < \tau^* < 1 \qquad \text{when} \qquad k < m\Phi$$

Scenario B

$$0 < c < \tilde{\tau}_{y3} < \tau^* < \tilde{\tau}_{y2} < 1 \qquad \text{when} \qquad m\Phi < k < \Phi,$$

where

$$\Phi = \left[1 - c - \left(1 - p + pA^{\gamma - 1}\right)^{\frac{1}{1-\gamma}}\right],$$

$$\tilde{\tau}_{y3} = \frac{1 - \left[\frac{1}{p}(1 - c - k)^{1-\gamma} - \frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s},$$

and

$$\tilde{\tau}_{y2} = \frac{1 - \left[\frac{1}{p}\left(1 - c - \frac{k}{m}\right)^{1-\gamma} - \frac{(1-p)}{p}\right]^{\frac{1}{1-\gamma}}}{1+s}. \qquad (3.22)$$

The values of $\tilde{\tau}_{y3}$ and $\tilde{\tau}_{y2}$ have been obtained substituting $y_3 = 1$ and $y_2 = m$ in expression (3.19).

Scenario A presents a situation where the fixed cost k is low enough to induce both the richest individuals and the individuals with income m to prefer avoiding at $\tau = \tau^*$. Using the same arguments as in Sections 5 it is easy to see that for $\tau \in \left[0, \tilde{\tau}_{y3}\right]$ all individuals are full evaders. For $\tau \in \left(\tilde{\tau}_{y3}, \tilde{\tau}_{y2}\right]$ the richest individuals avoid their income while the rest of the individuals still are full evaders. Finally, nobody pays taxes for $\tau \in \left(\tilde{\tau}_{y2}, 1\right]$. Hence, the tax revenue function is given by

$$G(\tau) = \begin{cases} p\tau(1+s)(\alpha_2 m + \alpha_3) & \text{if} \quad 0 \le \tau \le \tilde{\tau}_{y3} \\ p\tau(1+s)\,\alpha_2 m & \text{if} \quad \tilde{\tau}_{y3} < \tau \le \tilde{\tau}_{y2} \\ 0 & \text{if} \quad \tilde{\tau}_{y2} < \tau \le 1. \end{cases}$$

In scenario B, the fixed cost is not low enough for allowing the individuals with income m to avoid their income when $\tau = \tau^*$, then they evade all their true income. Then, using the arguments seen in the previous sections it is easy to see that for $\tau \in \left[0, \tilde{\tau}_{y3}\right]$ everybody will evade. For

$\tau \in \left(\tilde{\tau}_{y3}, \tau^*\right]$ the richest individuals (with income y_3) are avoiders and the individuals with income m are full evaders, for $\tau \in \left(\tau^*, \hat{\tau}_{y2}\right]$ the individuals with income m still being evaders but now they are partial evaders. Finally, the tax revenue is zero for $\tau \in \left(\hat{\tau}_{y2}, 1\right]$ because the only individuals who do not avoid their income (individuals with income y_1) do not even pay taxes. Summarizing, we have that the tax revenue function is the following four-part function:

$$G(\tau) = \begin{cases} p\tau(1+s)(\alpha_2 m + \alpha_3) & \text{if} \quad 0 \le \tau \le \tilde{\tau}_{y3} \\ p\tau(1+s)\,\alpha_2 m & \text{if} \quad \tilde{\tau}_{y3} < \tau \le \tau^* \\ \alpha_2 m\left[\tau(1-\phi(\tau) + p(1+s)\,\phi(\tau))\right] & \text{if} \quad \tau^* < \tau \le \hat{\tau}_{y2} \\ 0 & \text{if} \quad \hat{\tau}_{y2} < \tau \le 1. \end{cases}$$

The main result concerning the existence of the Laffer curve both in scenario A and in scenario B is that the Laffer curve always exists, and its specific form depends on the values taken by the proportions α_1, α_2 and by the income m. Next proposition summarizes the main results about the Laffer curve under the two scenarios considered above:

Proposition 22. *Assume that* $s < \frac{1-p}{p}$. *Then,*

(a) if $k < m\Phi$ *and the income* m *is large enough, the tax revenue function achieves its unique maximum at* $\tau = \tilde{\tau}_{y3}$,

(b) if $k < m\Phi$ *and the proportion* α_3 *is small enough, the tax revenue function achieves its unique maximum at* $\tau = \tilde{\tau}_{y2}$,

(c) if $m\Phi < k < \Phi$ *and the proportion* α_3 *is large enough, the tax revenue function achieves its unique maximum at* $\tau = \tilde{\tau}_{y3}$,

(d) if $m\Phi < k < \Phi$ *and the proportion* α_3 *is small enough, the tax revenue function achieves its unique maximum at* $\tau = \tilde{\tau}_{y2}$.

Proof. See the Appendix.

The intuition behind the statement (a) is quite clear because, if m is close to one, it means that the individuals with income y_2 are rather rich and in consequence they will resort to tax avoidance almost for the same tax rates as the richest individuals. Thus, the tax revenue becomes zero for $\tau > \tilde{\tau}_{y2}$ when $\tilde{\tau}_{y2} \in \left(\tilde{\tau}_{y3}, \tilde{\tau}_{y3} + \varepsilon(m)\right)$, where ε is decreasing in m and $\lim_{m \to 1} \varepsilon(m) = 0$. Hence, for m large enough we will have that the tax revenue will be zero before the penalties paid by the inspected individuals with income y_2, can offset the loss in the tax revenue caused by the richest individuals. The intuition of the statements (b), (c) and (d) is supported by the role that the proportion α_3 of richest individuals plays. If α_3 is large enough the loss in the tax revenue can not be offset by the resources obtained from the individuals with income m. When α_3 is small enough, the previous argument works in the opposite direction since the loss due to the tax avoidance by the richest individuals is very small and it can be offset by the payments made by the individuals with income y_2.

Finally, we have to point out that the scenarios considered when $\tau^* > c$ do not constitute a good approach of the situation in the real world since the available data of tax evasion reveals that tax evasion takes place in a partial way, i.e., $e < y$.

3.7. CONCLUSIONS AND EXTENSIONS

We have shown that the possibility of choosing between avoiding and evading brings about a tax revenue function exhibiting the shape of a Laffer curve. That is, the relationship between tax rates and government revenue is non-monotonic. We have carried out the analysis in a partial equilibrium context where the individuals have the same utility function but differ in their incomes. In all the scenarios studied we have found that the tax revenue always displays a Laffer curve under some conditions. This fact has to be taken into account when the different governments design the fiscal policies because, when the tax avoidance phenomenon is present, raising the tax rate might not result in an increase of the tax revenue. Moreover, when the costs associated with tax avoidance are low, to set high tax rates and to implement a strong anti-evasion policy is not only ineffective but also regressive because all the rich individuals will avoid their incomes and will pay no taxes. Under this policy only the poorest individuals will pay taxes (and/or penalties associated with tax evasion) since they cannot afford the cost of avoidance. In fact, a government could reduce the negative impact on tax revenues accruing from the possibility of tax avoidance by implementing the kind of policies analyzed by Chu (1990). He proposes a new policy called FATOTA which allows a target group of corporations to choose between two options: to pay a fixed amount of taxes set by the tax authorities and thus to be exempted from tax inspection, or to pay only an amount chosen by the corporation but be subject to a positive probability of being audited. This type of policies would become effective provided the fixed amount to be paid by the potential avoiders were smaller than the costs associated with avoidance.

Some extensions of the present work are possible. We comment on some of them. We could introduce a progressive tax function in our framework and to analyze the effects on government tax revenue of a modification in the progressivity of the tax function. The result one should expect is that a higher degree of progressivity will stimulate the avoidance of rich individuals and, therefore, the decreasing part of the Laffer curve will appear at lower average tax rates.

We could also consider other continuous income distributions, like the lognormal or the Pareto ones, which could better fit the empirical income distributions. Nevertheless, such functional form would prevent us from getting explicit results and we should rely instead on simulations.

Finally, in our model all the individuals have the same utility function and they only differ in their income. It could be interesting to consider heterogenous preferences. To this end, we could assume a distribution on the relevant parameters characterizing the indexes of risk aversion of the individuals.

3.8. APPENDIX

Proof of Lemma 9. Since the function $u(e)$ defined in (3.1) is strictly concave, we obtain the corner solution $e = 0$ whenever $u'(0) \le 0$. This weak inequality is in fact equivalent to $s \ge \frac{1-p}{p}$. Notice that when $\tau(1+s) \ge 1$, it is impossible to have $e = y$ since, then, the income of an inspected individual $y - \tau(1+s)\,y$ cannot be non-positive as follows from the Inada condition. For $\tau(1+s) < 1$, to obtain the corner solution $e = y$, we need that $u'(y) \ge 0$. This inequality becomes

$$s \le \left(\frac{1-p}{p}\right)\frac{U'(y)}{U'(y - \tau y - s\tau y)}.$$

Proof of Lemma 10. For $0 \le \tau \le \bar{\tau}(\overline{Y})$ the tax revenue is composed by the taxes that all the individuals pay. Thus, we have

$$G_1(\tau) = \int_0^{\overline{Y}} \tau y \frac{1}{\overline{Y}} dy .$$

For $\bar{\tau}(\overline{Y}) < \tau \le 1$ the tax revenue is given by the total taxes paid for the individuals who are honest. That is

$$G_2(\tau) = \int_0^{\frac{k}{\tau - c}} \tau y \frac{1}{\overline{Y}} dy .$$

Performing the integrals $G_1(\tau)$ where $i = 1, 2$, we obtain the expression appearing in the statement of the Lemma.

Proof of Proposition 11. Differentiating $G_i(\tau)$ respect to τ we have $\frac{dG_1}{d\tau} = \frac{\overline{Y}}{2}$ which is unambiguously positive. On the other hand, differentiating $G_2(\tau)$ respect to τ we obtain

$$\frac{dG_2}{d\tau} = \frac{k^2(\tau + c)}{2\overline{Y}(\tau - c)^3},$$

which is unambiguously negative. Obviously, the maximum value achieved by $G(\tau)$ is just the kink point $\tau = c + \frac{k}{\overline{\overline{Y}}}$.

Proof of Lemma 12. The proof adapts some of the arguments in Chu (1990)

Step 1. We will prove that $V(\tau, y)$ and $U(y - cy - k)$ intersect at least once.

As when the true income tends to zero, the optimal evasion also tends to zero, we have that at least for $y < \frac{k}{\tau - c}$, $U(y - cy - k) < V(\tau, y)$, since k is positive and by assumption $\tau > c$. Then, we only have to see that there exists one income such that $U(y - cy - k) > V(\tau, y)$ for a given τ. Consider the level evasion $e_0(\tau, y) = \frac{(1-\tau)\, y}{\tau s}$, which is less than y since we were assuming that $(1+s)\tau > 1$ holds. It can be seen that

$$y - \tau y + \tau e_0 = \frac{(1+s)(1-\tau)\, y}{s}$$

and

$$y - \tau y - \tau s e_0 = 0 .$$

Therefore, we get that

$$u'(e_0) = (1-p)\, U'\left(\frac{(1+s)(1-\tau)\, y}{s}\right)\tau - pU'(0)\, s\tau .$$

Since, by assumption, $\lim_{I \to 0} U'(I) = \infty$ we have

$$\lim_{I \to 0} u'(e_0) = -\infty ,$$

which implies that $\hat{e}(\tau, y) < e_0(\tau, y)$, where $\hat{e}(\tau, y)$ is the optimal evasion given in Lemma 9. Obviously, this means that

$$y - \tau y + \tau \hat{e}(\tau, y) < y - \tau y + \tau e_0(\tau, y). \qquad (3.23)$$

Now define y_0 as the income that makes equal the net income of avoiding with the net income of evading for $e_0(\tau, y)$ when the inspection does not occurs. By definition we have

$$y_0 - cy_0 - k = y_0 - \tau y_0 + \tau e_0(\tau, y_0).$$

After rearranging terms we have that the value y_0 becomes

$$y_0 = \frac{sk}{s(1-c)-(s+1)(1-\tau)}.$$

Condition $(1+s)\tau > 1+sc$ guarantees that $y_0 > 0$. Then, for $y > y_0$ we get

$$y\left[(1-c)-\frac{(s+1)(1-\tau)}{s}\right] > y_0\left[(1-c)-\frac{(s+1)(1-\tau)}{s}\right] = k.$$

Hence, we have

$$y(1-c)-k > \frac{y(s+1)(1-\tau)}{s} = y - \tau y + \tau e_0(\tau, y).$$

Finally by (3.23) it holds that

$$y(1-c)-k > y - \tau y + \tau\hat{e}(\tau, y).$$

Therefore for $y > y_0$ the following inequality must also hold:

$$y(1-c)-k > y - \tau y + \tau\hat{e}(\tau, y) > V(\tau, y).$$

Summarizing, we have that the individuals with income $y > y_0$ prefer avoiding to evading while the individuals with an income y small enough prefer evading. This guarantees then, that $V(\tau, y)$ and $U(y - cy - k)$ intersect at least once.

Step 2. We will prove that $V(\tau, y)$ and $U(y - cy - k)$ intersect only once.

Let be y^* any intersection of $V(\tau, y)$ and $U(y - cy - k)$. This implies that $V(\tau, y^*) = U(y^* - cy^* - k)$ and consequently we have that

$$y^* - \tau y^* - s\tau\hat{e}(\tau, y^*) < y^* - cy^* - k < y^* - \tau y^* + \tau\hat{e}(\tau, y^*)$$

must hold. This inequality implies that

$$U'(y^* - cy^* - k) > U'(y^* - \tau y^* + \tau\hat{e}(\tau, y^*)), \tag{3.24}$$

since the function $U(\cdot)$ is concave. By the envelop theorem we get that

$$\frac{\partial V(\tau, y^*)}{\partial y} = (1-p)\,U'(y^* - \tau y^* + \tau\hat{e}(\tau, y^*)) + pU'(y^* - \tau y^* - s\tau\hat{e}(\tau, y^*))(1-\tau-\tau s),$$

which can be simplified using (3.2) as

$$\frac{\partial V(\tau, y^*)}{\partial y} = \left(\frac{(1-p)(1+s)(1-\tau)}{s}\right)U'(y^* - \tau y^* + \tau\hat{e}(\tau, y^*)). \tag{3.25}$$

The condition $(1+s)\tau > 1+sc$, implies that $(1+s)\tau > 1$ also holds, and this is a sufficient condition for ensuring that $\left(\frac{(1-p)(1+s)(1-\tau)}{s}\right) < 1$. Thus, using (3.24) and (3.25) we get that

$$U'\left(y^* - cy^* - k\right) > \frac{\partial V\left(\tau, y^*\right)}{\partial y}. \quad (3.26)$$

It is immediate to check that $V(\tau, y)$ is a concave function, then the inequality (3.26) means that $U(\cdot)$ and $V(\cdot)$ intersect once at most.

Proof of Proposition 13. When the tax rate is zero the tax revenue is also zero because the government can not collect neither taxes nor fines. For $\tau \in (0, c]$ all the individuals prefer being evaders, therefore the government gets some revenue at least from the fines collected by the inspection. Moreover, for $\tau \in (0, c]$, $G(\tau)$ is increasing in τ since the optimal evasion is decreasing in τ (see Yitzhaki, 1974). In particular evaluating the tax revenue function given by (3.4) at $\tau = c$, we have that $G(c) > 0$. Nevertheless, we do not know which is the exact value of $G(c)$ because, although all the individuals are evaders, we ignore if they evade all his income or only a part of it. This decision depends on the own income, the tax rate and the values of p and s. Our proof leans on showing that $G(c) > G(1)$, because this implies that the tax revenue function can not be monotonic. Let us first investigate what happens with the optimal evasion when $\tau = 1$. From the expression (3.2), we have that

$$U'(y - \tau y - s\tau e(\tau, y)) = \frac{(1-p)}{ps} U'(y - \tau y + \tau e(\tau, y)).$$

We know that $y - \tau y + \tau e(\tau, y) > 0$ for $\tau \in (0, 1)$ so that condition $\lim_{I \to 0} U'(I) = \infty$ implies that

$$y - \tau y - s\tau e(\tau, y) > 0.$$

Rearranging the last inequality we have that

$$0 \leq e(\tau, y) < \frac{(1-\tau)\, y}{\tau s}. \quad (3.27)$$

So that,

$$0 \leq \lim_{\tau \to 1} e(\tau, y) \leq \lim_{\tau \to 1} \frac{y(1-\tau)}{\tau s} = 0.$$

We see that when the tax rate tends to one the optimal tax evasion tends to zero or, in other words, the individuals tell the truth independently of their respective incomes. This result allows us to say that only the individuals who can not face up to the fixed cost of avoidance will pay taxes, whereas the others will be avoiders. Thus, the tax revenue when $\tau = 1$ will be

$$G(1) = \lim_{\tau \to 1} \int_0^{y^*(\tau)} [(1-p)\,\tau(y - e(\tau, y)) + p\tau(y + se(\tau, y))] \frac{1}{\overline{Y}} dy = \int_0^{\frac{k}{1-c}} y \frac{1}{\overline{Y}} dy = \frac{k^2}{2\overline{Y}(1-c)^2},$$

since $\lim_{\tau \to 1} y^*(\tau) = \lim_{\tau \to 1} y_H^*(\tau) = \frac{k}{(1-c)}$ when $e(\tau, y) = 0$. Note that $\lim_{\tau \to 1} y^*(\tau) \in (0, \overline{Y})$ whenever $k \in (0, (1-c)\overline{Y})$. As $G(c)$ does not depend on k, we can make $G(c) > G(1)$ for a small enough value of k.

Proof of Lemma 14.

(a) Fix γ and then consider D defined in (3.14) as a function of s and p. After tedious differentiation, it can be proved that $\frac{\partial D}{\alpha s} = 0$ and $\frac{\partial D}{\partial p} = 0$ only when $s = \frac{1-p}{p}$. Moreover, the Hessian of D evaluated at $s = \frac{1-p}{p}$ is

$$H = \begin{pmatrix} \frac{p^3}{\gamma(1-p)} & \frac{p}{\gamma(1-p)} \\ \frac{p}{\gamma(1-p)} & \frac{1}{\gamma p(1-p)} \end{pmatrix},$$

which is positive semi-definite. Finally, we get that $D(s, p) = 1$ whenever $s = \frac{1-p}{p}$. We conclude thus that $D(s, p) > 1$ for all $s \neq \frac{1-p}{p}$. In particular, $D > 1$ for $ps < 1-p$.

(b) To see that $c < \hat{\tau}(\overline{Y}) < 1$ we only have to prove that $0 < \left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] < 1$. The assumption that $k \in (0, (1-c)\overline{Y})$ ensures that $\left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] > 0$, while the part (a) of this lemma implies that $\left[\frac{(1-c)}{D} - \frac{k}{D\overline{Y}}\right] < 1$.

Proof of Lemma 15. For $0 \le \tau \le \tau^*$ the tax revenue is only composed by the fines paid by the inspected individuals. Thus, we have

$$G_1(\tau) = \int_0^{\overline{Y}} p\tau(1+s)\, y \frac{1}{\overline{Y}} dy.$$

For $\tau^* \le \tau \le \hat{\tau}(\overline{Y})$ the tax revenue is given by the taxes and penalties paid for all the individuals. That is

$$G_2(\tau) = \int_0^{\overline{Y}} \left[(1-p)\,\tau(1-\phi(\tau)) + p\tau(1+s\,\phi(\tau))\right] y \frac{1}{\overline{Y}} dy = \int_0^{\overline{Y}} \left[\tau(1-\phi(\tau)) + p\tau(1+s)\,\phi(\tau)\right] y \frac{1}{\overline{Y}} dy .$$

Finally, for $\hat{\tau}(\overline{Y}) < \tau \le 1$ the tax revenue is

$$G_3(\tau) = \int_0^{y_P^*(\tau)} \left[(1-p)\tau(1-\phi(\tau)) + p\tau(1+s\,\phi(\tau))\right] y \frac{1}{\overline{Y}} dy = \int_0^{y_P^*(\tau)} \left[\tau(1-\phi(\tau)) + p\tau(1+s)\,\phi(\tau)\right] y \frac{1}{\overline{Y}} dy ,$$

since the tax authorities can only collect payments from the individuals who do not avoid.

Performing the integrals $G_i(\tau)$ where $i = 1, 2, 3$ we obtain the expression appearing in the statement of the Lemma.

Proof of Proposition 16. We prove the proposition by stating two claims.

Claim 1: $\frac{dG_1(\tau)}{d\tau} > 0$ for $\left(0, \tau^*\right]$ and $\frac{dG_2(\tau)}{d\tau} > 0$ for $\left(\tau^*, \hat{\tau}(\overline{Y})\right]$.

Calculating $\frac{dG_1(\tau)}{d\tau}$ we have

$$\frac{dG_1(\tau)}{d\tau} = \frac{p(1+s)\overline{Y}}{2} > 0 ,$$

which is unambiguously positive. In a similar way, we obtain

$$\frac{dG_2(\tau)}{d\tau} = \frac{\overline{Y}}{2}\left[(1-\phi(\tau) + p(1+s)\,\phi(\tau)) + \tau(p(1+s)-1)\frac{d\phi(\tau)}{d\tau}\right]. \tag{3.28}$$

From expression (3.11) we have $\frac{d\phi(\tau)}{\tau} = -\alpha\frac{1}{\tau^2} < 0$, where $\alpha > 0$ is given by

$$\alpha = \frac{(A-1)}{(1+As)} . \tag{3.29}$$

Note that the first term inside the square brackets of (3.28) is positive since the parameters satisfy the interior condition $s < \frac{1-p}{p}$. On the other hand, the second term inside the square brackets is also positive since $\frac{d\phi(\tau)}{d\tau} < 0$ and $(p(1+s)-1) < 0$. Then, $G_2(\tau)$ is also increasing in the tax rate.

Claim 2: $\frac{dG_3(\tau)}{d\tau} < 0$ for $\left(\hat{\tau}(\overline{Y}), 1\right]$.

We have the following derivative:

$$\frac{dG_3(\tau)}{d\tau}=\frac{1}{\overline{Y}}\left[\left(\tau(1-\phi(\tau))+\tau p(1+s)\,\phi(\tau)\right)\frac{dy_P^*(\tau)}{d\tau}y^*(\tau)\right]+$$
$$+\frac{1}{\overline{Y}}\left[\frac{1}{2}\left(y^*(\tau)\right)^2\left(1-\phi(\tau)+p(1+s)\,\phi(\tau)+\tau(p(1+s)-1)\frac{d\phi(\tau)}{d\tau}\right)\right]. \tag{3.30}$$

To evaluate the sign of the previous expression we need to calculate $\frac{dy^*(\tau)}{d\tau}$. Using expression (3.16), we get

$$\frac{dy_P^*(\tau)}{d\tau}=-\frac{D}{E}y_P^*(\tau), \tag{3.31}$$

where $E=[(1-c)-(1-\tau)D]$. Since $y_P^*(\tau)>0$, $D>0$ and $E>0$ for $\tau\geq\hat{\tau}(\overline{Y})$, we can conclude that $\frac{dy_P^*(\tau)}{d\tau}<0$. Then, plugging (3.31) in (3.30) and rearranging terms, we have

$$\frac{dG_3(\tau)}{d\tau}=\frac{\left(y_P^*(\tau)\right)^2}{\overline{Y}}\left[\left[\tau(1+\alpha(1-p(1+s)))-\alpha(1-p(1+s))\right]\left(\frac{-D}{E}\right)\right]+\frac{\left(y_P^*(\tau)\right)^2}{\overline{Y}}\left[\frac{1}{2}(1+\alpha(1-p(1+s)))\right].$$

Note that the sign of the previous derivative depends only on the sign of the expression into the brackets. Define

$$F\equiv\left(\tau(1+\alpha(1-p(1+s)))-\alpha(1-p(1+s))\right)\left(\frac{-D}{E}\right)+\frac{1}{2}(1+\alpha(1-p(1+s))),$$

which can be rewritten as

$$F\equiv\frac{-D[1-\alpha(1-ps)+\tau(1+\alpha(1-p(1+s)))]+(1-c)(1+\alpha(1-p(1+s)))}{2E}$$

The sign of F is the same as that of its numerator since $E>0$ for $\tau\geq\hat{\tau}(\overline{Y})$. Therefore if our objective is to prove that $\frac{dG_3(\tau)}{d\,\tau}<0$, we have to prove that the following inequality holds:

$$D[1-\alpha(1-p(1+s))+\tau(1+\alpha(1-p(1+s)))]>(1-c)\,(1+\alpha(1-p(1+s))). \tag{3.32}$$

Note that the tax revenue $G_3(\tau)$ is obtained when the tax rate moves between $\hat{\tau}(\overline{Y})<\tau\leq 1$. Then, if

$$D[1-\alpha(1-p(1+s))+\hat{\tau}(1+\alpha(1-p(1+s)))]>(1-c)\,(1+\alpha(1-p(1+s))), \tag{3.33}$$

we can guarantee that inequality (3.32) also holds for $\tau\in\left[\hat{\tau}(\overline{Y}),1\right]$ since the left term of inequality (3.32) is increasing in τ. Thus, substituting

$$\hat{\tau}(\overline{Y}) = 1 + \frac{k}{D\overline{Y}} - \frac{(1-c)}{D}$$

into (3.33) we get

$$D\left[1-\alpha(1-p(1+s)) + \left(1+\frac{k}{D\overline{Y}} - \frac{(1-c)}{D}\right)(1+\alpha(1-p(1+s)))\right] > (1-c)(1+\alpha(1-p(1+s))).$$

Rearranging and simplifying, we obtain

$$2D + \frac{k}{\overline{Y}}(1+\alpha(1-p(1+s))) > 2(1-c)(1+\alpha(1-p(1+s))). \tag{3.34}$$

According to lemma 14, $D > 1$. Hence,

$$(1-c)(1+\alpha(1-ps)(1-p(1+s))) < 1, \tag{3.35}$$

becomes a sufficient condition for (3.34). Rearranging the inequality (3.35) we have

$$\alpha(1-c) - \alpha p(1+s)(1-c) - c < 0. \tag{3.36}$$

It is easy to see that inequality (3.36) holds if

$$c > \frac{\alpha}{1+\alpha}. \tag{3.37}$$

Using (3.10) and (3.29) and rearranging terms, we get that the sufficient condition (3.37) becomes simply $c > \tau^*$, and that is always true by assumption. Hence, it follows that $\frac{dG_3(\tau)}{d\tau} < 0$.

Proof of Lemma 17. The richest individual wants to be avoider if the utility from avoidance is greater than the expected indirect utility from full evasion at $\tau = \tau^*$. This is

$$(1-p)\left[\overline{Y}\right]^{1-\gamma} + p\left[\overline{Y} - \tau^*\overline{Y} - s\tau^*\overline{Y}\right]^{1-\gamma} < \left(\overline{Y} - c\overline{Y} - k\right)^{1-\gamma}. \tag{3.38}$$

Rearranging the previous inequality we obtain that $\frac{k}{\overline{Y}} < (1-c) - \left[(1-p) + pA^{\gamma-1}\right]^{\frac{1}{1-\gamma}}$ has to hold for getting (3.38).

Proof of Lemma 18. For $0 \leq \tau \leq \tilde{\tau}(\overline{Y})$ the tax revenue is only composed by the fines paid by the inspected individuals. Thus, we have

$$G_1(\tau) = \int_0^{\overline{Y}} p\tau(1+s)\, y \frac{1}{\overline{Y}}\, dy.$$

For $\tilde{\tau}(\overline{Y}) \leq \tau \leq \tau^*$ the tax revenue is given by the penalties paid for the individuals which prefer being full evaders rather avoiders. That is

$$G_2(\tau) = \int_0^{y_T^*(\tau)} p(1+s)\, y \frac{1}{\overline{Y}}\, dy \,.$$

Finally, for $\tau^* < \tau \leq 1$ the tax revenue collected by the tax authorities comes both from the taxes voluntarily paid and from the penalties paid by the inspected partial evaders, that is

$$G_3(\tau) = \int_0^{y_P^*(\tau)} \left[(1-p)\tau(1-\phi(\tau)) + p\tau(1+s\,\phi(\tau))\right] y \frac{1}{\overline{Y}}\, dy = \int_0^{y_P^*(\tau)} \left[\tau(1-\phi(\tau)) + p\tau(1+s)\,\phi(\tau)\right] y \frac{1}{\overline{Y}}\, dy \,.$$

Performing the integrals $G_i(\tau)$ where $i = 1, 2, 3$ we obtain the expression appearing in the statement of the Lemma.

Proof of Proposition 19. We know that $G(0) = 0$ and then, we only have to prove that $G(1)$ is less than some positive value taken by the function $G(\tau)$ on $(0, 1)$. Computing the value of $G(1)$ we obtain

$$G(1) = \frac{1}{2\overline{Y}} \frac{k^2}{(1-c)^2} > 0 \,,$$

that can be as small as we want, taking values of k low enough. Similarly, evaluating the tax revenue function $G(\tau)$ at $\tau = c$, we have

$$G(c) = \frac{1}{2} c(1-\phi(c) + p(1+s)\,\phi(c))\overline{Y} > 0 \,.$$

Thus, it is easy to check that $G(c) > G(1)$, for $k < (1-c)\,\overline{Y}\sqrt{c(1-\phi(c)+p(1+s)\,\phi(c))}$.

Proof of Lemma 20. For $\tau \in [0, \tau^*]$ all the individuals evade all their income, and then the tax revenue is given by the penalties paid by the evaders inspected. Formally, this is

$$G_1(\tau) = \alpha_1 p\tau(1+s)\, y_1 + \alpha_2 p\tau(1+s)\, y_2 + \alpha_3 p\tau(1+s)\, y_3 \,.$$

Simplifying the last expression we have

$$G_1(\tau) = p\tau(1+s)\,(\alpha_2 m + \alpha_3) \,.$$

For $\tau \in (\tau^*, \hat{\tau}_{y3}]$ everybody is a partial evader. In this case, the tax revenue is composed both by the taxes voluntary paid and by the penalties from inspected individuals. After rearranging the terms the tax revenue is given by

$$G_2(\tau) = (\alpha_2 m + \alpha_3)\left[\tau(1-\phi(\tau) + p(1+s)\,\phi(\tau))\right].$$

Finally for $\tau \in (\hat{\tau}_{y3}, \hat{\tau}_{y2}]$, the tax revenue is equal to the taxes voluntarily paid by the individuals with income y_2 and to the fines paid by the inspected ones. This is

$$G_3(\tau) = \alpha_2 m[\tau(1-\phi(\tau)+p(1+s)\,\phi(\tau))].$$

Proof of Proposition 21.

(a) We need to prove that $G(\hat{\tau}_{y3})$ is the maximum value that the tax revenue function $G(\tau)$ can achieve. Computing $\frac{dG_1(\tau)}{d\tau}$ we have

$$\frac{dG_1(\tau)}{d\tau} = p(1+s)\,(\alpha_2 m + \alpha_3) > 0\,.$$

Similarly differentiating $G_2(\tau)$, we obtain after rearranging the terms

$$\frac{dG_2(\tau)}{d\tau} = (\alpha_2 m + \alpha_3)\left[1+\frac{(A-1)\,(1-p(1+s))}{1+sA}\right],$$

which is unambiguously positive. Then, for $\tau \in (0, \hat{\tau}_{y3}]$, we have that $G_2(\hat{\tau}_{y3})$ is the highest value of the tax revenue.

On the other hand, if we compute $\frac{dG_3(\tau)}{d\tau}$ we obtain that

$$\frac{dG_3(\tau)}{d\tau} = \alpha_2 m\left[1+\frac{(A-1)\,(1-p(1+s))}{1+sA}\right] > 0\,.$$

Then, we only have to prove that $G_2(\hat{\tau}_{y3}) > G_3(\hat{\tau}_{y2})$. Evaluating these expressions, we get

$$G_2(\hat{\tau}_{y3}) = (\alpha_2 m + \alpha_3)\,\Psi(\hat{\tau}_{y3}),$$

and

$$G_3(\hat{\tau}_{y2}) = \alpha_2 m\,\Psi(\hat{\tau}_{y2}),$$

where $\Psi(\tau) = [\tau(1-\phi(\tau)) + p\tau(1+s)\,\phi(\tau)]$. Comparing these two expressions is straightforward to see that, if $\alpha_3 \geq \alpha_2$ we only have to prove that

$$\left[\frac{m[\Psi(\hat{\tau}_{y2}) - \Psi(\hat{\tau}_{y3})]}{\Psi(\hat{\tau}_{y3})}\right] < 1,$$

to show that $G_2(\hat{\tau}_{y3}) > G_3(\hat{\tau}_{y2})$. Then, taking expressions (3.20) and (3.21) and plugging then into $\Psi(\tau)$, the inequality (3.39) becomes

$$\frac{D - B(1-c) + kBm}{D} > 0 \,, \qquad (3.40)$$

where $D = \left[\left(\frac{1+s}{1+As}\right)\left((1-p)A^{1-\gamma} + p\right)^{\frac{1}{1-\gamma}}\right]$, and $B = 1 + \alpha(1 - p(1+s))$. Lemma 14 and condition (3.35) ensure that $D - B(1-c) > 0$ so that the inequality (3.40) holds.

(b) Following the proof of (a), it is straightforward to see that the proof of (b) reduces to check when

$$\alpha_3 < \alpha_2\left[\frac{m(\Psi(\hat{\tau}_{y2}) - \Psi(\hat{\tau}_{y3}))}{\Psi(\hat{\tau}_{y3})}\right], \qquad (3.41)$$

holds. Since inequality (3.39) holds, we need a small enough value of α_3 to ensure the fulfillment of (3.41).

Proof of Proposition 22.

(a) It is immediate to see that $\frac{dG_1(\tau)}{d\tau} > 0$ and $\frac{dG_2(\tau)}{d\tau} > 0$, where

$$G_1(\tau) = p\tau(1+s)(\alpha_2 m + \alpha_3),$$

and

$$G_2(\tau) = p\tau(1+s)\,\alpha_2 m \,.$$

Then, we only have to prove that $G_1(\tilde{\tau}_{y3}) > G_2(\hat{\tau}_{y2})$, which is equivalent to prove that

$$\alpha_3 < \alpha_2 m\left[\frac{\tilde{\tau}_{y2} - \tilde{\tau}_{y3}}{\tilde{\tau}_{y3}}\right], \qquad (3.42)$$

after substituting the corresponding values of the tax rate. The expression (3.22) tells us how $\tilde{\tau}_{y2}$ depends on m. Then, we get

$$\lim_{m \to 1}\left[\alpha_2 m\left(\frac{\tilde{\tau}_{y2} - \tilde{\tau}_{y3}}{\tilde{\tau}_{y3}}\right)\right] = 0 \,.$$

In consequence we can conclude that for a m sufficiently close to one, the inequality (3.42) will hold.

(b) Following the proof of (a), it is straightforward to see that the proof of (b) reduces to check when

$$\alpha_3 < \alpha_2 m \left[\alpha_2 m \left(\frac{\tilde{\tau}_{y2} - \tilde{\tau}_{y3}}{\tilde{\tau}_{y3}} \right) \right], \qquad (3.43)$$

holds. Obviously, a value of α_3 small enough ensures that (3.43) holds.

(c) It is immediate to see that $\frac{dG_1(\tau)}{d\tau} > 0$, $\frac{dG_2(\tau)}{d\tau} > 0$ and $\frac{dG_3(\tau)}{d\tau} > 0$, where $G_1(\tau) = p\tau(1+s)(\alpha_2 m + \alpha_3)$, $G_2(\tau) = p\tau(1+s)\alpha_2 m$ and $G_3(\tau) = \alpha_2 m \Psi(\tau)$ with $\Psi(\tau) = [\tau(1-\phi(\tau)) + p\tau(1+s)\phi(\tau)]$. Then, we only have to prove that $G_1(\tilde{\tau}_{y3}) > G_3(\tilde{\tau}_{y2})$ since $G_2(\tau)$ and $G_3(\tau)$ are continuous on their respective domains. The last inequality is equivalent to

$$\alpha_3 > \alpha_2 m \left[\frac{\hat{\tau}_{y2} - \tilde{\tau}_{y3}}{\tilde{\tau}_{y3}} \right]. \qquad (3.44)$$

Thus, when the proportion α_3 is large enough, inequality (3.44) holds.

(d) Following the proof of (c), it is straightforward to see that the proof of (d) reduces to check when the following inequality is satisfied:

$$\alpha_3 < \alpha_2 m \left[\frac{\hat{\tau}_{y2} - \tilde{\tau}_{y3}}{\tilde{\tau}_{y3}} \right].$$

Obviously, such an inequality holds for a small enough value of α_3.

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