

A BI-STOCHASTIC NONPARAMETRIC ESTIMATOR

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ABSTRACT

Standard nonparametric techniques can be improved by using S-convex smoothing. This ensures that the variability of the estimation is reduced according to any dispersion measure, consistent with the second-order stochastic dominance criterion, and not only with respect to the variance. Moreover, a low-cost iterative proportional-fitting algorithm is applied to the original weight matrix of the estimator, which minimizes the Kullback-Liebler distance function. As a result, an overall S-convex nonparametric smoother, with a bistochastic matrix of weights, is obtained. Consistency and other practical properties are provided that highlight the potential use in applied economics.

JEL Classification: C14, D63.

Key Words: Nonparametric estimator, second-order stochastic dominance, bistochastic estimator.

1. INTRODUCTION

Nonparametric estimation of a regression curve has been proved to be a useful tool for applied researchers in econometrics. For instance, Diebold and Nason (1990) have investigated the presence of nonlinearities in forecasting asset prices; Bierens and Pott-Buter (1991) and Delgado and Miles (1997) applied nonparametric estimation of regression curves to the specification of Engel curves; and Bertschek and Entorf (1996) used the classic Nadaraya-Watson nonparametric estimator to study the Schumpeterian link between innovation and firm size.

However, we find possible improvements of the standard nonparametric techniques when the following interesting axiom is taken into consideration: nonparametric smoothing techniques should unambiguously reduce overall variability in a robust sense. Standard nonparametric methods concentrate only on particular variance-reducing smoothing. However, we think this is somehow arbitrary because there are other valid measures of dispersion. We suggest that the variability reduction accomplished by the nonparametric techniques has to be consistent with a wider class of dispersion measures that satisfy the second-order dominance criterion^{1,2}. Indeed, variance is a particular case of this class.

In this respect, we propose a *bistochastic* nonparametric smoothing technique, then prove that variability is reduced according to the second-order dominance criterion. We also look for a bistochastic estimator obtained from a simple low-cost modification of the existing nonparametric techniques. A basic relationship arises between the traditional and the proposed class of estimators.

Standard nonparametric smoothing is simply a *stochastic* estimator that only takes into account a single normalization of the weights matrix (summation across rows is equal to one). However, because we are dealing with a two-dimensional weights matrix³, we propose an estimator with a more natural double normalization (summations across rows and columns should be equal to one). As a result, we obtain the bistochastic estimator.

Among all possible mechanisms to do this double normalization of the weights matrix, we adopt a low-cost iterative proportional-fitting method,

¹ The second-order stochastic dominance criterion is well established in the risk (Rothschild and Stiglitz, 1970) and welfare (Atkinson, 1970) literature. It has counterpart equivalence with the mean-preserving spreads and Lorenz dominance criterion.

² A somehow related exercise can be found in Yitzhaki (1996), who points out the necessity of correcting the OLS estimator, in the welfare economics context, by a nonparametric estimator that is consistent with the extended Gini coefficient.

³ We concentrate on the bivariate regression case. Analogous extensions can be made for multivariate regressions.

which is a particular case of the algorithm proposed by Deming and Stephan (1940), that minimizes the Kullback-Liebler distance function (Ireland and Kullback, 1968). We find an interesting symmetry condition for this particular algorithm. The convergence result of this algorithm is the same, irrespective of whether we start normalizing by rows or by columns.

Two important implications of this feature are pointed out, which are not shared by typical nonparametric estimation:

- 1) S-convex smoothing is ensured under bistochastic nonparametric estimation, independent of the number of observations. As a consequence, variability reduction, measured in terms of the unambiguous decrease of every cumulation of the distribution function, is ensured. This is a desirable property for nonparametric smoothing techniques to satisfy, because variability reduction is defined in terms of a robust criterion (and not in the particular variance-reducing one). Moreover, we show that the bistochastic estimator, at any point, can be seen as a convex combination of the original stochastic smoother values.
- 2) A second property, shared by the OLS parametrical estimation, is that the mean of the estimated values is always equal to the mean of the observation values, independent of the number of observations. This gives rise to useful potential applications in empirical work.

In the field of public and taxation economics, the application of nonparametric techniques is appealing. Descriptive or positive aspects of taxation, such as overall increasing tax rates, are very well tested. In fact, Perrote *et al.* (2001) propose the use of this reformulated nonparametric technique to decompose the redistributive effect of the tax system into vertical and horizontal components⁴. Nevertheless, the use can be generalized to other cases, taking advantage of the fact that the Lorenz curve for the estimated variable always lies above the Lorenz curve for the actual explanatory variable. This implies important applications in the field of welfare and financial economics.

2. STANDARD NONPARAMETRIC SMOOTHING

Given any two-dimensional random sample, $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ ⁵, the random variables, X and Y , denote the explanatory and the response va-

⁴ In the field of inequality economics, nonparametric techniques have been used to estimate density curves in Hildenbrand and Hildenbrand (1986), Cowell *et al* (1996), Cowell and Victoria-Fesser (1996), Duclos and Lambert (2000).

⁵ We are referring to the stochastic design sample model, different from the alternative fixed design sample model.

riables, respectively. Suppose the theoretical regression curve at the point x is defined as:

$$m(x) = E(Y | X = x) \quad (1)$$

Let us write the nonparametric estimator of this curve, at point x , as a weighted average of the observations of the Y -variable:

$$M(x) = \sum_{j=1}^n W_j(x) Y_j \quad (2)$$

such that it satisfies the following consistency property (the estimated function converges in probability to the actual one).

$$\text{plim } M(x) = m(x) \quad (3)$$

The weights W_j can be any probabilistic weights⁶ that dampen the Y_j s if the corresponding X_j -value is far from x . For instance, the Nadaraya-Watson weights (Nadaraya, 1964; Watson, 1964),

$$W_j^{N-W}(x) = \frac{K_h\left(\frac{x - X_j}{h}\right)}{\sum_{j=1}^n K_h\left(\frac{x - X_j}{h}\right)} \quad (4)$$

where kernel K is a continuous, bounded and symmetric real function that integrates to one. The smoothing parameter h *tends to zero* as $n \rightarrow \infty$ and, for consistency, it is also assumed that $nh \rightarrow \infty$ as $n \rightarrow \infty$. The shape of the kernel weights is determined by K , whereas the size of the weights is parameterized by h . However, many other weights could be chosen like other kernel weights (Härdle, 1990) or the k -th nearest-neighbour weights (Stone, 1977) because it is imposed that the weights are probabilistic.

3. SECOND-ORDER STOCHASTIC DOMINANCE PROPERTY

There is a potential improvement from the previous standard nonparametric setting. Standard nonparametric methods concentrate on a particular variance-reducing smoothing. However, we think that this is somehow arbitrary, because there also exist other valid measures of dispersion. We require that the variability reduction accomplished by the nonparametric techniques has to be consis-

⁶ The weight function W_n is said to be a probability weight function if it is normalized ($\sum_j W_{nj}(x) = 1$) and nonnegative.

tent with a wider class of dispersion measures that satisfy the second-order dominance criterion. Indeed, the variance is a particular case of this class.

Definition 1: Given a distribution function, $F(x)$, where $x \in [0, +\infty)$, we define the *second-degree distribution function* as $F_2(x) = \int_0^{+\infty} F(t) dt$, which is the cumulation of the distribution function, $F(x)$.

Definition 2: Given any two distribution functions, $F(x)$ and $G(x)$, where, $x \in [0, +\infty)$, the distribution, $F(x)$, *second-degree stochastically dominates* the distribution $G(x)$ iff $F_2(x) \leq G_2(x)$ for all $x \in [0, +\infty)$.

We consider the following axiom:

Axiom 1 (Second-order stochastic dominance smoothing consistency): If a nonparametric smoothing achieves an unambiguous decrease of the second-degree distribution function, $F_2^{\hat{Y}}(x) \leq F_2^Y(x)$, for all $x \in [0, +\infty)$, then the smoothing technique is said to be consistent with the second-order stochastic dominance.

The relevance of this axiom is the well-known result of the equivalence between stochastic dominance and mean-preserving spreads. These generate well-established unambiguous variability-reducing movements, from a very general point of view, wider than the simple variance-reducing movements. If the smoothing technique is consistent with the second-order stochastic dominance, then the estimated values can be obtained from the original ones as a set of unambiguous variability-reducing mean-preserving spreads.

4. BISTOCHASTIC NONPARAMETRIC ESTIMATOR

We define the bistoochastic nonparametric estimator:

Definition 3: A nonparametric estimator, expressed in vector notation, by $\mathbf{M} = \mathbf{W} \cdot \mathbf{Y}$, is said to be *bistoochastic* if and only if \mathbf{W} is a bistoochastic weight matrix, which is doubly normalized by rows and by columns, that is $\sum_{i=1}^n W_{ij} = 1$ and

$$\sum_{j=1}^n W_{ij} = 1.$$

That is, if we estimate the curve at n points, and the weights matrix, represented by $\mathbf{W} = \{W_{ij}\}_{i,j=1,\dots,n}$, is bistoochastic, then the estimator is bistoochastic. The main difference between this estimator and the standard *stochastic* nonparametric estimator is that the latter is only normalized by rows.

There is a particular method to obtain a bistoochastic estimator. Given a nonparametric estimator, denoted by $\mathbf{M} = \mathbf{W} \cdot \mathbf{Y}$, we propose the following low-

cost method of obtaining a bistochastic smoother. Among all possible mechanisms to do this double normalization of the weights matrix, we adopt an iterative proportional-fitting method, which is a particular case of the algorithm proposed by Deming and Stephan (1940), that minimises the Kullback-Liebler distance function.

The algorithm is an iterative-fitting method applied to the initial elements W_{ij} and proceeds by row and column adjustments, such that at iteration t ($\forall t \in \mathbf{N}$), the new elements of the matrix of weights are:

$$\overline{W}_{ij}^{(0)} = W_{ij}$$

If t is odd,

$$\overline{W}_{ij}^{(t)} = \frac{\overline{W}_{ij}^{(t-1)}}{\overline{W}_{i+}^{(t-1)}} = \frac{W_{ij}}{\overline{W}_{i+}^{(t-1)} \times \dots \times \overline{W}_{+j}^{(1)} \times \overline{W}_{i+}^{(0)}}$$

if t is even,

$$\overline{W}_{ij}^{(t)} = \frac{\overline{W}_{ij}^{(t-1)}}{\overline{W}_{+j}^{(t-1)}} = \frac{W_{ij}}{\overline{W}_{+j}^{(t-1)} \times \dots \times \overline{W}_{+j}^{(1)} \times \overline{W}_{i+}^{(0)}}$$

where

$$\overline{W}_{+j}^{(t)} = \sum_{i=1}^n \overline{W}_{ij}^{(t)}$$

$$\forall j = 1, \dots, n; \quad t \in \mathbf{N}$$

and

$$\overline{W}_{i+}^{(t)} = \sum_{j=1}^n \overline{W}_{ij}^{(t)}$$

$$\forall i = 1, \dots, n; \quad t \in \mathbf{N} \tag{5}$$

We also find an interesting symmetry condition of this particular algorithm. The convergence result of this algorithm is $\mathbf{W}^{\mathbf{B}} = \{\overline{W}_{ij}\}_{1 \leq i \leq n, 1 \leq j \leq n}$ which is the same irrespective of whether we start normalizing by rows or by columns⁷.

We analyse the consistency property of the reformulated estimator.

Proposition 1: Let $\{W_n\}$ be a consistent sequence of probability weights, then $\{\overline{W}_n\}$ (as defined above) is consistent.

⁷ More generally, we can apply this method to any $r \times n$ -dimensional non-square weight matrix \mathbf{w} . Then, \mathbf{w} is not properly bistochastic, but the double normalization property is retained.

Proof: The bistochastic sequence of weights can be written as follows:

$$\bar{W}_n = f(W_n) \cdot W_n$$

where f is a bounded function because $\{\bar{W}_n\}$ is a sequence of normal weights. If we apply the result from Stone (1977, Corollary 2, p. 598), it is straightforward to see that the sequence $\{\bar{W}_n\}$ is consistent.

We establish the following properties:

Property 1: The smoothing technique is consistent with the second-order stochastic dominance, and therefore the estimated values can be obtained from the original ones as a set of unambiguous variability-reducing mean-preserving spreads, if and only if the estimator is bistochastic (Dasgupta, Sen and Starret, 1973 and Rothschild and Stiglitz, 1973)⁸.

Property 2: The estimator \mathbf{Z} is an S-convex combination of the variable \mathbf{Y} , independently of the sample size, because of the bistochastic link between them⁹. As a consequence, the Lorenz curve of Z always lies above the Lorenz curve of Y , $L(\mathbf{Z}) = L(\mathbf{W}^B \cdot \mathbf{Y}) \geq L(\mathbf{Y})$. This property has potentially important empirical applications in the field of welfare economics.

Property 3: The reformulated bistochastic estimator vector, to which this algorithm converges, is $\mathbf{Z} = \mathbf{W}^B \cdot \mathbf{Y}$, where \mathbf{W}^B is the closest bistochastic matrix to \mathbf{w} , according to the Kullback-Liebler distance function (Ireland and Kullback, 1968).

Property 4: An implicit property is that the estimator \mathbf{Z} and the variable \mathbf{Y} have the same means, $\mu(\mathbf{Z}) = \mu(\mathbf{W}^B \cdot \mathbf{Y}) = \mu(\mathbf{Y})$, independently of the sample size, because of the bistochastic matrix link between them.

Property 5: Given $\mathbf{Z} = \mathbf{W}^B \cdot \mathbf{Y}$, the estimated values by the proposed algorithm are overall convex combinations of the actual observations. The classical stochastic estimation are within-intervals convex combinations. In our case, observations are treated symmetrically in the sense that they all have the same weight on the process of construction of the non-parametric smoother¹⁰.

Property 6: Let $(X_1, Y_1), (X_2, Y_2), \dots, (X_n, Y_n)$ be a two-dimensional random sample; \mathbf{M} be a nonparametric estimator of the regression curve at n different

⁸ In fact, there is a naive nonparametric technique, the regressogram (Tukey, 1947), that guarantees that the weight matrix is bistochastic (see the proof in Appendix 1).

⁹ By definition, \mathbf{Z} is an S-convex combination of \mathbf{Y} iff \mathbf{Z} can be written as $\mathbf{Z} = \mathbf{A}\mathbf{Y}$, where \mathbf{A} is a bistochastic matrix (Dasgupta, Sen and Starret, 1973).

¹⁰ The intervals are truncated in the boundaries so that, in general, points in these intervals have less importance in the construction of the estimator than the "interior" points. Since the new smoother gives greater weights to points near the boundary, it can improve the performance of the estimator with respect to the so-called *boundary bias problem*. Simulations that were carried out over a set of different distributions of the variable \mathbf{X} confirm this result.

points; and \mathbf{Z} the bistochastic reformulation of \mathbf{M} . Then each element of \mathbf{Z} is a convex combination of the \mathbf{M} elements.

Proof: See Appendix 2.

5. CONCLUDING REMARKS

In this paper, we propose an improvement of the standard nonparametric techniques by using an S-convex smoothing. Standard nonparametric methods concentrate on stochastic estimation and variance-reducing smoothings.

Our method ensures that the variability of the estimation is reduced according to any dispersion measure, consistent with the second-order stochastic dominance criterion, and not only with respect to the variance. Moreover, a low-cost iterative proportional-fitting algorithm is applied to the original weight matrix of the estimator, which minimizes the Kullback-Liebler distance function with respect to the original weight matrix. As a result, an overall S-convex nonparametric smoother, with a bistochastic matrix of weights, is obtained. Consistency, preservation of the dependent variable mean value and Lorenz dominance are some other practical properties of the bistochastic estimator, which highlights the potential use in applied economics.

APPENDIX 1: THE REGRESSOGRAM AS A BISTOCHASTIC ESTIMATION

The regressogram is defined as the arithmetic mean of the Y-variable across the corresponding h non-overlapping intervals. Then, the regressogram estimation guarantees that the weights assigned to each observation of the response variable sum to one, not only across rows but also across columns, i.e., the weights matrix is bistochastic. However, the reverse is obviously not true.

Proof: Let $S = \{s_1(X), \dots, s_h(X)\}$ be the non-overlapping partition into h subgroups under consideration, $U = \{n_1, \dots, n_h\}$, the within-groups population set, and $V = \{\mu_1, \dots, \mu_h\}$ the associate response mean variable set. Under the regressogram estimation, we find:

$$z_1^i = \dots = z_{n_i}^i = \mu_i, \quad \forall i = 1, \dots, h$$

In vector notation, $\mathbf{M} = \mathbf{B}\mathbf{Y}$, where \mathbf{B} is the following n -dimensional bistochastic matrix:

$$\mathbf{B} = \begin{pmatrix} N_1 & 0 & \dots & 0 \\ 0 & N_2 & \dots & 0 \\ \dots & \dots & \ddots & 0 \\ 0 & 0 & \dots & N_h \end{pmatrix}$$

and N_i is the n_i -dimensional square matrix:

$$N_i = \begin{pmatrix} 1/n_i & \dots & 1/n_i \\ \dots & \ddots & \dots \\ 1/n_i \dots & \dots & 1/n_i \end{pmatrix} \quad \forall i = 1, \dots, h$$



APPENDIX 2: PROOF OF PROPERTY 6

The bistochastic estimator vector can be written as

$$\mathbf{Z} = \mathbf{W}^{\mathbf{B}} \cdot \mathbf{Y} = \mathbf{W}^{\mathbf{B}} \cdot \mathbf{W}^{-1} \cdot \mathbf{W} \cdot \mathbf{Y} = \mathbf{W}^{\mathbf{B}} \cdot \mathbf{W}^{-1} \cdot \mathbf{M}$$

which, in desegregate terms, can be written as

$$\begin{aligned} Z(x_i) &= [\bar{W}_{i1} \cdot W_{i1}^{-1} + \dots + \bar{W}_{ij} \cdot W_{ij}^{-1} + \dots + \bar{W}_{in} \cdot W_{in}^{-1}] \cdot M(x_1) + \dots \\ &+ [\bar{W}_{i1} \cdot W_{1j}^{-1} + \dots + \bar{W}_{ij} \cdot W_{ij}^{-1} + \dots + \bar{W}_{in} \cdot W_{nj}^{-1}] \cdot M(x_i) + \dots \\ &+ [\bar{W}_{i1} \cdot W_{1n}^{-1} + \dots + \bar{W}_{ij} \cdot W_{jn}^{-1} + \dots + \bar{W}_{in} \cdot W_{nn}^{-1}] \cdot M(x_n) \end{aligned}$$

Hence, the sum of terms within the square brackets

$$[\bar{W}_{i1} \cdot W_{i1}^{-1} + \dots + \bar{W}_{ij} \cdot W_{ij}^{-1} + \dots + \bar{W}_{in} \cdot W_{in}^{-1}] + \dots + [\bar{W}_{i1} \cdot W_{1n}^{-1} + \dots + \bar{W}_{ij} \cdot W_{jn}^{-1} + \dots + \bar{W}_{in} \cdot W_{nn}^{-1}]$$

must be one. Rearranging the expression above as

$$\bar{W}_{i1} [W_{i1}^{-1} + \dots + W_{1j}^{-1} + \dots + W_{in}^{-1}] + \dots + \bar{W}_{in} [W_{n1}^{-1} + \dots + W_{nj}^{-1} + \dots + W_{nn}^{-1}]$$

We only need to demonstrate that the inverse of a stochastic matrix sums to unity across rows.

Let $A = (a_{ij})_{i,j=1,\dots,n}$ be a stochastic matrix and B its inverse. As $B \cdot A = I_n$ (I is the identity matrix) we know that

$$I_{ij} = b_{i+} \cdot a_{+j} = \sum_{k=1}^n b_{ik} \cdot a_{kj}$$

So, because of the stochastic property of matrix A , we obtain:

$$\sum_{j=1}^n I_{ij} = 1 = \sum_{j=1}^n \sum_{k=1}^n b_{ik} a_{kj} = \sum_{k=1}^n b_{ik} \sum_{j=1}^n a_{kj} \Rightarrow \sum_{k=1}^n b_{ik} = 1$$

Q.E.D.

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Together with the original copy of the working paper a brief two-page summary highlighting the main policy implications derived from the research is also requested.

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