A PROBABILISTIC NONPARAMETRIC ESTIMATOR (*)

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ABSTRACT

This paper explores the adoption of a probabilistic nonparametric estimator in economics. First, it satisfies the *probabilistic coherence principle*, which ensures that the estimated variable can be generated by probabilistic assignment modeling from the observed variable. Second, it is proved to reduce variability in terms of noise, majorization and Lorenz dominance principles if, and only if, the estimator is probabilistic. The latter principles are universal criteria in risk and welfare economics, which expands the applicability of the estimator; for instance, to the measurement of economic discrimination. It also guarantees the symmetrical treatment of observations, a process that can produce smaller errors than positive-weight nonparametric estimators in terms of the bias-variance trade-off. This is verified by a general simulation exercise, with improvement due to the significant reduction in boundary bias. Finally, the estimator displays some other useful properties including consistency, preservation of the mean value, and multidimensional extension.

Key words: Nonparametric smoothing, majorization, Lorenz dominance, probabilistic estimator, noise.

1. INTRODUCTION

Nonparametric estimation of a regression curve has proved to be a useful tool for applied researchers in economics. For instance, Diebold and Nason (1990) have investigated the presence of nonlinearities in forecasting asset prices, Bierens and Pott-Buter (1991) and Delgado and Miles (1997) have applied nonparametric estimation of regression curves to the specification of Engel curves, and Bertschek and Entorf (1996) have used the classic Nadaraya-Watson nonparametric estimator to study the Schumpeterian link between innovation and firm size. Delgado *et al*. (2002) have also examined nonparametrically total factor productivity differences between exporting and nonexporting firms, ranking firms on the basis of stochastic dominance.

However, this paper suggests that improvements to standard nonparametric techniques are still possible. We propose a *probabilistic* nonparametric smoothing technique with the following properties. First, it satisfies the *probabilistic coherence principle*, which ensures a probabilistic link between the estimated and the observed variables. Standard nonparametric smoothers typically restrict the probabilistic link to every particular element of the estimated variable taken one by one, because of the row normalization of the weight matrix of the estimation. In the current probabilistic context, this paper extends the probabilistic link to the vector of the estimated variable as a whole, because of the double (rows and columns) normalization of the weight matrix. Therefore, it is able to generate the estimated variable from the observed variable through a probabilistic assignment modeling.[1]

The second property is that the estimator unambiguously reduces overall noise variability of the observed variable if, and only if, it is probabilistic. The estimated values are less dispersed (or more smoothed) than the observed ones in the sense that the estimated variable can be obtained from the observed one by subtracting the noise.

Moreover, the estimator reduces variability according to the robust majorization principle (Hardy *et al*., 1934; Marshall and Olkin, 1979, and Arnold, 1987), which is equivalent to the mean-preserving second-order stochastic dominance. As the mean value of the estimated variable remains unchanged, the estimation also achieves *Lorenz smoothing*, meaning that the Lorenz curve for the estimated variable dominates that of the observed variable. Accordingly, the estimated values are less dispersed than the observed ones in the sense that the observed variable can be obtained as a set of mean-preserving spreads of the estimated variable. Mean-preserving second-order stochastic dominance and Lorenz dominance are already well established in the risk (Rothschild, and Sti-

[1] We concentrate on the bivariate regression case. Analogous extensions apply to multivariate regressions. See property 8 below.

glitz, 1970) and welfare literature (Atkinson, 1970. Dasgupta; Sen, and Starret, 1973. Rothschild, and Stiglitz, 1973. Kolm, 1976. And Shorrocks, 1983).

This process allows a set of ordinal dispersion measures of the observed variable to be defined around the fitted variable, consistently with the second-order stochastic dominance criterion or with the distance between the Lorenz curves for both variables. The set of measures includes the difference of all cardinal dispersion measures for both variables, consistently with this criterion (*i.e.*, the difference of extended Gini coefficients (Donaldson, and Weymark, 1983, and Yitzhaki, 1983), Atkinson indices (Atkinson, 1970), and general entropy indices (Cowell, 1995) up to a positive monotone transformation.[2] Rodríguez *et al*. (2004), for example, propose this idea behind the probabilistic nonparametric technique to measure tax discrimination associated with a fiscal system.[3] Furthermore, its usefulness may be generalized to the measurement of economic discrimination.

Another practical property, which is a consequence of satisfying the probabilistic coherence principle, is that the estimator is observed to perform better than alternative standard nonparametric methods that use positive weights, on the basis of the bias-variance trade-off. This concentration on nonparametric methods that use positive weights is most useful because of their clear economic interpretation (see section 2) and is related to the improvement in boundary bias. The intervals are typically truncated at the boundaries so that, in general, observations in these intervals are given less importance in the construction of the estimator than the 'interior' points. As the new smoother treats all observations symmetrically, in the sense that they all receive the same aggregate weight in the process of construction of the nonparametric smoother, it gives greater weights to points near the boundary. Therefore, the estimator can alleviate the so-called *boundary bias problem*. Simulations over a set of different distributions of the explanatory variable confirm this result.

The proposed estimator has a number of additional desirable properties. First, it is consistent. Second, the mean of the estimated values is always equal to the mean of the observed values, irrespective of the number of observations (as in OLS estimation) so that the expected mean error equals zero. Third, the probabilistic estimator can be generalized to the multidimensional regression case. Moreover, the proposed probabilistic estimator is obtained from a simple low-cost modification of existing nonparametric techniques.

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[2] Such as the coefficient of variation, which is an increasing transformation of the general entropy index for $c = 2$, or the variance when the mean of the distribution remains unchanged.

[3] In the field of inequality economics, nonparametric techniques have been used to estimate density curves. See, for instance, Hildenbrand, and Hildenbrand (1986); Cowell *et al*. (1996); Cowell, and Victoria-Fesser (1996), and Duclos, and Lambert (2000).

This paper itself is organized as follows. Section 2 provides a brief review of nonparametric smoothing. Section 3 defines the probabilistic coherence principle, and the majorization, Lorenz dominance and noise properties are examined in section 4. Section 5 deals with probabilistic nonparametric estimation and its properties while the simulation exercises are conducted in section 6. The final section includes some concluding remarks.

2. STANDARD NONPARAMETRIC SMOOTHING

Given any two-dimensional random sample, (X_1, Y_1), (X_2, Y_2), ... (X_n, Y_n),[4] the random variables, X and $Y \in \Re^n$, denote vectors of the explanatory and response variables, respectively. The *theoretical* regression curve $m(x_i)$ is defined as the expected value of Y at point $x_i \in \Re$,

$$m(x_i) = E(Y \mid X = x_i).$$

The *nonparametrically estimated* regression curve at point x_i, $M(x_i)$, can then be written as a weighted average of the observations on Y, such that:

$$M(x_i) = \sum_{j=1}^{n} W_j(x_i) Y_j$$

where the weights W_j, which downwardly weight the Y_j s if the corresponding X_j value is far from x_i, are probabilistic.[5] Reasons for concentrating on nonparametric methods that use nonnegative weights are their clear economic interpretation. For instance, when estimating an Engel curve, negative weights are either difficult to interpret or contribute to the generation of implausible (read negative) consumption values.[6] This restriction prevents working with estimators such as the local linear smoother, which optimizes the minimax risk criterion (Fan, 1993) but uses negative weights. For instance, W_j could be the Nadaraya-Watson weights (Nadaraya, 1964, and Watson, 1964):

$$W_j^{N-W}(x_i) = \frac{K\left(\dfrac{x_i - X_j}{h}\right)}{\sum_{j=1}^{n} K\left(\dfrac{x_i - X_j}{h}\right)}$$

[4] This refers to the stochastic design sample model. However, extension to the fixed design sample model is straightforward.

[5] A weight function is said to be a probability weight function if it is normal ($\Sigma_j W_j(x_i) = \Sigma_j W_{ij} = 1$) and nonnegative (see, for example Stone, 1977).

[6] In fact, Härdle (1990, p. 142) comments: "... it is highly recommended to use a positive kernel even though one has to pay a price in bias increase."

where kernel K is a continuous, bounded, and symmetric real function that integrates to unity (such as, for example, the normal density function) and h is the bandwidth smoothing value. The smoothing parameter h tends to zero as $n \to \infty$ and, for consistency, it is assumed that $nh \to \infty$ as $n \to \infty$. Consistency ensures that the estimated function converges to the theoretical one. The shape of the kernel weights is determined by K, whereas the size of the weights is parameterized by h.

However, many other nonparametric weights could be chosen, including the Priestley-Chao (1976) and Gasser-Müller (1979) smoothers or the k-th nearest-neighbor (k-NN) weights (Stone, 1977). The Priestly-Chao and Gasser-Müller estimators have more severe boundary bias problems than does the Nadaraya-Watson smoother, and for random designs has variances that are 50% higher than that of the local linear estimator (see, for example, Wand and Jones, 1995). Recall that the variance of the local linear smoother is higher than that of the Nadaraya-Watson estimator (this is confirmed by the simulation exercise below). Applying the simulation exercises used in section 6 to the Priestly-Chao and Gasser-Müller estimators obtains similar results.[7] This analysis does not, however, consider k-NN smoothing as this is equivalent to the kernel estimation when an appropriate bandwidth k parameter is used (see for example Härdle, 1990). Henceforth, nonparametric estimation is written in vector notation, $M = W \cdot Y$, where W is the weights matrix and M is the nonparametric smoother evaluated at any n points.

3. THE PROBABILISTIC COHERENCE PRINCIPLE

How should the estimated M and observed Y variables then be linked? Under the classical nonparametric estimation, the probabilistic weights criterion according to Stone's definition (see footnote 5) is satisfied. This is due to the assumption that W is stochastic; that is, the row sums are equal to unity (the row normalization feature). As a consequence, the probabilistic link is particularly restricted to any individual element of the estimated variable. Then we can write for every i:

$$M(x_i) = \sum_{j=1}^{n} W_j(x_i) Y_j = \sum_{j=1}^{n} W_{ji} Y_j, \quad \text{where} \quad \sum_{j=1}^{n} W_{ji} = 1.$$

In the current context, we extend the probabilistic link to the whole vector M with respect to Y. For every observation j we also have:

$$\sum_{i=1}^{n} W_{ji} = 1.$$

[7] The results can be obtained on request from the authors.

If (and only if) a probabilistic link over the whole vector M is assumed, we will be able to generate the estimated variable from the observed variable through a probabilistic assignment modeling. Formally we define this as follows.

DEFINITION 1. A bistochastic (or doubly stochastic) matrix is a square matrix in which all elements are nonnegative and all row and column sums are equal to unity. A particular case is a permutation matrix, which is a squared matrix with elements 0 and 1 and the row and column sums are equal to unity.

DEFINITION 2. *The probabilistic coherence principle*. Given any two variables $M, Y \in \Re^n$, M satisfies the probabilistic coherence principle, if and only if:

$$M = WY$$

where W is a bistochastic matrix.

In the nonparametric estimation context, this means that there is a probabilistic link between the observed variable Y and the estimated variable M. In other words, M can be generated from Y using probabilistic assignment modeling. Formally, we apply the following theorem.

THEOREM 1 (Birkhoff, 1946, and von Neumann, 1953). The set of bistochastic matrices is the convex hull of all permutation matrices. Formally, an n by n matrix is bistochastic if and only if it can be written in the form:

$$W = \sum_{i=1}^{n} p_i P_i$$

for some set of probabilities p_i and permutation matrices P_i; that is, the matrix is bistochastic if and only if it can be expressed as a convex combination of permutation matrices (the latter define the set of extreme points of the convex hull). A consequence of this theorem is that it is possible to decompose any bistochastic matrix additively in terms of a probability assignment model.

For example, let us assume that under a classical stochastic nonparametric estimator, the following stochastic matrix is obtained:

$$W = \begin{pmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.3 & 0.7 \end{pmatrix}, \tag{1}$$

and the following bistochastic weight matrix W^B is derived from the stochastic one (according to a procedure explained below):

$$W^B = \begin{pmatrix} 0.81 & 0.11 & 0.08 \\ 0.19 & 0.68 & 0.13 \\ 0 & 0.21 & 0.79 \end{pmatrix}. \tag{2}$$

Accordingly, the bistochastic matrix W^B can be decomposed (making use of Birkhoff's algorithm; see Appendix A for details) as follows.

$$W^{B}=0.68\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+0.13\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}+0.11\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}+0.08\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

A first interpretation is that the estimated variable coincides with the observed variable under a probability equal to 0.68. An adding-up property is that the overall effect of Y_1 on $M(x_1)$ of 0.81 can be now decompose on the additive 0.68 and 0.13 subeffects. Furthermore, the probabilistic coherence principle property guarantees a symmetrical treatment of observations that has a clear economic interpretation.

Symmetrical treatment of observations

The probabilistic coherence principle ensures a symmetrical (balanced) treatment of the observations, in the sense that they have the same aggregate weight, in contrast with the classical estimators that do not. This property rules out the possibility of over- or underweighted observations. For example, the weights in (1) are unbalanced (or not symmetrical) as the aggregate weight on M for observation Y_1 is 0.8, while for observation Y_2 it is 1.3 and for observation Y_3 it is 0.9. However, all observations in (2) have a symmetrical aggregate weight on M equal to unity [*i.e.*, the overall effect of Y_1 on $M(x_1)$, $M(x_2)$ and $M(x_3)$ is 0.81; 0.19 and 0.00, respectively]. As there is no *a priori* economic justification for an unequal treatment of observations, problems may arise because of difficulties in economic interpretation (see property 2 in section 5).

4. THE MAJORIZATION AND SECOND-ORDER STOCHASTIC DOMINANCE PROPERTIES

(Mean-preserving) second-order stochastic and Lorenz dominance are broadly defined criteria for smoothing or dispersion reduction in welfare and risk literature (Rothschild and Stiglitz, 1970, and Atkinson, 1970). Analogous concepts in the mathematical literature include the general majorization principle (Hardy *et al*., 1934; Marshall and Olkin, 1979, and Arnold, 1987).[8] Here there is a potential for improvement over the standard nonparametric approach, which comes from insisting that this majorization criterion be satisfied by nonparametric smoothing.

[8] Majorization is also broadly used in quantum mechanics in connection with the measurement of entropy or disorder, and it provides a natural language for expressing sharp fundamental constraints on the ability of quantum measurements to acquire information about a quantum system (see, for example, Nielsen, 2002).

We establish these principles formally. Let F and G be two n-dimensional real vectors. We use the notation $F^{\uparrow}$ to denote the vector whose elements are the entries of F reordered into increasing order, $F_1^{\uparrow} \leq F_2^{\uparrow} \leq \cdots \leq F_n^{\uparrow}$.

DEFINITION 3 (*Majorization*). Given any two vectors F and $G \in \Re^n$, *F is majorized by G*, written $F \overset{m}{\succ} G$, if and only if:

$$\sum_{i=1}^{k} F_i^{\uparrow} \geq \sum_{i=1}^{k} G_i^{\uparrow}$$

for $k = 1, ..., n$, with equality when $k = n$.[9] The last equality implies that both vectors have the same mean value. Majorization has a counterpart in economics under the title of *mean-preserving second-order stochastic dominance* (Rothschild and Stiglitz, 1970).

DEFINITION 4 (*Lorenz dominance*). Given any two vectors F and $G \in \Re^n$, and their relative transformations, say $F^R = \frac{1}{n\mu_F} F$ and $G^R = \frac{1}{n\mu_G} G$, where μ_F and μ_G denote the mean values of F and G, and sign $(\mu_F) = \text{sign}\,(\mu_G)$, *$F$ Lorenz dominates G*, written $F \overset{l}{\succ} G$, if and only if:

$$\sum_{i=1}^{k} F_i^{R\uparrow} \geq \sum_{i=1}^{k} G_i^{R\uparrow} \quad \text{for} \quad \mu_F \quad \text{and} \quad \mu_G \neq 0$$

for $k = 1, ..., n$. This Lorenz criterion is linked to majorization over relative vectors (which have the same mean value by construction).[10]

An important proposition that is used in what follows is:

THEOREM 2. Given two vectors F and $G \in \Re^n$, where $\mu_F = \mu_G \neq 0$, the following statements are equivalent.[11]

(a) $F \overset{m}{\succ} G$.

(b) $F \overset{l}{\succ} G$.

(c) F can be written as $F = V^B G$, where V^B is a bistochastic matrix.

(d) $\sum_{i=1}^{n} \Psi(F_i) \geq \sum_{i=1}^{n} \Psi(G_i)$ for every concave function $\Psi : \Re \rightarrow \Re$ (or convex function for the $\leq$ case).

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[9] Formally, this concept is originally established in the equivalent version $\sum_{i=1}^{k} F_i^{\downarrow} \leq \sum_{i=1}^{k} G_i^{\downarrow}$, where vector F is re-ordered into decreasing order, $F_1^{\downarrow} \geq F_2^{\downarrow} \geq \cdots \geq F_n^{\downarrow}$. We adopt the definition in the main text for convenience.

[10] Note that this is a slight extension of the original Lorenz dominance criterion, which is defined for strictly positive elements. However, this extension only affects the generality of statement (b) in Theorem 2. Under the original Lorenz dominance criterion the domain should be restricted to $F, G \in \Re_{++}^n$, where $\mu_F = \mu_G \neq 0$.

[11] This theorem is, in fact, more general. The non-zero mean value is inessential to the proofs of the equivalence between (a), (c), (d), (e), (f) and (g), but it is required in the proof of the equivalence with (b).

(e) $\Phi(F) \geq \Phi(G)$ for every Schur-concave function $\Phi : \Re^n \rightarrow \Re$ (or Schur-convex for the $\leq$ case).[12]

(f) G has the same distribution as $F + R$, written $G =^d F + R$, where R is a random n-dimensional real vector that satisfies $E[R_i \mid F_i] = 0$ for all F_i.

(g) G is a mean-preserving spread of F, according to Rothschild and Stiglitz's definition (1970).

PROOF. Statement (a) is equivalent to (b) because $\mu_F = \mu_G$ by definition; statements (b), (c) and (e) are equivalent, see Dasgupta *et al*. (1973); (b) is equivalent to (f) and (g), see Rothschild, and Stiglitz (1970). Finally, propositions (a) and (d) are proved to be equivalent in Hardy *et al*. (1934), and also in Atkinson (1970).

The binary relations "$\overset{m}{\succ}$" and "$\overset{l}{\succ}$" are not complete, and they generate partial rather than total orderings of vectors. However, if F is majorized by (is more smoothed than) G, it achieves, in this mean-constant case, either a Lorenz or a mean-preserving spreads smoothing (by statements a, b and g). This is a general criterion that ensures a smoothing according to the wide class of convex, or the (even wider) class of Schur-convex, dispersion measures (statements d and e). Mean-constant variance-reducing smoothing is a particular case. Nonetheless, mean-constant variance-reducing smoothing does not imply Lorenz smoothing. In this respect, majorization and Lorenz dominance are more general or robust criteria for smoothing. Moreover, the majorization principle has a counterpart in terms of a noise-free distribution as established in statement (f). Notice that R, as defined, is usually interpreted as a noise term in the literature. Therefore, F can be viewed as a noise-free vector distribution of G.

In the next section, a nonparametric estimator is stated to satisfy majorization, Lorenz dominance and it is a noise-free estimator if and only if it is probabilistic.

5. THE PROBABILISTIC NONPARAMETRIC ESTIMATOR

The probabilistic nonparametric estimator is defined as follows.

DEFINITION 5. A nonparametric estimator, expressed in vector notation by $Z = W^B \cdot Y$, is said to be *probabilistic* if and only if W^B is a bistochastic weights matrix, which is normalized by both rows and by columns; that is $\sum_{i=1}^{n} W_{ij} = 1$ and $\sum_{j=1}^{n} W_{ij} = 1$.

[12] A function $f : \Re^n \rightarrow \Re$ is said to be Schur-concave if $x \overset{m}{\succ} y \Rightarrow f(x) \geq f(y)$. Notice that $f(\cdot)$ is Schur-convex if $-f(\cdot)$ is Schur-concave.

That is, if the curve is estimated at any n points and the weights matrix, represented by $W^B = \{W_{ij}\}_{i,j=1,\ldots,n}$, is bistochastic, then the estimator is probabilistic. The main difference between this estimator and the standard *stochastic* nonparametric estimator is that the latter is only normalized by rows.

A particular method of obtaining a probabilistic estimator is now proposed.[13] Given a nonparametric stochastic estimator, denoted by $M = W \cdot Y$, the following low-cost method of obtaining a probabilistic smoother, denoted by $Z = W^B \cdot Y$, is proposed. Of all the potential methods that could be used for double normalization of the weights matrix, the so-called iterative proportional-fitting method is adopted. In turn, this method is a special case of the algorithm proposed by Deming and Stephan (1940), which minimizes the Kullback-Liebler distance function between W and W^B (see property 5). Moreover, the algorithm converges when the observed weights are all nonnegative and estimates are consistent and asymptotically normal.

The algorithm is an iterative-fitting method applied to the initial elements W_{ij}, and it proceeds by row and column adjustments, such that at iteration t ($\forall t \in \mathbb{N}$), the new elements of the matrix of weights are:

$$W_{ij}^{(0)} = W_{ij}.$$

If t is odd,

$$W_{ij}^{(t)} = \frac{W_{ij}^{(t-1)}}{W_{i+}^{(t-1)}} = \frac{W_{ij}}{W_{i+}^{(t-1)} \times \cdots \cdots \times W_{+j}^{(1)} \times W_{i+}^{(0)}},$$

and if t is even,

$$W_{ij}^{(t)} = \frac{W_{ij}^{(t-1)}}{W_{+j}^{(t-1)}} = \frac{W_{ij}}{W_{+j}^{(t-1)} \times \cdots \cdots \times W_{+j}^{(1)} \times W_{i+}^{(0)}},$$

where $W_{+j}^{(t)} = \sum_{i=1}^{n} W_{ij}^{(t)}$, $\forall j = 1, \ldots, n$; $W_{i+}^{(t)} = \sum_{j=1}^{n} W_{ij}^{(t)}$, $\forall i = 1, \ldots, n$; and $t \in \mathbb{N}$.

The following example clarifies the proposed iterative method. Let us start with the matrix W in (1). Iteration 1 normalizes by columns (by dividing every element in the column by the column sum), but now the normalization by rows is not verified (row sums are not equal to unity). Iteration 2 renormalizes by rows (by dividing every element in the row by the row sum) and so on. Finally, after 11 iterations, the bistochastic matrix in (2) is obtained:

[13] Another nonparametric technique is the regressogram (Tukey, 1947), which guarantees that the weights matrix is bistochastic (see the proof in Appendix B). However, this estimator is rarely used because of a lack of desirable properties.

$$\begin{array}{ll} W^{(0)} = \left[\begin{array}{ccc} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0 & 0.30 & 0.70 \end{array}\right]\begin{array}{c} 1.00 \\ 1.00 \\ 1.00 \end{array} \rightarrow & W^{(1)} = \left[\begin{array}{ccc} 0.88 & 0.15 & 0.11 \\ 0.12 & 0.62 & 0.11 \\ 0 & 0.23 & 0.78 \end{array}\right]\begin{array}{c} 1.14 \\ 0.85 \\ 1.01 \end{array} \rightarrow \\ \qquad\quad \begin{array}{ccc} 0.80 & 1.30 & 0.90 \end{array} & \qquad\quad \begin{array}{ccc} 1.00 & 1.00 & 1.00 \end{array} \end{array}$$

$$\begin{array}{ll} W^{(2)} = \left[\begin{array}{ccc} 0.77 & 0.13 & 0.10 \\ 0.15 & 0.72 & 0.13 \\ 0 & 0.23 & 0.77 \end{array}\right]\begin{array}{c} 1.00 \\ 1.00 \\ 1.00 \end{array} \rightarrow \ldots \rightarrow & W^{B} = \left[\begin{array}{ccc} 0.81 & 0.11 & 0.08 \\ 0.19 & 0.68 & 0.13 \\ 0 & 0.21 & 0.79 \end{array}\right]\begin{array}{c} 1.00 \\ 1.00 \\ 1.00 \end{array} \\ \qquad\quad \begin{array}{ccc} 0.92 & 1.08 & 1.00 \end{array} & \qquad\quad \begin{array}{ccc} 1.00 & 1.00 & 1.00 \end{array} \end{array}$$

There is an interesting symmetry condition relating to this particular algorithm. The convergence result for this algorithm is $W^B = \{W_{ij}^{(T)}\}_{1 \le i \le n, 1 \le j \le n}$, which applies whether one begins by normalizing by rows or columns, where *T* denotes the final iteration according to a sufficiently accurate stopping rule.[14]

What, then, is the computational cost of this algorithm? In principle, as the sample size is *n*, the curve must be evaluated at *n* points to obtain a square weight matrix. Then, $O(n^2)$ kernel evaluations are necessary before the iterative method is applied to the stochastic nonparametric estimator to obtain the bistochastic (probabilistic) one. This may make the computation of the probabilistic estimator very slow for a sufficiently large value of *n*. One way of dramatically increasing the computational speed is to compute the *binned kernel regression estimator* (see, for example, Georgiev, 1986; Fan, and Marron, 1994) before the bistochastic smoothing is applied. The binned regression smoothing technique replaces kernel estimators by approximations that can be computed quickly by using the fast Fourier transform. The intent is to replace the data by a mesh of *R* grid counts, where each grid count is a weight that represents the amount of data near the corresponding grid point. The approximation is generally very good for moderate values of *R*, and it can be made arbitrarily better by increasing the value of *R*.

Next, we analyze the consistency property of the reformulated estimator.

PROPERTY 1 (consistency of the estimator). Let $\{W_n\}$ be a consistent sequence of probability weights (as defined in footnote 5) and let $\{W_n^{(T)}\}$ be a sequence of weights of the proposed probabilistic estimator. Then, it follows that $\{W_n^{(T)}\}$ is consistent.

PROOF. The bistochastic sequence of weights can be written as:

$$W_n^{(T)} = f(W_n) \cdot W_n,$$

[14] More generally, this method can be applied to any *r* x *n*-dimensional nonsquare weights matrix *W*. Then, *W* is not properly bistochastic, but the double normalization property is retained.

where *f* is a bounded function because $\{W_n^{(T)}\}$ is a sequence of normal weights. Applying the result of Stone (1977, Corollary 2, p. 598) reveals that the sequence $\{W_n^{(T)}\}$ is consistent.

In addition, the probabilistic estimator has the following properties.

PROPERTY 2 (probabilistic coherence). The estimator satisfies the probabilistic coherence principle by construction; therefore, we can interpret the estimator in terms of a probabilistic assignment model (see section 3). A relevant implication of this property is the symmetrical treatment of observations. All observations have the same aggregate weight in the construction of the nonparametric estimator *Z*, in the sense that the sum of the (*across-interval*) weights assigned to any observation Y_i is the same (and is eventually unity). This is equivalent to imposing normalized summation across columns in the weights matrix *W*, which is what the probabilistic estimator does by definition. Note that classical stochastic estimators simply normalize weights *within intervals* because the weights matrix *W* is only normalized by rows. In the probabilistic case, both *across*- and *within-interval* weights are normalized to unity. Note that under this methodology, asymptotic unbiasedness is retained, as row normalization is verified.

Furthermore, a related issue is that the estimator can alleviate the so-called *boundary bias problem*. Aggregate weights of the extreme observations at the boundaries are typically less than unity, while those at the central distribution typically exceed unity; as in the initial *W* matrix in expression (1). The reason lies in the truncation of the intervals at the boundaries. As the new smoother gives greater weights to points near the boundary, it may improve the goodness of the fit at the boundaries. The empirical exercise used in this analysis, and applied to a set of different distributions of the variable *X*, confirms this result.

PROPERTY 3 (Majorization and Lorenz dominance consistency). The bistochastic smoothing technique is consistent with majorization, mean-preserving second-order stochastic dominance and Lorenz dominance. For instance, the Lorenz curve for *Z*:

$$L_Z(k/n) = \sum_{i=1}^{k} Z_i^{R\uparrow},$$

always lies above the Lorenz curve for *Y*, when $\mu_Z = \mu_Y \neq 0$:

$$L_Z(k/n) \geq L_Y(k/n) \quad \forall k = 1,\cdots,n.$$

PROOF. This proof applies the Theorem 2 of Dasgupta *et al.* (1973). A necessary and sufficient condition for $L_Z(k/n) \geq L_Y(k/n)\ \forall k = 1,\cdots,n$ is that $Z = W^B Y$, where W^B is bistochastic. This result is even more general as it is also second-order stochastic dominance consistent. The reasoning is that the mean of the dependent variable remains constant (see property 6 below).[15]

[15] *Second-order stochastic dominance*, *generalized Lorenz dominance* (Shorrocks, 1983) and *supermajorization* are equivalent criteria. The only difference with respect to majorization in definition 3 is the omission of the equality requirement at $k = n$.

As a consequence of this property, the estimated values can be obtained from the original values as a set of unambiguous Lorenz-variability-reducing mean-preserving spreads, if and only if the estimator is probabilistic. In particular, they can be obtained as a set of variance-reducing mean-preserving spreads. Moreover, Lorenz dominance is a general criterion that ensures the variability reduction according to the wide set of Schur-convex measures of dispersion (see section 4), such as the extended Gini coefficients, Atkinson indices or the general entropy measures.

Application

This property has potentially important applications in economics, particularly applications concerning the measurement of discrimination. In a recent paper (see Rodríguez *et al*., 2004), the probabilistic estimator has been applied to the measurement of tax discrimination (horizontal inequity associated with a tax system).

Let X and $Y \in \Re^n_{++}$ be the pre- and post-tax equivalent income distributions, respectively, and let Z be the estimated post-tax equivalent income by the nonparametric bistochastic technique. The higher the dispersion of Y around Z, the greater the discrimination associated with tax. In Rodríguez *et al*. (2004), it is shown that dispersion of Y around Z exists if and only if similar individuals with close-to-equal pre-tax income levels (associated to a particular bandwidth h) pay different taxes; that is, there is tax discrimination. In this approach, Z becomes the discrimination-free benchmark distribution.

This notion of dispersion or inequality of Y around Z can be measured by any monotone transformation of any distance function between the Lorenz curves of Y and Z. This implies, in fact, an ordinal representation of discrimination and it satisfies the important property of consistency with the standard Lorenz and mean-preserving second-order stochastic dominance criteria. The Lorenz smoothing property of the probabilistic nonparametric estimator ensures that Z Lorenz dominates Y and therefore ensures that the distance between both Lorenz curves is nonnegative. Notice that negative discrimination values have no economic meaning.

Extensions to more general discrimination frameworks, such as wage discrimination, may be done by computing the distance between the actual and the estimated wage Lorenz curves for workers with close-to-equal attributes and the same properties.[16] Both curves only differ whenever workers with close-to-equal attributes and similar productivity levels receive different wages. With im-

[16] Jenkins (1994) also uses a Lorenz curve-based methodology to measure wage discrimination.

perfect information, the unintended dispersion due to incomplete information may be discounted. This is the basis of the measurement of discrimination that the authors intend to develop in future research.

PROPERTY 4. $Z =^{d} Y - R$ where $E[R_i | Z_i] = 0$ for all Z_i. Noise is the sole element that is eliminated by the smoothing process if and only if the estimator is probabilistic. Otherwise, the estimator would probably eliminate some signal (or fail to remove all noise). See Theorem 2 in section 4 for the proof.

PROPERTY 5. The probabilistic estimator vector, to which this algorithm converges, is $Z = W^B \cdot Y$, where W^B is the closest bistochastic matrix to W, according to the Kullback-Liebler distance function $D(W^B, W)$:

$$D(W^B, W) = \sum_{i=1}^{n} \sum_{j=1}^{n} W_{ij}^B \ln\left(\frac{W_{ij}^B}{W_{ij}}\right).$$

The proof applies the result of Ireland and Kullback (1968). This distance function is widely used in the information theory approach for both generation of models and estimation of parameters.

PROPERTY 6 (zero expected estimation error). An implicit property is that the estimator Z and the variable Y have the same mean, $\mu(Z) = \mu(W^B \cdot Y) = \mu(Y)$, whatever the sample size (as in OLS estimation), because of the bistochastic matrix link between them. Hence, the expected estimated error is zero, unlike in standard stochastic estimation. This property is, in fact, implied by property 4.

In the following example, assume that $Y = (6000, 10000, 15000)$ is a vector of individual taxes. Then, suppose that the estimated stochastic W and bistochastic W^B are as in (1) and (2). In this case, the estimated taxes are $M = (7700, 11000, 13500)$ and $Z = (7160, 9890, 13950)$, respectively. The inherent adding-up property in the probabilistic estimator guarantees that the overall estimated taxes equal the actual figure of 31000, irrespective of the sample size. However, under the stochastic estimator, we obtain an overall amount of 31300. This adding-up accountant property can be also useful in the nonparametric smoothing of earnings, profits, savings, GDP, etc.

PROPERTY 7. Let (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n) be a two-dimensional random sample, let M be a nonparametric estimator of the regression curve at n different points, and let Z be the bistochastic reformulation of M. Then, each element of Z is a convex combination of the M elements. See Appendix B for the proof.

PROPERTY 8. The probabilistic estimator can be generalized to the multivariate regression case given that the Deming-Stephan algorithm can be applied to higher dimensions. Property 8 allows one to generalize the application of the probabilistic nonparametric technique to a multidimensional framework.

6. A SIMULATION EXERCISE

In this section, we test the efficiency of the proposed probabilistic estimator in terms of the bias-variance trade-off, as measured by the conditional quadratic error for different sample sizes, and compare it with standard stochastic estimators, including the Nadaraya-Watson estimator and local linear estimator.

We show that the probabilistic smoother performs better than the alternative standard nonparametric methods that employ positive weights, on the basis of the conditional quadratic error. The mean integrated square error (MISE) and other asymptotically equivalent measures are not considered in this analysis because there is no explicit expression for the probabilistic estimator described in section 5. For this reason, we undertake the following simulation exercise.

Design of the Exercise

In this simulation exercise, we evaluate the performance of the probabilistic estimator according to the conditional quadratic error (d_c),

$$d_c = E\left[n^{-1}\sum_{j=1}^{n}\left(Z(x_j) - m(x_j)\right)^2 \mid x_1,\ldots,x_n\right]$$

where $m(x)$ is the true curve and $x_1, \ldots, x_n$ is a particular sample. It is well known that d_c can be decomposed additively into bias and variance components (see, for example, Härdle, 1990, p. 148).

We compute the bias, variance and conditional quadratic error for three nonparametric smoothers. These are the Nadaraya-Watson, the bistochastic Nadaraya-Watson and the local linear estimator. Data on X have been generated from two different distributions, the standard normal, $X_i \sim N(0,1)$, and the uniform, $X_i \sim U(0,1)$. Data on Y were generated from the model, $Y_i = m(X_i)+\varepsilon_i$, where $\varepsilon_i \sim N(0,\sigma^2)$, with $m(X_i)$ being either $m(X_i) = \mathrm{Sin}(2\pi X_i)$ or $m(X_i) = \exp(X_i)$. These two different specifications imply two quite different models. We also perform simulations using two different values for σ^2, namely $\sigma^2 = 0.2$ and $\sigma^2 = 0.8$, in both models. The simulations were developed from three different sample sizes, $n = 30$, $n = 100$ and $n = 1000$. In total, we performed $2\times2\times2\times3\times3\times3 = 216$ basic computations. The results were then obtained by taking the mean of 200 independent samples or repetitions of each basic computation.[17]

Results

Results for the experiment are presented in Table 1 for the N(0,1) case. The results for the U(0,1) distribution are very similar and for the purposes of bre-

[17] Different specifications for the distribution function (lognormal) and for the $m(X)$ function (polynomials) were tested, but did not significantly alter the main results.

vity are not presented. The results are evaluated for the optimal bandwidth obtained according to the cross-validation function. However, we show below that this is not an important aspect of the analysis. The results also show that in all cases examined, the conditional quadratic error for the bistochastic smoother is lower than that for the Nadaraya-Watson estimator. In fact, the bias is substantially reduced by the bistochastic reformulation, while the increase in the variance is insufficient to offset this reduction.[18] Consequently, the probabilistic estimator provides a better fit.

Table 1

BIAS, VARIANCE AND d_c FOR THE DIFFERENT ESTIMATORS UNDER $X_i \sim N(0,1)$ (*)

n=30												
	$\sin(2\pi X_i)$						$\exp(X_i)$					
	$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$			$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$		
	Bias	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**
N-W	0.0226	0.0750	0.0976	0.1236	0.2145	0.3380	0.0929	0.0350	0.1279	0.2160	0.1085	0.3245
B N-W	0.0192	0.0762	0.0954	0.1184	0.2185	0.3370	0.0592	0.0361	0.0953	0.1153	0.1135	0.2288
L-L	0.0099	0.0911	0.1010	0.1035	0.2619	0.3654	0.0134	0.0440	0.0574	0.0214	0.1464	0.1678
n=100												
	$\sin(2\pi X_i)$						$\exp(X_i)$					
	$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$			$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$		
	Bias	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**
N-W	0.0100	0.0364	0.0464	0.0257	0.1131	0.1388	0.0469	0.0143	0.0612	0.1189	0.0453	0.1642
B N-W	0.0079	0.0371	0.0450	0.0221	0.1151	0.1372	0.0249	0.0148	0.0397	0.0567	0.0477	0.1044
L-L	0.0044	0.0421	0.0465	0.0161	0.1297	0.1459	0.0043	0.0174	0.0217	0.0116	0.0584	0.0701
n=1000												
	$\sin(2\pi X_i)$						$\exp(X_i)$					
	$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$			$\sigma_\varepsilon^2 = 0.2$			$\sigma_\varepsilon^2 = 0.8$		
	Bias	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**	**Bias**	**Variance**	**d_c**
N-W	0.0018	0.0068	0.0087	0.0037	0.0228	0.0266	0.0097	0.0028	0.0125	0.0266	0.0086	0.0353
B N-W	0.0015	0.0069	0.0084	0.0032	0.0231	0.0263	0.0039	0.0029	0.0067	0.0088	0.0090	0.0178
L-L	0.0011	0.0074	0.0085	0.0027	0.0247	0.0274	0.0006	0.0032	0.0038	0.0019	0.0103	0.0122

(*) Results are the average of 200 independent samples, and are evaluated for the optimal bandwidth according to the cross-validation function. N-W, B N-W and L-L denote the Nadaraya-Watson, bistochastic Nadaraya-Watson and local linear estimators, respectively.

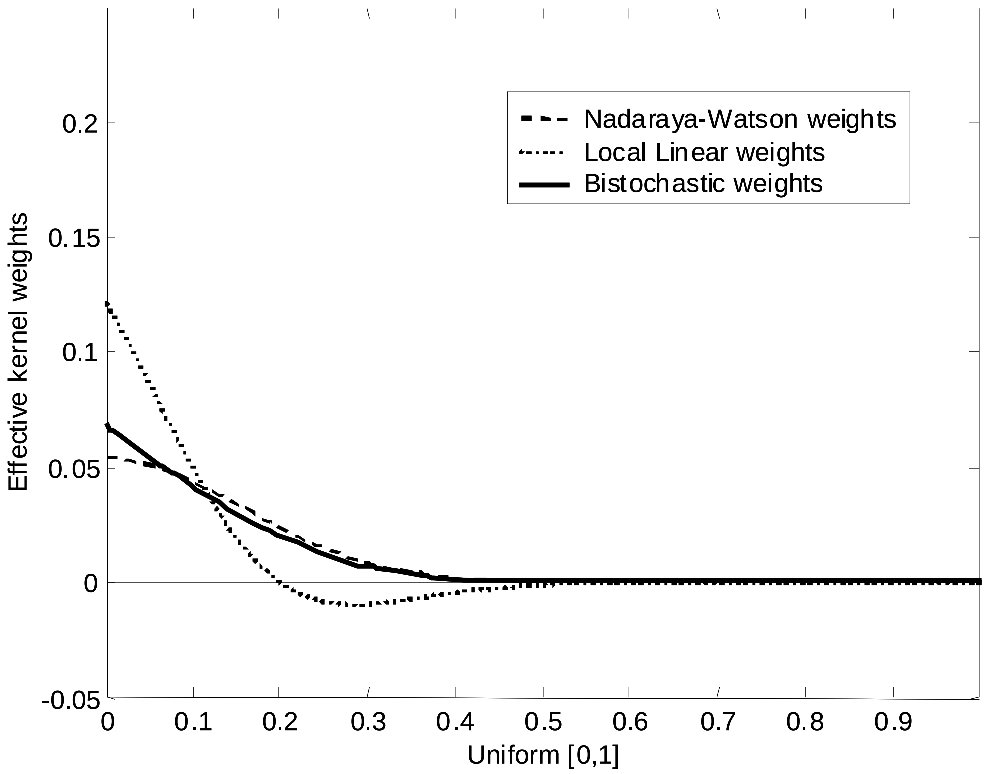
[18] Note that this increase in the variance does not contradict second-order stochastic dominance, because in this case, there is no stochastic dominance between the Nadaraya-Watson and bistochastic methods.

How are these results explained? The results are due to the way in which the bistochastic method corrects the boundary bias, which is apparent from Figure 1. Figure 1 shows the effective kernel weights associated with the lowest boundary value, for the whole range of values.[19] Probabilistic estimators give greater weights to boundary values than do standard stochastic methods. Hence, they tend to alleviate the boundary bias problem. Figure 1 shows that local linear estimators do even better (in Table 1, bias and d_c are even lower). However, we do not consider these estimators because they use negative weights elsewhere, as shown in Figure 1.

This raises the question of why the probabilistic smoother changes the boundary weights appropriately. The answer is related to the symmetrical treatment of the observations that is due to the double normalization (see property 2). The intervals are truncated at the boundaries so that, in general, observations in these intervals have less importance in the construction of the estimator than the 'interior' points. Since the new smoother gives greater weights to points near the boundary, it can improve the performance of the estimator with respect to the so-called *boundary bias problem*.

Figure 1

EFFECTIVE KERNEL WEIGHTS EVALUATED AT THE LOWEST X_i VALUE



Exercise design: $Y_i = m(X_i) + \varepsilon_i$, where $X_i \sim U(0,1)$, $m(X_i) = \exp(X_i)$ and $\varepsilon_i \sim N(0,0.2)$.

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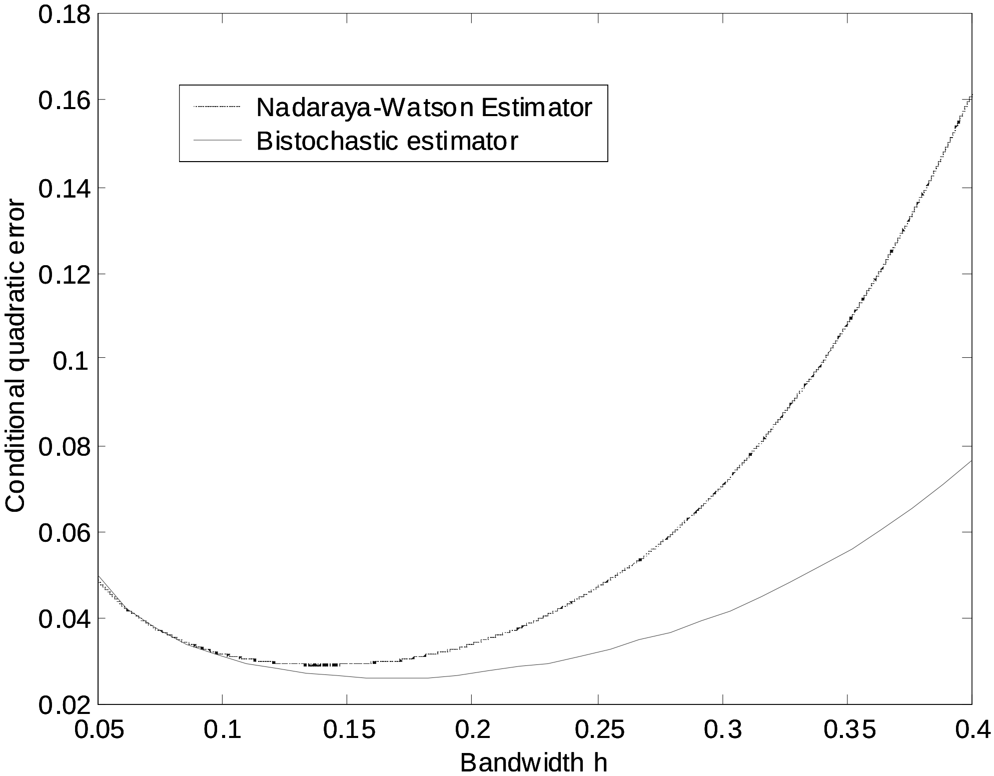
19 We present the uniform distribution case to avoid negative values in the *X*-variable axes. Results are similar for the normal distribution case.

The results are even more general. Figure 2 shows the conditional quadratic error for a wide range of bandwidth values. The greater efficiency of the bistochastic estimator, relative to the classical estimator, seems independent of the bandwidth value used. In particular, note especially the greater efficiency in the neighborhood of the optimal bandwidth for the stochastic estimator, whatever the optimal bandwidth criteria used. Also note that the efficiency gain may be even higher, since the optimal bandwidth for the probabilistic estimator differs from that of the stochastic smoother. This is the case in Figure 2. We further suggest computing an algorithm process for the whole range of bandwidths to achieve the most efficient probabilistic estimator.

Figure 2

CONDITIONAL QUADRATIC ERROR (d_c) FOR DIFFERENT h BANDWIDTH VALUES



Exercise design: $Y_i = m(X_i) + \varepsilon_i$, where $X_i \sim N(0,1)$, $m(X_i) = \exp(X_i)$ and $\varepsilon_i \sim N(0,0.2)$. The d_c function values correspond to the mean value for 200 independent samples.

Furthermore, the probabilistic estimator converges more quickly to the true curve than does the classical Nadaraya-Watson smoother. Note that the (negative) rate of variation of d_c for the bistochastic estimator is greater with respect to n than for the Nadaraya-Watson in all cases. For instance, the rates of variation of the bistochastic and Nadaraya-Watson estimators, between $n = 30$ and $n = 100$, are -58.3 and -52.2 percent, respectively, in the exponential and $\sigma^2 = 0.2$ case. Corresponding values between $n = 100$ and $n = 1000$ are -83.0 and -78.5, respectively, in the exponential and $\sigma^2 = 0.8$ case. By contrast, we find no evidence that the probabilistic estimator is superior to the local linear smoother in this respect.

7. CONCLUDING REMARKS

This paper analyzes the implications of adopting probabilistic nonparametric smoothing which satisfies a basic property that has been overlooked in the literature. The smoothing ensures that all noise is eliminated in the estimation if, and only if, it is probabilistic. Otherwise, the smoothing cannot avoid either eliminating some signal or leaving some noise.

The probabilistic or bistochastic smoother has also the following theoretical advantages. A probabilistic assignment model can be established between the estimated and the dependent variables. It implies an important additive decomposition property. It is also proved that the estimator reduces variability according to the robust criterion of majorization and Lorenz dominance if, and only if, it is probabilistic. In addition, it is shown that the estimator displays some other useful properties including consistency, symmetrical treatment of observations, and zero expected mean error irrespective of the number of observations (as the OLS case). Moreover, the smoothing can be generalized to the multidimensional case.

The probabilistic smoother itself imposes a double normalization of the weights matrix of the estimator, which is performed by using the low-cost iterative proportional-fitting algorithm proposed by Deming and Stephan (1940). This algorithm minimizes the Kullback-Liebler distance function with respect to the original weights matrix of the stochastic estimator.

Among the practical advantages of the probabilistic estimator, we find that the underlying Lorenz smoothing property of the estimator enhances the usefulness of the smoother in applied economics (in particular, in economic discrimination measurement). Moreover, there is an improvement over standard nonparametric methods that use positive weights in terms of the bias-variance trade-off as it is confirmed in a general simulation exercise. This raises a paradox. How can it be possible that the imposition of a new restriction (column normalization) improves the efficiency of the estimator? The probabilistic estimation takes into consideration information across intervals (see property 2 in section 5) that standard stochastic methods do not. Therefore, in the simulation exercise, the efficiency gain associated with the better use of information more than offsets the efficiency loss from the imposition of the new restriction. However, this line of inquiry requires extensive research beyond the scope of the present paper.

APÉNDIX A. THE BIRKHOFF ALGORITH (Birkhoff, 1946)

The way the Birhkoff algorithm operates can be easily seen from the example in (2):

$$W^B = \begin{bmatrix} \langle 0.81\rangle & 0.11 & 0.08 \\ 0.19 & \langle 0.68\rangle & 0.13 \\ 0 & 0.21 & \langle 0.79\rangle \end{bmatrix} = 0.68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} \langle 0.13\rangle & 0.11 & 0.08 \\ 0.19 & 0 & \langle 0.13\rangle \\ 0 & \langle 0.21\rangle & 0.11 \end{bmatrix}$$

$$= 0.68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.13\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & \langle 0.11\rangle & 0.08 \\ \langle 0.19\rangle & 0 & 0 \\ 0 & 0.08 & \langle 0.11\rangle \end{bmatrix}$$

$$= 0.68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.13\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.11\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \langle 0.08\rangle \\ \langle 0.08\rangle & 0 & 0 \\ 0 & \langle 0.08\rangle & 0 \end{bmatrix}$$

$$= 0.68\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.13\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + 0.11\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 0.08\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

APÉNDIX B. THE REGRESSOGRAM AS A BISTOCHASTIC ESTIMATOR

The regressogram is defined as the arithmetic mean of the Y variable across the corresponding h nonoverlapping intervals. The regressogram estimator ensures that the weights assigned to each observation of the response variable sum to unity, not only across rows but also across columns; that is, the weights matrix is bistochastic.

PROOF. Let $S = \{s_1(X), \ldots, s_h(X)\}$ be the nonoverlapping partition into h subgroups under consideration, let $U = \{n_1, \ldots, n_h\}$ be the within-groups population set, and let $V = \{\mu_1, \ldots, \mu_h\}$ be the associated response mean variable set. For the regressogram estimator,

$$z_1^i = \ldots = z_{n_i}^i = \mu_i, \quad \forall\, i = 1, \ldots, h.$$

In vector notation, $M = BY$, where B is the n-dimensional bistochastic matrix,

$$B = \begin{pmatrix} N_1 & 0 & \cdots & 0 \\ 0 & N_2 & \cdots & 0 \\ \cdots & \cdots & \ddots & 0 \\ 0 & 0 & \cdots & N_h \end{pmatrix},$$

and N_i is the n_i-dimensional square matrix,

$$N_i = \begin{pmatrix} 1/n_i & \cdots & 1/n_i \\ \cdots & \ddots & \cdots \\ 1/n_i \cdots & \cdots & 1/n_i \end{pmatrix} \quad \forall i = 1, \ldots, h.$$

APÉNDIX C. PROOF OF PROPERTY 6

The bistochastic estimator vector can be written as

$$Z = W^B \cdot Y = W^B \cdot W^{-1} \cdot W \cdot Y = W^B \cdot W^{-1} \cdot M.$$

In desegregated terms, it can be written as

$$\begin{aligned} Z(x_i) = & \left[W_{i1} \cdot W_{11}^{-1} + .. + W_{ij} \cdot W_{i1}^{-1} + .. + W_{in} \cdot W_{n1}^{-1}\right] \cdot M(x_1) + ... \\ & + \left[W_{i1} \cdot W_{1j}^{-1} + .. + W_{ij} \cdot W_{ij}^{-1} + .. + W_{in} \cdot W_{nj}^{-1}\right] \cdot M(x_i) + ... \\ & + \left[W_{i1} \cdot W_{1n}^{-1} + .. + W_{ij} \cdot W_{in}^{-1} + .. + W_{in} \cdot W_{nn}^{-1}\right] \cdot M(x_n) \end{aligned}$$

Hence, the sum of terms within the square brackets,

$$\left[W_{i1} \cdot W_{11}^{-1} + .. + W_{ij} \cdot W_{i1}^{-1} + .. + W_{in} \cdot W_{n1}^{-1}\right] + ... + \left[W_{i1} \cdot W_{1n}^{-1} + .. + W_{ij} \cdot W_{in}^{-1} + .. + W_{in} \cdot W_{nn}^{-1}\right]$$

must be unity. The above expression can be rewritten as

$$W_{i1}\left[W_{11}^{-1} + .. + W_{1j}^{-1} + .. + W_{1n}^{-1}\right] + ... + W_{in}\left[W_{n1}^{-1} + .. + W_{nj}^{-1} + .. + W_{nn}^{-1}\right]$$

We only need demonstrate that the inverse of a stochastic matrix sums to unity across rows.

Let $A = (a_{ij})_{i,j=1,..,n}$ be a stochastic matrix and let B be its inverse. Since $B{\cdot}A = I_n$ (I is the identity matrix), we know that

$$I_{ij} = b_{i+} \cdot a_{+j} = \sum_{k=1}^{n} b_{ik} \cdot a_{kj}.$$

Hence, from the stochastic property of matrix A, we obtain

$$\sum_{j=1}^{n} I_{ij} = 1 = \sum_{j=1}^{n}\sum_{k=1}^{n} b_{ik} a_{kj} = \sum_{k=1}^{n} b_{ik} \sum_{j=1}^{n} a_{kj} \Rightarrow \sum_{k=1}^{n} b_{ik} = 1.$$

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