INSTITUTIONALIZING UNCERTAINTY: THE CHOICE OF ELECTORAL FORMULAS(1)

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P. T. N.º 19/08

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(1) We wish to thank Serguei Kaniovski, Adam Przeworski, and Kanchan Chandra for valuable comments to earlier versions of this draft. We acknowledge the financial support of a resarch grant by the Spanish Instituto de Estudios Fiscales.

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Edita: Instituto de Estudios Fiscales
N.I.P.O.: 602-08-004-3
I.S.S.N.: 1578-0252
Depósito Legal: M-23772-2001

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ABSTRACT

Taken from an infinite set of divisors methods, the D'Hondt formula is the unique rule that maximizes the minimum number of seats for parties exceeding average size but not surpassing absolute majority of the votes. This property is also shared, in the quota set of methods, by the Droop quota. At the same time, these methods are those most commonly observed in practice. This paper relates the property stated with the stylized fact observed. If parties try to maximize the minimum number of seats for a given share of votes, then the D'Hondt formula should be chosen. This choice is consistent with rational parties that make institutional choices in an uncertain environment.

1. INTRODUCTION

This paper holds that if parties care for their worst results possible, as it is likely under conditions of uncertainty, then we should observe that the D'Hondt method is frequently chosen. This rule is shown to maximize, together with the Droop quota and other affine methods, the minimal number of seats for parties which are large enough to be larger than average but which do not command a majority of the votes, a property that has gone so far unnoticed in the literature. As a matter of empirical fact, the D'Hondt method is the most commonly observed formula, specially in newly established democratic regimes. According to our hypothesis, this fact should be related to the uncertainty surrounding the institutional choices in regime transitions and to the decisive role that parties above average size are likely to play in that choice. Parties smaller than average are likely to prefer formulas more biased towards the minority than D'Hondt is, whereas parties above majority prefer rules more biased towards the majority. If parties want to maximize their minimal result, intermediate parties should choose D'Hondt or a rule affine to it.

The most common dichotomy in electoral studies is the opposition of proportional vs. majoritarian systems, which is usually taken to mean multimember districts versus single member districts. The conclusion of this paper holds for districts of any size, and for any number of parties, but it is irrelevant for single seat districts or strictly bipartisan competition. We do not take any particular stance in the debate on the origins of proportional representation. However, we should mention that uncertainty about electoral results may be singled out as a background condition, when not a decisive factor, in the explanation of the choice of proportional systems in some important pieces of research (e.g. Andrews and Jackman 2005; Benoit 2004; Boix 1999; Blais, Dobrzynska and Indiridason 2004). Uncertainty plays a very specific role in our explanation: if everything that parties know is how to place themselves, along the continuum of possible vote results, in the intervals bound by the average size and the absolute majority of the votes, then the D'Hondt method of apportionment, together with other methods with strong affinity with this formula, should be the preferred rule by parties within those bounds if they use the maximin rule to guide their choices under uncertainty.

The paper is organized as follows: Section 2 presents the empirical regularity to be explained, the frequency of the D'Hondt method, and some alternative explanations. Section 3 makes a succinct case for a set of apportionment rules (comprising divisors and quotas) as the choice set of electoral formulas under democratic conditions. Building on previous work, the section also makes a summary description of the choice set, including the threshold functions of the electoral rules. Section 4 shows that the D'Hondt method, or any other method sharing the higher threshold function of this method, maximizes the minimal number of seats for parties above average but below majority. Section 5 compares the seat allocations produced by D'Hondt and by the most common method that shares the same higher threshold function, the Droop quota, and provides an argument for the hegemony of the former. Section 6 briefly concludes.

2. EMPIRICAL MOTIVATION

In the period of early democratization, from 1890 to 1940, nearly every country which adopted a list system adopted the D'Hondt formula, the Droop quota formula, or a variation thereof. Strict exceptions were, on the one hand, Spain in 1931-36, which employed a limited vote system, and, on the other hand, the Netherlands, where the simple (Hare) quota method was adopted from 1918-1933, although it moved to D'Hondt in 1937. Another, brief, exception to the general rule in this period is France, which experimented with a complicated variant of multi-seat majority rule in 1919-24,with no relation to the earlier process of democratization. Belgium introduced D'Hondt (the first country ever) in 1900, and moved to a modified version of the formula in 1919 (coincidentally with full male suffrage). Ireland introduced the Droop quota in 1922 and Greece, in a complicated multi-tier variant, in 1928.The following countries introduced the D'Hondt method coincidentally with suffrage extensions or democratization: Austria (1919), Denmark (1918, reform completed in 1920), Finland (1907), Germany (1919), Iceland (for the list seats since 1916), Italy (1919), Luxemburg (1919), Norway (1921), Sweden (1911), Switzerland (1919). The remaining early democracies kept single member majoritarian systems during their transitions: Australia, Canada, France, Great Britain, New Zealand and the United States (Penadés 2005).

Table 1 reports the number of electoral systems introduced during the 20th century in new democracies, or in new periods of democratization after non democratic rule, classified by the type of electoral formula used. The data for 1940 onwards come from Matt Golder's data set (Golder 2005). Although the choice of either D'Hondt or Droop has become relatively less frequent along the century, especially in the 80s and 90s, the sum of both methods represents 64,2% of the newly introduced proportional methods in the entire century, and 55,3% of the list (multimember) systems. In addition, the D'Hondt method is 4 times more frequent than the Droop quota method.

Table 1

ELECTORAL FORMULAS IN NEW DEMOCRACIES OR NEW DEMOCRATIC PERIODS

Electoral Formula	1900-1940	40s-50s	60s-70s	80s-90s	Total
D'Hondt method	12	6	9	15	41
Droop Quota method	1	3	2	4	10
Hare Quota or similar	1	5	4	19	30
Multimember non PR	1	3	4	5	13
Single member districts	6	5	16	31	58
Total	21	22	35	74	152

Source: Author's compilation with data from Golder (2005) and Penadés (2005.)

A case for contagion perhaps could be made for the strong early start of the D'Hondt method, but it should explain why the D'Hondt method frequently came under different names and algorithms in different countries (to this date, the D'Hondt method and the Hagenbach-Bischoff method are often reported as different methods, although they only denote different algorithms to find solutions for the same function). In any case, it should be explained why this and not other method was diffused. The habitual explanation of why D'Hondt is this: because it favors large parties. This explanation is, at best, incomplete, since there are infinite formulas which favor large parties more than the D'Hondt method does. That potential explanation could be completed by remarking that the D'Hondt formula is the exact (i.e. proportional when possible) method which is most favorable to majorities. This is true, but it incorporates a normative prejudice for proportionality which does not suit well the accompanying picture of seats-greedy large parties. At any rate, the normative ideal of proportionality is not universally accepted. A more subtle explanation could refer to the fact that the D'Hondt method is the only exact method which, in addition, encourages coalitions and, hence, discourages fragmentation (Balinski and Young 2001). Those are beautiful properties of D'Hondt, but they are normative ideals. We would like to show them to be by-products of interested choice under conditions of uncertainty.

3. THE CHOICE SET

The choice set for our problem consists only of electoral formulas, for the properties we want to highlight hold for any district magnitude. In particular, the choice set consists of the infinite number of allocation formulas based either on quotas of any size or on stationary divisors methods. Stationary divisor methods are given this name in Balinski and Rachev (1997) and can be succinctly described as methods for which there is an allocation algorithm that uses an arithmetic progression as a sequence of divisors (Penadés 2006). It includes any empirically known simple formula that is used for the allocation of seats to parties. The set is not constrained to proportional or exact methods; it ranges from strict equality to winner-takes-all (Penadés 2000, 2006).

To justify the choice set, we assume that the problem of democratization is solved, that is, we assume that parties agree to submit themselves to the rule of majorities, and, in that sense, that democracy is an equilibrium (Przeworski 2003). May (1952) demonstrated long ago that simple majority rule is the unique rule which is anonymous, neutral, decisive and positive responsive. Briefly, this characterization says that under majority rule there are no privileged individuals and no privileged options; and that decisions are always taken and respond to the relative sizes of parties. We assume that, if majority rule is politically accepted, then only representation methods satisfying these conditions are accepted.

The problem of representation, as understood in this paper, consists in the problem of apportioning a finite integer number m of seats to a set P of parties according to their share of the vote. Parties are indexed $i = \{1,\dots,p\}$, with $p \geq 2$. The voting process yields a vector of claims which is the votes vector $v = (v_1,\dots,v_p)$ where v_i is the claim of party i to a share of representation. An apportionment vector is denoted $s = (s_1,\dots,s_p)$, all integer, where s_i is the number of seats allocated to i. A representation function is a correspondence which assigns a set of apportionments to every votes vector. Such sets are called solution sets.

Single seat *Majority-plurality* rule can be written as follows:

$$F(v,1) = \left\{ s \,\middle|\, s_i = 1 \text{ and } s_j = 0 \text{ implies } v_i \geq v_j, \text{ and } \sum_{i=1}^{p} s_i = 1 \right\},$$

where simple majority rule is obviously a particular case when $p = 2$.

Call weak votes monotonicity to the property of rules such that, for any two parties, the smaller party can never obtain more seats than the larger party (Balinski and Young 2001). May's characterization of simple majority rule may be adapted to the context of representation and also be stated as follows: Majority-Plurality is the only single seat allocation rule which is neutral, anonymous, decisive, and weakly votes monotone (Penadés 2006).

To solve apportionment problems for $m > 1$ two further properties seem natural: homogeneity and balance (Balinski and Young 2001). Homogeneity guarantees that what counts for apportionments are the relative sizes of parties;

balance requires that, if parties are equal, seats should be distributed equally or as equally as possible. In Penadés (2006) it is demonstrated that, for two party problems, divisor methods are the only methods which are neutral, anonymous, decisive, weakly votes monotone, homogeneous and balanced. In addition, it can be easily shown that, for two party problems, quota methods are equivalent to the stationary subset of divisor methods. Hence, divisors and quota methods seem to be a natural extension of simple majority rule for the problem of multi-seat apportionment for two party problems.

In the domain of multiparty problems further properties are required to characterize either divisor or quota methods, but we do not have a complete characterization of the set that contains both (Balinski and Young 2001; Penadés 2006). However, we can single out quotas and stationary divisors as the only methods for which the seats expectations can be bounded for a party regardless of the vote distribution for other parties in multiparty contests. To put it differently, those are the methods for which the minimal and maximal price in votes for the marginal seat, for any seat-gaining party, can be determined ex-ante (Penadés 2006). This seems to us a reasonable property for formulas given the predicament of uncertainty. At any rate, every empirically observed electoral formula possesses this property. The divisor methods that lack this property, like those based on the geometric or harmonic means, are never used as electoral formulas.

3.1. Description of methods: divisors and quotas

Divisor functions are based on divisor criteria. A *stationary divisor criterion* α denotes, in these pages, any function $f(s) = a + bs$ defined for all integers $s \geq 0$, such that $a/b = \alpha$. We use the normalized form of the function $d(s) = s + \alpha$.The divisor method based on d can be described as the correspondence:

$$F(v,m) = \left\{ s \left| \min_{s_i \geq 1} \frac{v_i}{d(s_i - 1)} \geq \max_{s_j \geq 0} \frac{v_j}{d(s_j)}, \sum_{i=1}^{p} s_i = m \right. \right\}$$

This just follows the description of divisor methods by Balinski and Young (2001), but with one important difference: the set of divisor criteria is not limited here to the exact criteria such that $s \leq d(s) \leq s + 1$. In particular, the set of stationary divisor methods is defined as the set of methods based on criteria $d(s) = s + \alpha$ for $\alpha \in [0, \infty)$. Exact methods are those which produce perfectly proportional apportionments when this is possible; this set includes the Adams method $(\alpha = 0)$, the Sainte-Laguë method $(\alpha = 1/2)$ and the D'Hondt method $(\alpha = 1)$. Larger values of α induce greater bias in favor of larger parties (Balinski and Young 2001; Penadés 2006). At the limit, for $\alpha \to \infty$, we obtain multiple-seat majority rule (Penadés 2006).

General quota methods are based on a *quota scaling criteria* $Q(m,\sum v_i)=\frac{1}{m+\delta}\sum v_i$, where δ is a rational number $\delta > -m$. We call this number the *quota modifier* on which a particular method is based. The set of general quota methods is described by the following (Penadés 2006):

$$F(v,m)=\left\{ s \left| \min_{s_i \geq 1}\left(\frac{v_i(m+\delta)}{\sum v_i}-s_i+1\right) \geq \max_{s_j \geq 0}\left(\frac{v_j(m+\delta)}{\sum v_i}-s_j\right), \sum s_i = m \right. \right\}$$

The quota scaling criterion $\frac{1}{m}\sum v_i$ is the simple quota (also called proportional quota, or Hare's, or Hamilton's); the scaling criterion $\frac{1}{m+1}\sum v_i$ is usually called the Droop quota. Quota methods are exact if $\delta \in [-1,1]$. Largest remainders methods belong to the exact subset of quota methods. Beyond the bounds of exactness, at the limit, we obtain multiseat-majority for $\delta \to \infty$ and equality rule for $\delta \to -m$ (Penadés 2006).

In the case of bipartism, divisor methods are equivalent to quota methods. In multipartism, for every allocation problem and every divisor method there is at least one quota method that produces the same solution for that allocation problem.

3.2. Threshold functions

This section summarizes results in Penadés (2000, 2006). Define the set A as the set of all p -tuples of rational proportions of all possible votes distributions; and define the set B as the set of all possible distributions of m seats among p parties, that is $A=\left\{v \in Q^p \middle| v_i \in (0,1) \wedge \sum_{i=1}^{p} v_i = 1\right\}$ and $B=\left\{s \in N^p \middle| s_i \in [0,m] \wedge \sum_{i=1}^{p} s_i = m\right\}$. The fact that we move from the natural number of votes to proportions will be inconsequential in dealing with representation functions, because homogeneity is assumed. In what follows, v denotes a rational proportion and s a non negative integer.

Now, assume a representation function $F: A \to B$ with the properties of being decisive, impartial, weakly votes monotone, balanced and homogeneous; we define the subset $C \subseteq B$ as the set of all possible apportionments of m to p parties under F, that is $C = \mathrm{Ran}(F)$. We can now define the *lower threshold function* LT as a function yielding the necessary votes v for any of the p parties to obtain at least s seats under F:

$$LT=\left\{ \begin{array}{l} (s,v) \in [0,m] \times [0,1] \\ \left| \forall s \in C \, \forall v \in A \, \forall i \in P (s_i \geq s \to v_i \geq v) \wedge (\nexists x > v | (s_i \geq s \to v_i \geq x) \right. \end{array} \right\}.$$

Again, we can define the *higher threshold function* HT as a function yielding the votes v that, for any of the p parties, is it necessary not to exceed to obtain at most s seats:

$$HT = \left\{ \begin{array}{l} (s,v) \in [0,m] \times [0,1] \\ \left| \forall s \in C \, \forall v \in A \, \forall i \in P (s_i \leq s \rightarrow v_i \leq v) \wedge (\nexists x < v | (s_i \leq s \rightarrow v_i \leq x) \right. \end{array} \right\}.$$

Both functions express necessary conditions. The contrapositive of the implications in any of the functions expresses an equivalent sufficient condition. It may seem more natural to write the higher threshold's condition in this manner: $v_i > v$ implies $s_i > s$, or, equivalently $v_i > v$ implies $s_i \geq s+1$. Hence, we can redefine the upper threshold as a *sufficient votes threshold* $ST(s) = HT(s-1)$. Stated in this way, the upper threshold is read as the sufficient condition to obtain a number of seats, while the lower threshold still indicates the necessary condition for obtaining that number of seats. This is how thresholds are often referred to in the literature.

From the threshold functions, it is immediate to define the functions which yield the minimal and maximal number of seats that a given party may expect with its votes, in a competition of p parties for m seats to be apportioned by formula F. We define the *upper bound* of the expected seats of party i as

$$UB_i = \left\{ \begin{array}{l} (v_i, s) \in (0,1) \times [0,m] \\ \left| \forall v \in A \forall s \in C \forall i \in P(s_i \geq s \rightarrow v_i \geq v = LT(s)) \right. \end{array} \right\}$$

and the *lower bound* as

$$LB_i = \left\{ \begin{array}{l} (v_i, s) \in (0,1) \times [0,m] \\ \left| \forall v \in A \forall s \in C \forall i \in P(s_i \leq s \rightarrow v_i \leq v = HT(s)) \right. \end{array} \right\}.$$

For any given linear divisor method based in α, any total number of seat m, and any number p of parties, the necessary or minimal votes to obtain s seats is given by

$$LT(s; m, p, \alpha) = \frac{s - 1 + \alpha}{m - 1 + p\alpha},$$

and the maximal number of votes to obtain s seats is given by

$$HT(s; m, p, \alpha) = \begin{cases} \frac{s+\alpha}{m+1+p\alpha-p} & \text{if} \quad \alpha \in [0,1] \text{ and } s \leq m-p+1 \\ \frac{s+\alpha}{\alpha m+\alpha+s-s\alpha} & \text{if} \quad \alpha \in [0,1] \text{ and } s \geq m-p+1 \\ \frac{s+\alpha}{m-1+2\alpha} & \text{if} \quad \alpha \geq 1 \end{cases}$$

For any general quota method based on scaling criterion with quota modifier δ, any total number of seats m, and any number p of parties, the necessary or minimal votes to obtain s seats is given by

$$LT(s; m, p, \delta) = \frac{sp - p + 1 + \delta}{p(m + \delta)},$$

and the maximal number of votes to obtain s seats is given by

$$\mathrm{HT}(s;m,p,\delta)=\begin{cases}\frac{sp+p-1+\delta}{p(m+\delta)} & \text{if} \quad \delta\le 1 \text{and}\, s\le m-p+1\\ \frac{-sm+s^2-m-\delta}{(-m+s-1)(m+\delta)} & \text{if} \quad \delta\le 1 \text{and}\, s\ge m-p+1\\ \frac{1}{2}\frac{1+\delta+2s}{m+\delta} & \text{if} \quad \delta\ge 1\end{cases}$$

4. MAXIMIZING THE MINIMAL NUMBER OF SEATS

We assume that the party of reference has enough information to place itself on a certain range as regards the vote distribution in future elections, namely, that the party is above average but cannot secure an absolute majority, that is $v\in$ $[1/p,1/2]$. We further assume that there are at least three parties and two seats to be distributed according to their votes, and both numbers are known to any party. We do not assume that the parties have any information about the vote that their rivals may obtain. Our main result is that the D'Hondt or the Droop formulas maximize the minimal number of seats for a party placed in the above interval. The results follows directly from the equations for the threshold functions.

Remark 1: *The higher threshold functions for either divisors or quota methods form two bundles. First, for* $\alpha\le 1$ *the pair* $(1/p,(m-p+1)/p)$ *always belongs to* HT *if* $p\le m+1$, *and the pair* $(1/(m+1),0)$ *always belongs to* HT *if* $p\ge m+1$. *The same is true for* $\delta\le 1$. *Second, for* $\alpha\ge 1$ *the pair* $(1/2,(m-1)/2)$ *always belongs to* HT, *and the same is true for* $\delta\ge 1$.

Lemma 1: *In the set of stationary divisor methods, the D'Hondt method minimizes* HT, *that is, the maximal proportion of the votes, for any numbe* s *of seats in the interval* $(m-p+1)/p\le s\le(m-1)/2$, *if* $p\le m+1$, *or in the interval* $0\le s\le(m-1)/2$ *if* $p\ge m+1$. *Equivalently, the D'Hondt method minimizes the sufficient votes threshold* ST *for seats in the interval* $(m+1)/p\le s\le(m+1)/2$ *if* $p\le m+1$, *or in the interval* $1\le s\le(m+1)/2$ *if* $p\ge m+1$.

Proof: Check the sign of the High Threshold $(\mathrm{HT}(\alpha))$ function to determine the increasing-decreasing patterns with respect to α. We show conditions on s, so that the $\mathrm{HT}(\alpha)$ function reaches a global minimum w.r.t α in its domain. From the definition:

$$\mathrm{HT}(s,m,p,\alpha)=$$

$$\frac{s+\alpha}{m+1+p\alpha-p} \text{ if } \alpha\in(0,1] \text{ and } s\le m-p+1$$

$$\frac{s+\alpha}{\alpha(m+1)+s(1-\alpha)} \text{ if } \alpha\in(0,1] \text{ and } s\ge m-p+1$$

$$\frac{s+\alpha}{m-1+2\alpha} \text{ if } \alpha\ge 1$$

we check that the sign of HT is positive for all $\alpha > 1$, and negative for all $\alpha < 1$, since the function HT is continuous at $\alpha = 1$ we conclude that there is a global minimum at this point.

For the first part of the High Threshold equation we have that: δ

$$\frac{\partial HT}{\partial \alpha} = \frac{(m+1+p\alpha - p) - p(s+\alpha)}{(m+1+p(\alpha-1))^2} =$$

$$= \frac{m+1-p(s+1)}{(m+1+p(\alpha-1))^2} \leq 0$$

$$m+1-p(s+1) \leq 0 \Leftrightarrow s \geq \frac{m+1}{p} - 1$$

Next, find a condition for the for the function defined as the second part of the equation to be negative, that is:

$$\frac{\partial HT}{\partial \alpha} = \frac{(\alpha m + \alpha - s\alpha + s) - (s+\alpha)(m-s+1)}{(\alpha(m+1) + s(1-\alpha))^2} =$$

$$= \frac{s(s-m)}{(\alpha(m+1) + s(1-\alpha))^2} \leq 0$$

Since $s(s-m) = 0$, has two roots $(s = 0,$ and $s = m)$, this expression is always negative in the domain of s, because $s \leq m$.

Finally, for equation the third part of the equation

$$\frac{\partial HT}{\partial \alpha} = \frac{m-1-2s}{(m-1+2\alpha)^2} \geq 0$$

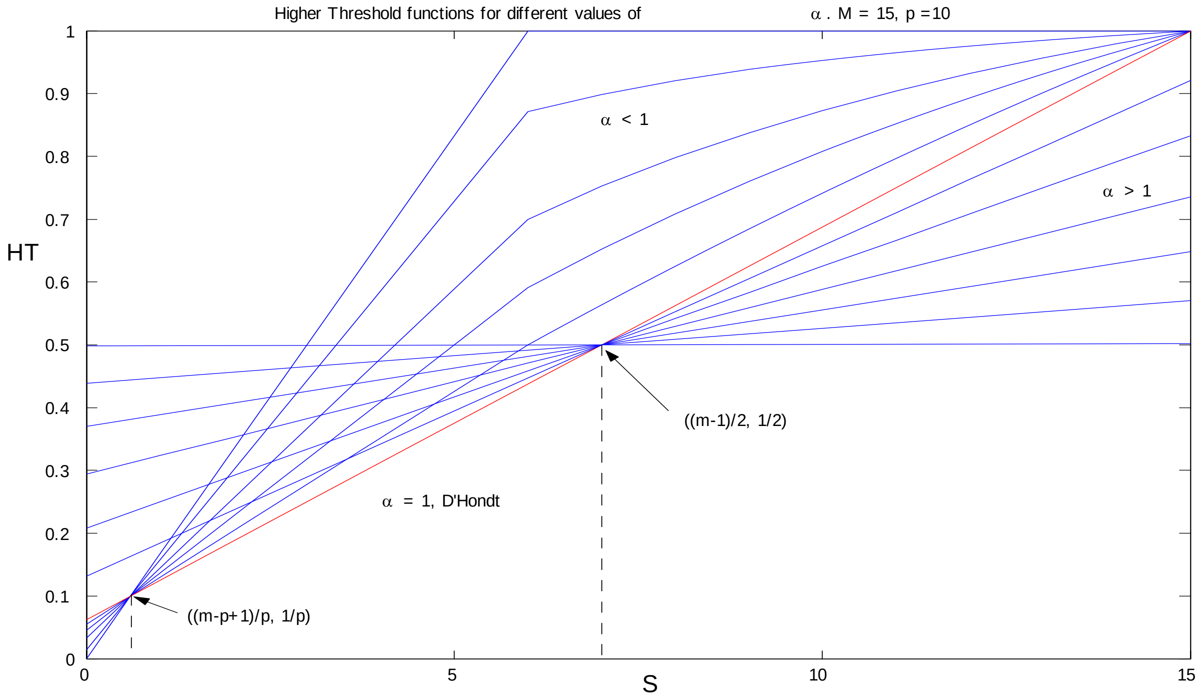
The denominator is always positive, therefore the sign of this expression is the sign of the numerator, so we find:

$$m-1-2s \geq 0 \Leftrightarrow s \leq \frac{m-1}{2}$$

Since all conditions onsare satisfied, the proof is concluded.

Figure I illustrates both remark I and lemma I for a case in which $p = 10$ and $m = 15$. For seats above $s = (m-p+1)/p$ and bellow $s = (m-1)/2$ the D'Hondt function strictly minimizes the maximal proportion of the votes that can be associated to those numbers of seats. For $s = (m-p+1)/p$ the D'Hondt method has the same higher threshold as any other divisor method based on $\alpha < 1$; for $s = (m-1)/2$ the D'Hondt method has the same higher threshold as any other divisor method based on $\alpha > 1$. Recall that $ST(s) = HT(s-1)$, so we may also state that for seats $(m+1)/p \leq s \leq (m+1)/2$ the D'Hondt function minimizes the sufficient votes threshold. If $p \geq m+1$, a possibility not shown in the figure, then the HT functions are minimized in the interval $0 \leq s \leq (m-1)/2$ and the ST functions are minimized in the interval $1 \leq s \leq (m+1)/2$.

Figure I



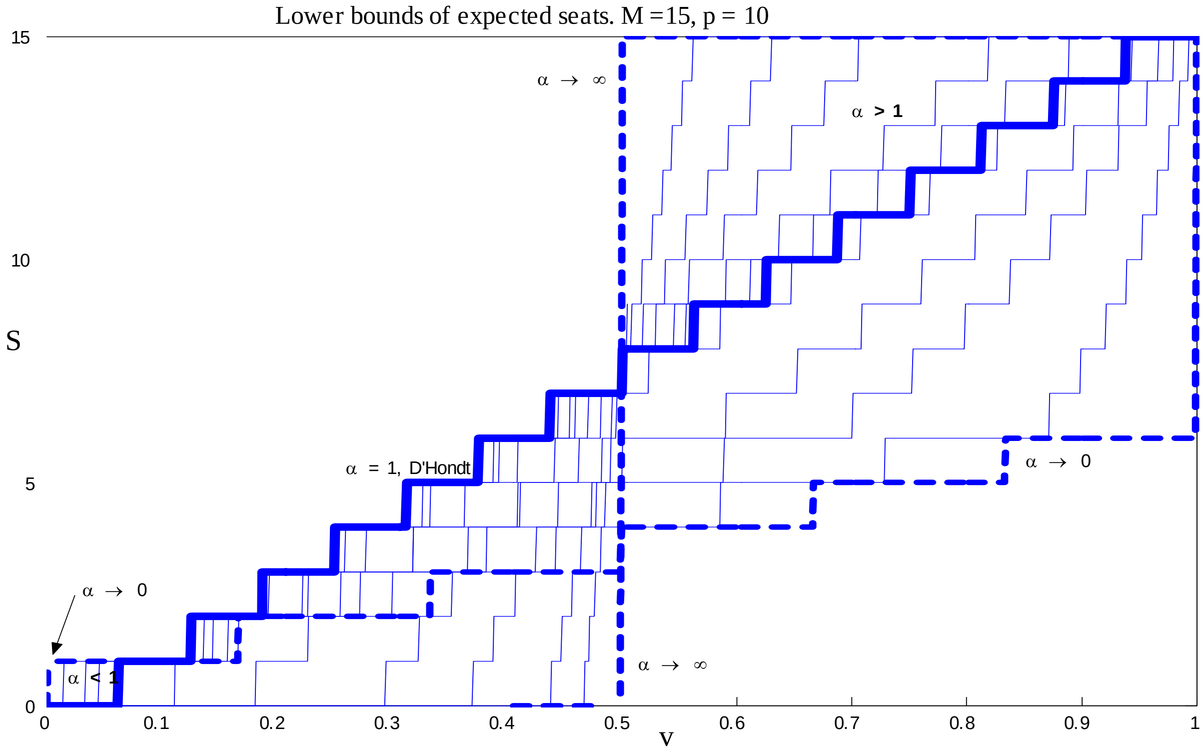
Theorem I: *In the set of stationary divisor methods, the D'Hondt method maximizes lower bound (LB), that is, the minimal expected number of seats, for parties obtaining a proportion of the votes in the interval* $1/p \leq v \leq 1/2$ *if* $p \leq m+1$, *or in the interval* $1/(m+1) \leq v \leq 1/2$ *if* $p \geq m+1$.

Proof: The theorem follows directly from the definition of the expected payoff functions, from remark bundles and from lemma 1.

Figure 2 illustrates theorem 1, also for the ten parties and fifteen seats case. Theorem 1 expresses our result most intuitively: for parties of sizes between average size and half of the votes, the D'Hondt method maximizes the minimally expected seats. In the figure, the minimal expectation of the D'Hondt method dominates the minimal expectation of any other formula for parties ranging between 10% of the vote and the vote majority. As the figure clearly depicts, parties obtaining less than the vote average would improve their base with more egalitarian formulas, that is, formulas having a flatter expectation, which are the set of formulas such that $\alpha < 1$.Again, a party obtaining a majority of the votes would obviously do better with more majority-biased formulas, those for which the expectation increases more sharply around majority. At the limit, as $\alpha \to \infty$, the expectation approaches the winner-takes-all apportionment.

The HT function of the D'Hondt and Droop methods are identical: $HT(s,m,p,\alpha)\big|_{\alpha=1} = HT(s,m,p,\delta)\big|_{\delta=1} = \frac{s+1}{m+1}$. Hence, it comes to no surprise that the Droop quota can be singled out among the set of general quotas for the same reasons stated in lemma minHTDhondt.

Figure 2



Lower bounds of expected seats. M =15, p = 10

Lemma 2: *In the set of general quota methods, the Droop quota minimizes* HT, *that is, the maximal proportion of the votes, for any number* s *of seats in the interval* $(m-p+1)/p \leq s \leq (m-1)/2$, *if* $p \leq m+1$, *or in the interval* $0 \leq s \leq (m-1)/2$ *if* $p \geq m+1$. *Equivalently, the Droop quota minimizes the sufficient votes threshold* ST *for seats in the interval* $(m+1)/p \leq s \leq (m+1)/2$ *if* $p \leq m+1$, *or in the interval* $1 \leq s \leq (m+1)/2$ *if* $p \geq m+1$.

Proof: The proof proceeds as in lemma 1.

Theorem 2: *In the set of general quota methods, the Droop method maximizes lower bound* (LB), *that is, the minimal expected number of seats, for parties obtaining a proportion of the votes in the interval* $1/p \leq v \leq 1/2$ *if* $p \leq m+1$, *or in the interval* $1/(m+1) \leq v \leq 1/2$ *if* $p \geq m+1$.

Proof: As in theorem 1.

Corollary 1: *In the set of methods for which the expectations can be bounded (including divisors and quota methods), either the D'Hondt or the Droop methods maximize the lower bound* (LB_i), *that is, the minimal expected number of seats, for parties obtaining a proportion of the votes in the interval* $1/p \leq v \leq 1/2$ *if* $p \leq m+1$, *or in the interval* $1/(m+1) \leq v \leq 1/2$ *if* $p \geq m+1$.

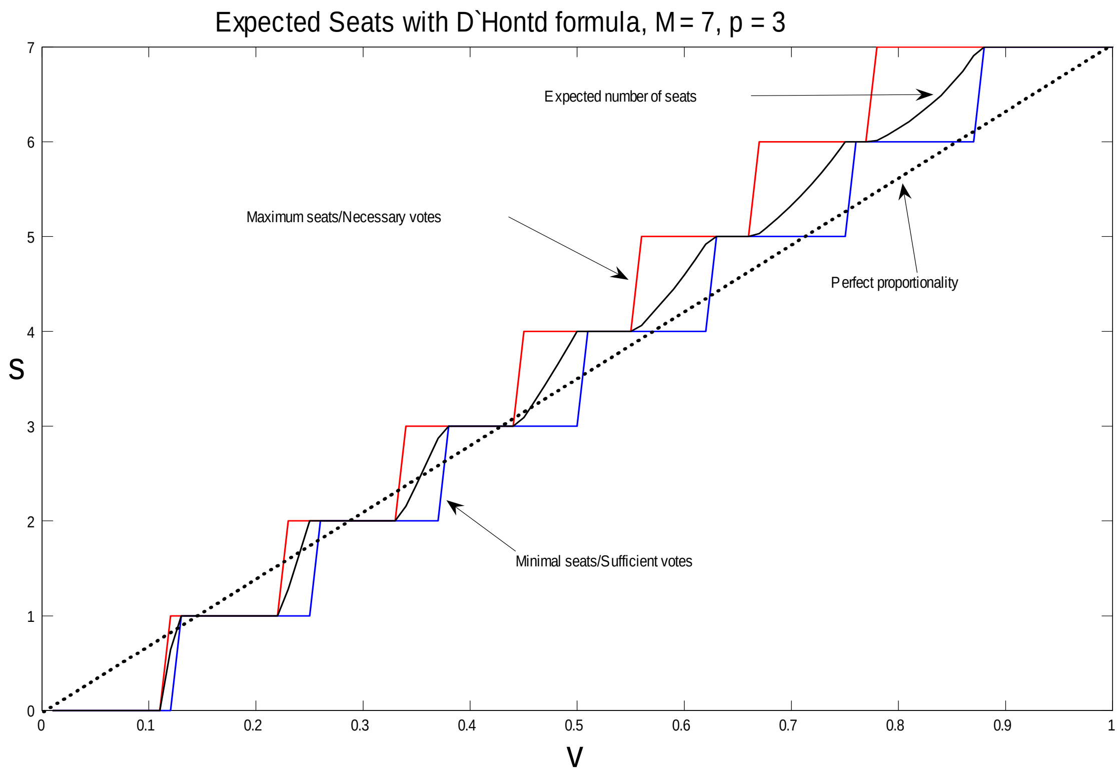
Under conditions of uncertainty, when parties know their likely electoral size only within a relatively ample range and do not know how the rest of the votes will be distributed, the choice of a rule that maximizes de minimal number of seats may be reasonable. Again, it is reasonable to suppose that precisely those parties above average (but bellow majority) will be determinant in the collective choice. Hence our prediction: with enough uncertainty on the results and with a reasonably fragmented party system, electoral rules will be chosen with higher thresholds given by $HT = \frac{s+1}{m+1}$, or, what is the same, with minimal seat expectation equal to $\lfloor v + mv \rfloor$, where v denotes the proportion of the votes. The D'Hondt and Droop method are the best known formulae that satisfy that condition, and, as we have seen, they are the most frequently observed formulas for proportional representation. Still, the D'Hondt method is far more common than its quota counterpart. In the next section we argue that the reason for this is that, as a rule, the D'Hondt method yields more seats than the Droop quota method for parties in the interval in which we have focused our interest.

5. EXPECTED SEATS: D'HONDT *VS*. DROOP

In the previous section we have stated that the D'Hondt divisors method and the Droop quota method display a property that makes them a likely choice for intermediate to large parties under conditions of uncertainty. Both methods are commonly observed in democracies with proportional representation as discussed in the introduction of the paper. However, the D'Hondt method delivers more expected seats.

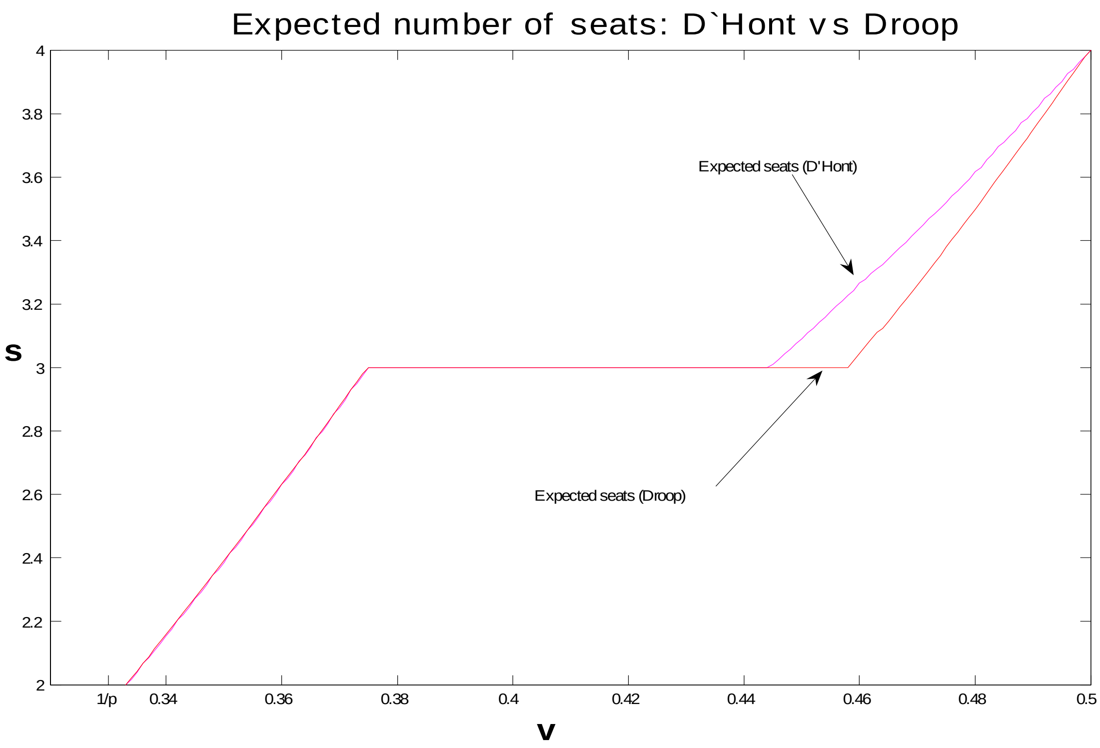
The expected number of seats is a random variable with unknown probability distribution, therefore we have resorted to numerical methods to obtain an approximation to the true value of the expected number of seats. In figure 3, the domain of votes range from 0% of the votes to 100%, and the number of seats range from 0 seats to the total number of seats to be allocated. In the example below, the number of seats is 7 to be allocated among 3 parties according to the D'Hondt rule. In addition to the minimum and maximum number of seats (upper and lower bounds), we have a representation of the expected number of seats given a proportion of votes obtained by a party. The procedure we have pursued to numerically approximate the true value of the expected number of seats is as follows: Consider a point ν_0 in the domain of votes, and associate this number to the votes obtained by a reference party. The number of votes that can be allocated to the remaining two parties is $1 - v_0$, and with this number of votes we generate all possible distributions in a discretization of the interval $[0, 1 - v_0]$ among the two remaining parties. For each distribution we compute the number of seats apportioned to each party, and compute the expected number of seats to the reference party as the arithmetic average of seats allocated in each distribution. Next we move from ν_0, to a neighbors point in the domain, v_1, and repeat the process.

Figure 3


Expected Seats with D`Hontd formula, M = 7, p = 3

In figure 4, we have plotted together the expected number of seats for the D'Hondt and the Droop methods in the interval $(1/p,1/2)$ of votes.

Figure 4


Expected number of seats: D`Hont vs Droop

Some salient features are apparent from the inspection of figure 4. First, the expected number of seats under the D'Hondt method is higher than the number of seats delivered by the Droop formula over the range under our consideration The average number of seats delivered under the Droop formula is 3 the D'Hondt formula delivers 3.04 on average. The maximal difference between D'Hondt and Droop is around 7.6% where D'Hondt delivers 3.23 seats and Droop still delivers 3 seats. Second, the picture above has two clearly differentiated areas, one where both formulas provide almost identical number of expected seats and another, where the D'Hondt formula clearly departs from the Droop, and then converges again at the point where 50% of the votes deliver 4 seats under both formulas. In the first area, one can single out certain distributions of votes where the expected number of seats delivered by Droop are slightly higher than the D'Hondt. A Monte Carlo simulation provides us with a numerical estimation of the number of distributions in the interval where one can find this particular result. In just about 6.2% of the distributions the Droop formula provided an average of0.0003 more seats than the D'Hondt formula. That is, the number of distributions where Droop provides more expected seats than D'Hondt is very small, and the difference in favor of Droop is extremely small. Our conclusion is that the D'Hondt and Droop methods are essentially identical although D´Hondt dominates Droop in the relevant interval.

6. CONCLUSION

This paper is only a first step towards a general model of electoral systems as equilibrium institutions. Since electoral systems are methods of choice, we could describe the institutional equilibrium as the set of rules that choose themselves, but this theoretical object is not yet within reach. An electoral system is defined at least by two variables: the magnitude of districts and the representation method; although several other variables, as it is well known, usually complicate the picture, like the variance of magnitude, legal thresholds, multiple tiers, adjustments seats, and so on. We have taken the first steps to develop a theory of preferences for representation methods (or electoral formulas). We have shown that (for a wide range of party configurations) the problem of choice of method can be separated from the choice of magnitude. There are several reasons to expect magnitudes to be large or small, but we can show the reasons to expect the D'Hondt method (or at least an affine method) to be chosen, under moderate fragmentation, for whatever the electoral magnitude, when parties are uncertain about the results but no party expect to reach a majority of the votes. This approach tackles only one aspect of the problem, in two senses: it considers the choice of electoral formula but not of magnitude (not to speak of further complications) and it considers only the theory of preferences, not the conditions of equilibrium for the chosen rule. However, we believe that this first step may contribute to clarify the wider problem. If democracy is the institutionalization of uncertainty, in Przeworski's felicitous phrase (1991), we need to understand, step by step, how politicians choose electoral rules that help them cope with an uncertain environment.

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SÍNTESIS

PRINCIPALES IMPLICACIONES DE POLÍTICA ECONÓMICA

Este trabajo demuestra la existencia de una propiedad analítica del método D'Hondt (compartida por el método de cuota Droop) que no ha sido observada antes: dicho método maximiza los escaños mínimos esperados para partidos que no son ni demasiado pequeños (superan a la media) ni demasiado grandes (no llegan a la mayoría absoluta). La elección de dicho método es coherente con la lógica de la decisión bajo incertidumbre en la mayoría de las situaciones típicas de diseño institucional. De este modo algunas propiedades bien conocidas de dicho método, como el hecho de que es proporcional pero favorece a los grandes partidos, aparecen como un subproducto de la elección bajo incertidumbre por parte de partidos que no pueden saber si son lo suficientemente grandes como para alcanzar la mayoría.

En los sistemas electorales que emplean distritos con múltiples candidatos, la fórmula electoral más frecuente es la fórmula de divisores de D'Hondt. De 152 casos contabilizados de decisión sobre la fórmula electoral en regímenes democráticos occidentales durante el siglo XX –de los que sólo en 98 se eligieron fórmulas para la representación proporcional– en 41 casos se eligió el método D'Hondt, y en 10 más el muy semejante método de cuota Droop. En 30 casos se eligieron métodos más favorables a los partidos menores, la mayor parte de ellos a partir de los años 80, y en 10 casos fórmulas más favorables a los partidos mayores.

El método D'Hondt ha sido valorado de muy distintas maneras en la literatura. De sus propiedades analíticas destacan dos que son complementarias: es el único método proporcional que nunca ofrece incentivos a la escisión de los partidos; es el método proporcional que más favorece a los partidos grandes. Con independencia de que estas propiedades sean o no deseables, ningún modelo de economía política predice la elección de este método de forma satisfactoria. Si el método D'Hondt fuera el resultado de elección de coaliciones dominantes, no se explica por qué no se escogen reglas aún más favorables a los grandes partidos. En definitiva, no se explica por qué el conjunto de elección debe limitarse a reglas que son aproximadamente proporcionales. Este trabajo propone un problema de elección de reglas en el que el conjunto de elección contiene los infinitos métodos de reparto que forman un continuo entre la igualdad de escaños entre todos los partidos y la asignación de todos los escaños al ganador. Los métodos proporcionales son un subconjunto de este continuo.

En este trabajo se demuestra que el método D'Hondt es el método de divisores que maximiza el mínimo número de escaños posibles para partidos cuyos votos esperados superan a la media de los partidos pero no alcanzan la mayoría absoluta de los votantes. El método de cuota Droop tiene esta misma propiedad, pero el número de escaños esperados para los partidos dentro de ese recorrido es, en general, menor con este método. Sostenemos que en un problema de elección con suficiente incertidumbre, los grandes partidos, cuando no están seguros de poder ser dominantes, revelan con su elección del método D'Hondt su preferencia minimax.

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