Papeles de Trabajo

THE GINI COEFFICIENT: MAJORITY VOTING AND SOCIAL WELFARE*

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ABSTRACT

Majority voting and social evaluation functions are the main alternatives proposed in the literature for aggregatingindividual preferences. Despite these being very different, this papershows that the ranking of income distributions, symmetric under the same transformation, by S-Gini consistent social evaluation functions and majority voting coincide if and only if the inequality index under consideration is the Gini coefficient. In this case, we show that the equally distributed equivalent income is equal to the median of the distribution. In addition, we find that the Gini coefficient is just an affine function of the median–mean ratio.

JEL: Classification: D31, D63, P16.

Key Words: social evaluation function; the Gini coefficient; majority voting; median–mean ratio; symmetric distribution.

1. INTRODUCTION

Traditionally, two strategies have been adopted by scholars to aggregate individual preferences and, hence, to rank distributions: 1) a political process like the majority voting mechanism (see, among others, Black, 1948, Romer, 1975 and Bishop *et al.*, 1991); and 2) a social evaluation function (SEF) derived from a set of “desirable” assumptions (Kolm, 1969, Atkinson, 1970 and Blackorby *et al*., 2002). The first procedure, majority voting, is the binary decision rule most commonly used in decision-making bodies, and involves selecting the distribution that receives more than half the votes. Meanwhile, a social evaluation function provides the set of axioms that has to be assumed in order to reach a particular social decision. Despite theirevident differences, the two approaches have recently been linked. Salas and Rodríguez (2012) have shown that for those distributions that are symmetric under the same strictly increasing transformation, the Atkinson**–**Kolm**–**Sen (AKS) class of utilitarian social evaluation functions (Kolm, 1969, Atkinson, 1970 and Sen 1973), consistent with the Kolm-Atkinson index of inequality, accords with the majority voting procedure.

In principle, the extension of this result to the class of rank-dependent SEFsconsistent with the widely used Gini coefficient isproblematic given the result in Newbery (1970). This author found that there is no differentiable strictly concave utility function such that a utilitarian SEF *W* accords with the Gini coefficient. Worse still, Dasgupta *et al.* (1973) generalized Newbery’s result from *W* to any strictly quasi-concave SEF and, later on, Lambert (1985) directly generalized Newbery’s result from *W* to any differentiable SEF.

Fortunately, some authors (see Sheshinski, 1972, Sen, 1973, Kakwani, 1985 and Lambert, 1985) have argued that a convincing rationale for the use of a SEF consistent with the Gini coefficient could still exist if we abandon the class of individualistic social evaluation functions[1]. In particular, Kakwani (1985) and Lambert (1985) provided a positive result by widening the domain for personal preferences to incorporate envy or altruism.

In this paper we extend the link between the AKS class of utilitarian SEFs and majority voting to thewider class of Kakwani–Lambert (KL) SEFs (Kakwani, 1985 and Lambert, 1985) consistent with the class of S-Gini indices. For this purpose, we first look for the transformation that makes the equally distributed equivalent income (EDE) of this class of rank-dependent KLSEFs equal to the median income. And we then show that majority voting and the class of rank-dependent KL SEFs are consistent if and only if the inequality index under consideration is the Gini coefficient.

A number of political economy models proposed in the literature rely on the (inverse) relationship between inequality and the median–mean ratio (see, for example, Meltzer and Richard, 1981 and Alesina and Rodrik, 1994). However, no theoretical proof of this link has been provided. A by-product of this paper is the result that the Gini coefficient can be written as a simple affine decreasing function of the median–mean ratio. Thus, the widely used Gini coefficient can be summarized by two common measures of position, namely the mean and median values. These results are illustrated with data drawn from the Survey on Income and Living Conditions (EU-SILC) dataset for the European Union for the period 2005-2007.

The structure of the paper is as follows. In Section 2, we provide our main resultsfor the class of rank-dependent KLSEFs. Section 3 illustrates these results with data drawn from 26 European countries over the period 2005-2007, while Section 4 presents some concluding remarks.

2. A CLASS OF RANK-DEPENDENT SEFS CONSISTENT WITH MAJORITY VOTING

Let’s assumean odd finite number of homogeneous income recipients $n \in N$ where N is the set of natural numbers greater than one. An income distribution for this population is represented by a positive ascending-ranked vector $Y = (Y_1, Y_2, \ldots, Y_n)$, so that $0 < Y_1 \leq Y_2 \leq \cdots \leq Y_n$. The set of all (positive ascending-ranked) income distributions is denoted by $D = \bigcup_{n \in N} D^n$.

[1] In 1978, Blackorby and Donaldson set out a list of properties that characterized (although not completely) the social evaluation functions which accord with the Gini coefficient. They should be homothetic, quasi-concave and additive but not separable. In 2001, Aaberge fully characterized the preference orderings related to the Gini coefficient and to the extended S-Gini coefficients.

A transformation τ of Y from D^{n} to $\mathbb{R}^n$: $(Y_1, Y_2, \ldots, Y_n) \rightarrow (\tau(Y_1), \tau(Y_2), \ldots, \tau(Y_n))$, is said to be symmetric if it satisfies the condition that

$$\tau(Y_m) - \tau(Y_{m-s}) = \tau(Y_{m+s}) - \tau(Y_m) \quad (1)$$

for all $s \in \{1, 2, \ldots, m-1\}$, where $m = \frac{n+1}{2}$ is the position of the median income[2]. An example of a symmetric transformation would be the log transformation for lognormal income distributions.

For all $Y \in D$ we define a continuous SEF W^{n} from D^{n} to $\mathbb{R}$: $(Y_1, Y_2, \ldots, Y_n) \rightarrow W^n(Y_1, Y_2, \ldots, Y_n)$, for all $n \in N$ that ranks the set of all income distributions according to welfare. The set of all continuous, increasing and S-concave SEFs W^n for all $n \in N$ is denoted by Ω.

We also define the EDE ξ corresponding to an income distribution $(Y_1, Y_2, \ldots, Y_n) \in D^{n}$ for all $n \in N$ as the value of ξ that solves:

$$W^n(\xi, \xi, \ldots, \xi) = W^n(Y_1, Y_2, \ldots, Y_n) \quad (2)$$

The continuity and monotonicity of $W^{n} \in \Omega$ guarantee a unique solution for $\xi = \Xi^n(Y_1, Y_2, \ldots, Y_n)$. The function Ξ^n is a particular numerical representation of W^{n}. More generally, any monotone transformation of Ξ^{n} represents the same ordinal preferences as in W^{n}.

Assuming that the inequality index ^{n} from D^n to $\mathbb{R}$: $(Y_1, Y_2, \ldots, Y_n) \rightarrow {}^{n}(Y_1, Y_2, \ldots, Y_n)$ for all $n \in N$ ranks the set of all income distributions, the set of all continuous and S-convex ^{n} for all $n \in N$ is denoted by Γ.

We concentrate on a particular subset of inequality indices, ${}^{n} \subset \Gamma$, that was introduced by Donaldson and Weymark (1980) andis called the family of single-parameter Ginis (or S-Ginis). This class of inequality indices is defined as ^{n} from D^{n} x $(1, \infty)$ to $\mathbb{R}$: $(Y_1, Y_2, \ldots, Y_n; \ ) \rightarrow {}^{n}(Y_1, Y_2, \ldots, Y_n; \ )$ where

$$^{n}(Y; \ ) = 1 - \frac{1}{(\ )} \sum_{1} (\ ) Y, \ Y \ D^n, n \ N, \ > 1, \quad (3)$$

(Y) is the mean of the distribution Y and $(\ ) = \frac{(n+1- \ ) - (n- \ )}{n}$, $\sum_{1} (\ ) = 1.$[3] This family of indices belongs to the class of linear rank-dependent inequality indices proposed by Mehran (1976) and coincides with the standard Gini coefficient when $= 2.$

Donaldson and Weymark (1980) proved that this family of inequality indices could be derived directly from a SEF as follows:

$$^{n}(Y; \ ) = 1 - \frac{(\ )}{(\ )} \text{ for all } Y \ D^{n} \text{ and } n \ N. \quad (4)$$

Therefore, for the S-Gini indices we obtain the following SEF:

$$W^n(Y; \ ) = \Xi^n(Y; \ ) = \ (Y)(1 - \ ^{n}(Y; \ )) = \sum_{1}^{n} (\ ) Y \quad (5)$$

whichbelongs to the Yaari family of rank-dependent SEFs (Yaari, 1987 and 1988).

More generally, Kakwani (1985 and 1986) and Lambert (1985) proposed the following class of SEFs:

$$W^n(Y; \ ) = \Xi^n(Y; \ ) = \ (Y) \ 1 - \ ^{n}(Y), \ Y \ D^n, n \ N, 0 < \ \leq 1, \quad (6)$$

where the parameter *k* denotes the trade-off between efficiency (mean) and inequality. This parameter can be justified under the Ebert (1987) approach which explicitly takes into account value judgments of the split-ups between mean and inequality. For this family of SEFs we can derive the following family of inequality indices (we shall call it the KL family of inequality indices):

$$^{n}(Y) = \ ^{1} \ 1 - \frac{(\ ; \ )}{(\ )}, \ Y \ D^n, n \ N, 0 < \ \leq 1. \quad (7)$$

By assuming that the class of S-Gini indices belongs to the KL family of inequality indices, equation (6) becomes:

$$W^n(Y; \ ; \ ) = \Xi^n(Y; \ ; \ ) = \ (Y) \ 1 - \ ^{n}(Y; \ ), \ Y \ D^n, n \ N, \ > 1, 0 < \ \leq 1. \quad (8)$$

From equations (3) and (8), we obtain this particular representation of the SEF:

[2] To avoid any discussion on exactly how to define the median, we have assumed that *n* is odd.

[3] Donaldson and Weymark (1983) and Yitzhaki (1983) provided a continuous version of the same family of indices.An axiomatic characterization of the S-Gini indices can be found in Aaberge (2001).

$$W^n(Y;v) = \Xi^n(Y;v) = \sum_{i=1}^{n} \omega(i;v) Y_i \tag{9}$$

where $\omega(i;v) = (1-\theta)\frac{1}{n} + \theta \, \omega(i)$. Thus, for the class of S-Gini indices the family of KL SEFs becomes a convex combination of a utilitarian SEF (see, for example, d'Aspremont and Gevers, 2002) and the subfamily of rank-dependent SEFs in equation (5) that belongs to the class of Yaari SEFs. For the particular case of the Gini coefficient (*v* = 2), expression (9) becomes:

$$W^n(Y;2) = \Xi^n(Y;2) = (1-\theta)\frac{1}{n}\sum_{i=1}^{n} Y_i + \theta \frac{1}{n}\sum_{i=1}^{n} \frac{2(n-i)+1}{n} Y_i \tag{10}$$

where all weights are clearly linear.

After obtaining the expression for the class of rank-dependent KL SEFs for the family of S-Ginis, we show in the following result that a Social Decision Maker (SDM) with a SEF belonging to this class will prefer the Condorcet winner of a set of income distributions if and only if the inequality index under consideration is the Gini coefficient.

Theorem

Let $Y^1, Y^2, \ldots, Y^T \in D^n$ be T income distributions, symmetric under the same transformation $\omega(Y) = \omega(i;v) Y_i$ where $\omega(i;v) = (1-\theta)\frac{1}{n} + \theta\,\omega(i)$, $\forall i = 1, 2, \ldots, n$, $v > 1$, $0 < \theta \leq 1$. Assume a SDM with a S-Gini-consistent Kakwani-Lambert SEF $W^n(Y;v)$ $v > 1$, $0 < \theta \leq 1$. Then, the SDM will prefer the Condorcet winner among $Y^1, Y^2, \ldots, Y^T$ if and only if its SEF is based on the Gini coefficient, i.e., v=2.

Proof: Let us assume the following transformation of the initial distribution Y:

$$\omega(Y_i) = \omega(i;v) Y_i = \left((1-\theta)\frac{1}{n} + \theta\,\omega(i)\right) Y_i, \; i = 1, \ldots, n; \; v > 1, 0 < \theta \leq 1. \tag{11}$$

From (9), it is true that

$$\sum_{i=1}^{n} \omega(Y_i) = \Xi^n(Y;v) \tag{12}$$

so the mean $\mu_\omega(Y)$ is $\frac{\Xi}{n}$. Moreover, because $\omega(Y)$ is symmetrically distributed, the median $m_\omega(Y)$ is equal to $\omega(Y_m)$ and to the mean $\mu_\omega(Y) = \frac{\Xi}{n}$.

By definition $\omega(Y_m) = \omega(m;v) Y_m$ where $\omega(m;v)$ is the value of $\omega(i;v)$ at the median. Therefore, the median of the original distribution Y, Y_m, is:

$$Y_m = \frac{\Xi}{n\,\omega(m;v)}, \; v > 1, 0 < \theta \leq 1. \tag{13}$$

From this expression, it can be observed that $Y_m = \Xi$ for any distribution Y if and only if $\omega(m;v) = \frac{1}{n}$.

By definition, we know that $\omega(m;v) = \overline{\omega}(i;v)$. Moreover, $\frac{n+1}{2}$ is the mean of the ranks provided that the ranks follow a uniform distribution. As a result, if and only if $\omega(i;v)$ is linear in *i*, $\overline{\omega}(i;v) = \frac{1}{n}\sum_{i=1}^{n} \omega(i;v)$. This occurs only when $v = 2$, which is the inequality aversion parameter that corresponds to the Gini coefficient. Given that $\sum_{i=1}^{n} \omega(i;v) = 1$, then $Y_m = \Xi$. Hence, the EDE of the class of rank-dependent KL SEFs is equal to the median income, and both approaches rank distributions in the same way, if and only if $v = 2$.

Now, we prove that the median voter always holds the deciding vote. A majority of voters prefer *Y* to *Z* if and only if a majority of voters prefer $\omega(Y)$ to $\omega(Z)$ because $\omega(i;v) > 0 \; \forall i = 1, \ldots, n$. From the definition of symmetry given above, we also know that

$$\omega(Y_m) - \omega(Y_{m-s}) > \omega(Z_m) - \omega(Z_{m-s}) \Leftrightarrow \omega(Y_{m+s}) - \omega(Y_m) > \omega(Z_{m+s}) - \omega(Z_m) \tag{14}$$

Therefore, if we assume without loss of generality that $\omega(Y_m) = \omega(Z_m)$ then for any $s \in \{1, 2, \ldots, m-1\}$ such that $\omega(Y_{m-s}) < \omega(Z_{m-s})$, we have $\omega(Y_{m+s}) > \omega(Z_{m+s})$. That is, any voter below the median who prefers *Z* over *Y* is matched by another voter above the median who prefers *Y* over *Z*. Likewise, any voter below the median who prefers *Y* would be matched by another voter above the median who prefers *Z*. Hence, majority voting over the set $Y^1, Y^2, \ldots, Y^T$ will yield the median voter's preferences.

Combining these two findings, we obtain the main result.

This result provides necessary and sufficient conditions under which majority voting and rank-dependent social welfare are consistent. In particular, it proves that the class of KL SEFs consistent with the Gini coefficient, $W^n(Y;2;) = (Y)(1 - \ ^n(Y;2)), 0 < \ \leq 1$ accords with the outcome of majority voting.

From this result, we can derive the exact relationship between inequality (measured by the Gini coefficient) and the median–mean ratio. The median–mean ratio has been used as a proxy for income equality in many studies (see, for instance, Meltzer and Richard, 1981 and Alesina and Rodrik, 1994), despite the fact that they are not equivalent concepts and the literature has not provided, as far as we are aware, an explicit expression for the exact relationship[4].In the following result we show the exact expression that connects both concepts.

Corollary

Let $Y \in D^n$ be an income distribution that is symmetric under the transformation $(Y) = (2;)Y, = 1, ..., n, 0 < \ \leq 1$, with median Y and mean . Then, the Gini coefficient is:

$$^n(Y,2) = \frac{1}{\ }\left(1 - \frac{\ }{\ }\right). \tag{15}$$

Proof:

From the proof of the previous theorem we know that $\Xi^n(Y;2;) = Y$. Moreover, we know that the KL family of inequality indices for the Gini coefficient is $^n(Y;2) = \frac{1}{\ }\left(1 - \frac{\Xi(\ ;2;\ )}{(\ )}\right), 0 < \ \leq 1$. By combining both results, we obtain straightforwardly the expression in (15).

The advantage of this result is threefold. First, inequality measurements (through the widely used Gini coefficient) can be summarized by just two common measures of position, the mean value and the median value. This simplification permits not only an easier calculation of inequality, but also a straightforward inclusion of inequality in macroeconomic modelling, in particular in political economy models. It is interesting to note that when the initial distribution *Y* is symmetric, $Y =$ which implies that $^n(Y;2) = 0$. However, this possibility is ruled out in our case because $Y =$ requires $(Y) = \frac{1}{n} \forall$ and, in turn, $= 1$.

Second, the result allows inequality to be estimated even when there is no available micro data. When a database with individual records is not available, inequality can still be calculated if two basic measures, the mean and median, are at hand, whichis the case for many aggregate databases.

Third, the median–mean ratio can be characterized as an equality index as follows:

$$\frac{\ }{\ } = 1 - \ ^n(Y,2). \tag{16}$$

This characterization raises the possibility of reinterpretingall those indices in the literature that make use of the median–mean ratio. For example, the Wolfson polarization index (Wolfson, 1994, Foster and Wolfson, 2010 and Rodríguez and Salas, 2003).

Let $(Y;2)$ be the between-groups Gini coefficient and $(Y;2)$ the within-groups Gini coefficientcalculated for an income distribution *Y* divided into two income groups separated by the median. Then, the Wolfson index of income bipolarization, *P*, can be written as[5]:

$$(Y) = \frac{2}{\ }\left((Y;2) - (Y;2)\right). \tag{17}$$

This index is consistent with the two basic axioms of bipolarization, namely, *increasing spread* and *increasing bipolarity*. However, it does not verify the basic axiom of inequality, the principle of progressive transfers. The mean–median income ratio is, in principle, a normalization term in this formula. However, given the result above we can rewrite the expression for the Wolfson bipolarization index as follows:

$$(Y) = \frac{2\left((\ ;2) - (\ ;2)\right)}{1 - (\ ;2)}. \tag{18}$$

[4] For example, Alesina and Rodrik (1994) considered the gap between the median and mean values in their theoretical model and interpreted it as a measure of inequality. They then used the specific Gini coefficient in the empirical exercise.

[5] An extension of this bipolarization index for $\in 2, 3$ has been proposed in Rodríguez and Salas (2003).

When the income distribution is split into two income groups by the median, the between-groups component is typically larger than the within-groups component. Given this fact, it is easy to see from the last expression that income polarization measured in this way and inequality measured by the Gini coefficient should be highly correlated. This is in fact what empirical studies have found (see for example Zhang and Kanbur, 2001).

3. AN ILLUSTRATION

We illustrate our results with data drawn from the EU-SILC dataset for the European Union over the period 2005-2007. This database is panel data for 26 European countries over three consecutive years (78 real distributions). In this manner, we can test the symmetry hypothesis using a set of heterogeneous countries, not only in economic terms but also in political, cultural and institutional terms. After-tax and transfer household income is the variable under analysis. Post-tax income is calculated as pre-tax income plus cash transfers from the government, minus income tax payments and social security contributions. Moreover, incomes are normalized by an adult-equivalence scale defined as $e^{0.5}$, where e is household size. Observations with negative incomes are removed and household observations are weighted by the sample weights.

First of all, we apply the transformation proposed in the previous section to the data. For this transformation, we set parameter v at two (v = 2) and consider that $k \in (0, 1]$ to an accuracy of two decimal places. We then formally test each transformed distribution for symmetry using the consistent nonparametric kernel-based test developed by Ahmad and Li (1997). The procedure we use tests the hypothesis that a distribution is symmetric about the median. In particular, the null hypothesis is expressed as $H_0: f(x) = f(-x)$ for all $x \in \mathbb{R}$, whereas the alternative is that H_0 is false, i.e. $H_1: f(x) \neq f(-x)$ for some $x \in \mathbb{R}$. For details about this test see the Appendix.

Assuming that v = 2, we calculate the critical values of the parameter k for which symmetry is not rejected. In Table 1, we reject symmetry when the values k_{min}andk_{max} are unspecified. We can see that the symmetry hypothesis is not rejected in 86% of cases. Therefore, we findthat the symmetry condition is not too restrictive in practice. After testing for symmetry, we calculate the optimal value k^* as the most probable k for which symmetry is not rejected. We show in Figure 1 the distribution of estimated k^*. It can be observed that the parameter k^* ranges from 0.1 to 0.6 with a value around 0.4 as the most frequent. Despite the fact that the AKS class of SEFs assumes implicitly that the parameter k^* is 1, our estimations point to a lower value.

In Table 1 we also provide the level of welfare (W) for the corresponding optimal parameter k^*, the median income (Y), the mean income (μ) and the Gini coefficient (G) for those cases where the symmetry condition has not been rejected. Median income tracks welfare very wellsince the coefficient of determination R^2 is 0.999 and the slope of the regression, 1.032, is close to one[6].

Finally, we test for the median–mean ratio as an index of equality. In Figure 2 we contrast the correlation between the median–mean ratio and $1 - I^{n}(Y,2)$ where k = k^*. The coefficient of determination R^2 is 0.946 so as expected the degree of correlation is high. The theoretical proposal developed above, that the median–mean ratio can be viewed as an equality index, can be used, therefore, not only in economic modelling, but also in empirical studies.

4. CONCLUDING REMARKS

In this paper, we find conditions under which a class of rank-dependent social evaluation functions is consistent with majority voting. Given a set of co-symmetricable income distributions, arecent result for the utilitarian case shows that there is always a transformation for which the median equals the EDE of the distribution and, therefore, the SEF is consistent with majority voting (Salas and Rodríguez, 2012). Parallel to this result, we checked for the existence of an analogue transformation for the case of (S-Gini-consistent) rank-dependent SEFs. For the class of S-Gini-consistent KL SEFs, however, we find that this result is not generalized. Only if the SEF is consistent with the Gini coefficient, does such a transformation exist.

6 Mean income does not track welfare quite as well since the coefficient of determination R^2 is 0.994, and the corresponding slope of the regression is 0.925.

As a by-product of this result, the Gini coefficient can be characterized as an affine decreasing function of the median–mean ratio, which allowsinequality, measured by the Gini coefficient, to be summarised by simply using the mean and median values. An alternative way to interpret this result is to say that the median–mean ratio is actually an equality index. This result formally justifies the extended use of this ratio as a proxy for equality in the literature on political economy.

Future research points toan extension of the result toother classes of rank-dependent SEFs. However, it seems that the result will be repeated for other families of rank-dependent SEFs, due to the impossibility of our result under convex or concave weights in the SEF.

Table 1

CRITICAL VALUES OF THE PARAMETER *K* IN EUROPE (2005-2007)

Country	Year	k_{min}	k_{max}	k*	W	m	μ	G
Austria	2005	0,22	0,35	0,29	19820	19286	21549	0,27674
Austria	2006	0,16	0,34	0,26	19496	18920	20971	0,27053
Austria	2007	0,21	0,38	0,30	20113	19575	21922	0,27511
Belgium	2005	0,29	0,32	0,30	17056	16550	18598	0,27631
Belgium	2006	0,28	0,31	0,29	17865	17422	19501	0,28916
Belgium	2007	—	—	—	—	—	—	—
Cyprus	2005	—	—	—	—	—	—	—
Cyprus	2006	—	—	—	—	—	—	—
Cyprus	2007	—	—	—	—	—	—	—
Czech Rep.	2005	0,33	0,53	0,43	4524	4425	5121	0,27138
Czech Rep.	2006	0,30	0,50	0,40	5133	4977	5741	0,26499
Czech Rep.	2007	0,35	0,54	0,43	5529	5373	6195	0,24982
Denmark	2005	0,15	0,33	0,26	21743	21533	23353	0,26518
Denmark	2006	0,14	0,31	0,24	22440	22178	23974	0,26653
Denmark	2007	0,03	0,21	0,12	29259	27629	30226	0,26648
Estonia	2005	0,48	0,59	0,55	2964	2938	3684	0,35536
Estonia	2006	0,46	0,57	0,52	3624	3538	4445	0,35532
Estonia	2007	0,36	0,44	0,41	4396	4288	5085	0,33051
Finland	2005	0,26	0,34	0,31	17772	17410	19384	0,26834
Finland	2006	—	—	—	—	—	—	—
Finland	2007	0,23	0,33	0,28	22171	21427	24138	0,29098
France	2005	0,28	0,40	0,35	17207	16700	19133	0,28757
France	2006	0,25	0,45	0,36	17311	16813	19272	0,28274
France	2007	0,24	0,43	0,35	18336	17943	20277	0,27356
Germany	2005	—	—	—	—	—	—	—
Germany	2006	0,13	0,33	0,24	16819	16422	18088	0,29219
Germany	2007	0,19	0,38	0,29	19354	18783	21321	0,31806
Greece	2005	0,29	0,46	0,38	10213	10000	11908	0,37471
Greece	2006	0,32	0,46	0,40	10699	10456	12509	0,36167
Greece	2007	0,31	0,47	0,40	10687	10423	12573	0,37495
Hungary	2005	0,20	0,38	0,30	3824	3684	4170	0,27658
Hungary	2006	0,25	0,46	0,36	4362	4112	4975	0,34219
Hungary	2007	0,19	0,41	0,31	4262	4158	4643	0,26438
Iceland	2005	0,20	0,35	0,28	25391	24498	27503	0,27421

(Continues)

(Continuation)

Country	Year	k_{min}	k_{max}	k*	W	m	μ	G
Iceland	2006	0,26	0,40	0,34	30846	29715	34062	0,27770
Iceland	2007	0,26	0,47	0,38	33051	31727	36993	0,28040
Ireland	2005	—	—	—	—	—	—	—
Ireland	2006	—	—	—	—	—	—	—
Ireland	2007	0,57	0,59	0,58	20875	20726	26241	0,35261
Italy	2005	0,26	0,40	0,33	15965	15245	17889	0,32597
Italy	2006	0,27	0,41	0,35	15983	15460	18012	0,32190
Italy	2007	0,26	0,38	0,32	17238	16490	19195	0,31857
Latvia	2005	0,43	0,59	0,53	2189	2189	2819	0,42171
Latvia	2006	0,51	0,54	0,53	2562	2528	3341	0,44021
Latvia	2007	0,58	0,65	0,63	2911	2952	3912	0,40608
Lithuania	2005	0,45	0,60	0,54	2085	2089	2674	0,40778
Lithuania	2006	0,46	0,60	0,54	2540	2508	3191	0,37792
Lithuania	2007	0,42	0,56	0,50	3394	3293	4121	0,35258
Luxembourg	2005	0,25	0,46	0,36	31833	31523	35179	0,26419
Luxembourg	2006	0,27	0,45	0,37	32819	31894	36687	0,28492
Luxembourg	2007	—	—	—	—	—	—	—
Norway	2005	0,05	0,20	0,13	27121	25974	28165	0,28508
Norway	2006	0,02	0,30	0,16	32143	30147	33892	0,32251
Norway	2007	0,02	0,25	0,12	32492	32056	33617	0,27889
Poland	2005	0,32	0,54	0,44	2913	2849	3467	0,36294
Poland	2006	0,31	0,48	0,40	3667	3537	4219	0,32725
Poland	2007	0,32	0,47	0,40	4050	3930	4641	0,31823
Portugal	2005	—	—	—	—	—	—	—
Portugal	2006	—	—	—	—	—	—	—
Portugal	2007	0,51	0,62	0,57	8390	7999	10814	0,39336
Slovakia	2005	0,23	0,44	0,34	3059	2994	3374	0,27452
Slovakia	2006	0,23	0,47	0,35	3616	3478	4004	0,27701
Slovakia	2007	0,24	0,42	0,35	4277	4186	4702	0,25814
Slovenia	2005	0,12	0,29	0,21	9582	9364	10155	0,26846
Slovenia	2006	0,11	0,32	0,23	11019	10769	11682	0,24683
Slovenia	2007	0,11	0,31	0,22	11990	11646	12663	0,24156
Spain	2005	0,31	0,44	0,39	11414	11300	13265	0,35774
Spain	2006	0,29	0,36	0,33	12344	12225	13950	0,34883
Spain	2007	0,31	0,41	0,36	12589	12465	14392	0,34808
Sweden	2005	0,16	0,35	0,27	17610	17585	19028	0,27600
Sweden	2006	0,02	0,23	0,13	19952	19531	20632	0,25346
Sweden	2007	0,08	0,26	0,18	20766	20374	21759	0,25354
The Netherlands	2005	0,23	0,43	0,34	18235	18029	20213	0,28782
The Netherlands	2006	0,26	0,44	0,36	18428	18187	20551	0,28698
The Netherlands	2007	0,22	0,42	0,33	21933	21189	24072	0,26936
United Kingdom	2005	0,37	0,49	0,44	19668	19129	23410	0,36328
United Kingdom	2006	0,36	0,50	0,44	20097	19795	23812	0,35457
United Kingdom	2007	0,36	0,46	0,41	22199	21660	25908	0,34913

Figure 1

RELATIVE FREQUENCIES OF *K IN EUROPE (2005-2007)**

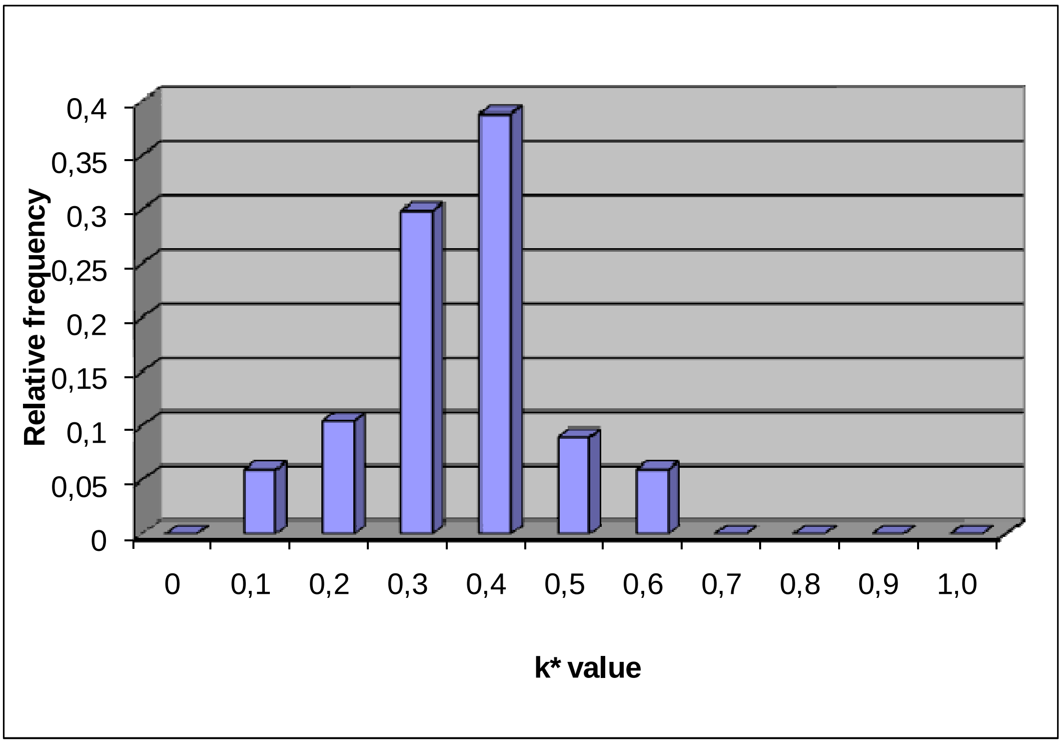
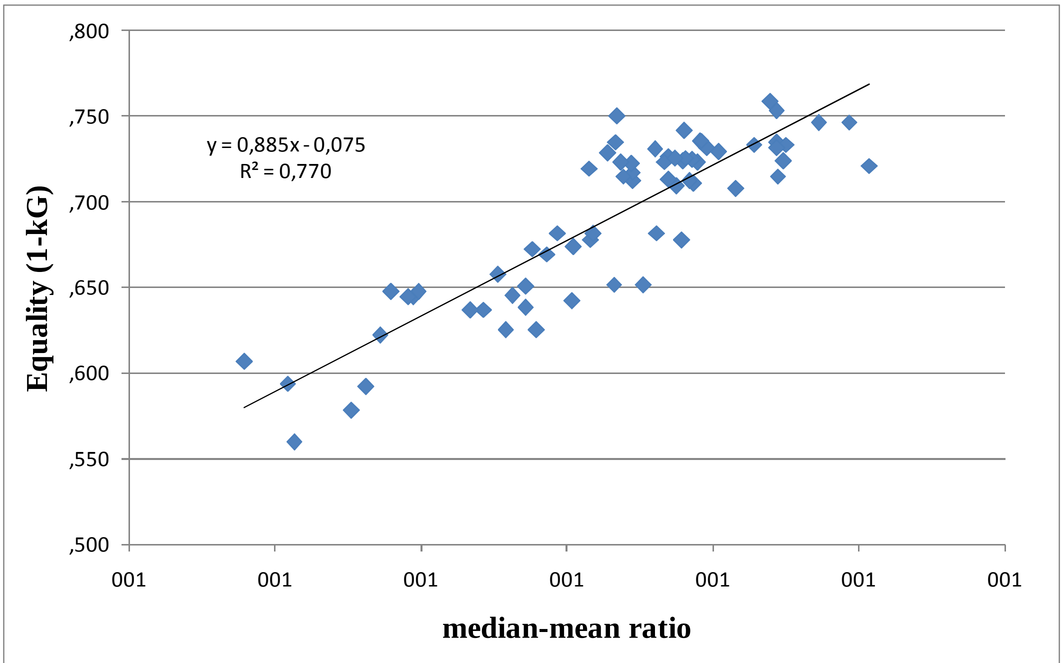


Figure 2

CORRELATION BETWEEN THE MEDIAN–MEAN RATIO AND (1-*K*G*)



APPENDIX

The nonparametric kernel-based test developed by Ahmad and Li (1997) is an intuitively appealing and very direct way of constructing a statistic for testing the symmetry of a distribution. One advantage of this method is that it is based on distributions and, consequently, it takes into account all the information that may be forthcoming from many moments without assuming the existence of such moments, and without much more difficulty in estimation.[7] For the same reason, the method is also less likely to suffer from the inadvertent introduction of special relationships between moments that only hold for special distributions (i.e. the Gaussian distribution). Finally, large samples are not required to estimate high order moments, assuming these exist.

Let's assume a random sample of *n*i.i.d. observations of income Y_i, i = 1,…, n, drawn from the distribution F and ordered such that $Y_1 \le Y_2 \le \cdots \le Y_n$. From Ahmad and Li (1997), we know that $n\sqrt{h}\,\hat{I}_{2n}$ converges to a normal distribution with mean 0 and variance $4\sigma^2$, where h is the smoothing parameter and $\hat{I}_{2n}$ is as follows:

$$\hat{I}_{2n} = \frac{1}{n^2 h}\sum_{i=1}^{n}\sum_{j\neq i}^{n}\left[K\left(\frac{Y_i - Y_j}{h}\right) - K\left(\frac{Y_i + Y_j}{h}\right)\right] \tag{A1}$$

where$K(\cdot)$ is the Gaussian kernel function. We estimate the variance σ^2 according to the following term:

$$\hat{\sigma}^2 = \frac{1}{2\sqrt{\pi}}\,\frac{1}{n^2 h}\sum_{i=1}^{n}\sum_{j=1}^{n}K\left(\frac{Y_i - Y_j}{h}\right) \tag{A2}$$

Following Ahmad and Li (1997), we chose $h = \delta n^{-\frac{1}{\gamma}}$, where δ denotes the standard deviation of the sample data, and the value γ = 6. This test is one-sided as the alternative hypothesis states that the statistic $\hat{I}_{2n}$ is positive. The critical value is 2.33 if a 1% significance level is adopted.

[7] Many of the existing symmetry tests examine high order moments (see for example Premaratne and Bera, 2005). Bai and Ng (2005) discuss the difficulties of estimating higher order moments, such as kurtosis, as well as the greater power of tests based simultaneously on several odd order moments.

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