A Revenue-raising Government Taxing a Firm with Private Information*

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Received: April, 2011
Accepted: November, 2012

Abstract

This paper examines, for a two-period signalling game played by a revenue-raising government and a monopolistic firm in an asymmetric information context, how the government behaves when it taxes the firm’s production. Information regarding the firm’s efficiency (or its potential to pay taxes) is private; therefore, only the firm knows its ‘type’. To prevent opportunism and lead the firm to disclose information through its period-1 output, the period-1 per-unit tax needs to be lower than it would if information were perfect (and tax revenue is consequently lower). As a result, expected taxes increase with time. This behaviour generally (but not always) reduces social welfare. In contrast, when the government prefers the firm’s information is not disclosed by the period-1 signal, the expected per-unit tax in such period is the same as it would be if information were perfect. Moreover, if the firm reveals no information, then welfare is generally (but not always) greater than it would be if information were perfect. The government would generally prefer information not to be revealed, because disclose would greatly reduce its period-1 tax income. From a social standpoint, information non-disclosure increases welfare, except when both the probability of the firm being efficient and its efficiency level relative to that of the inefficient firm are sufficiently high.

Keywords: Output-related Taxes, Revenue-raising Government, Asymmetric Information, Monopoly, Separating and Pooling Equilibrium.

JEL classification: H21, D82.

* The author would like to thank the following for their helpful comments and suggestions, which have greatly improved this paper: Lluís Bru; an anonymous referee of this journal; and Julio López-Laborda and Jorge Martínez-Vázquez (the executive editors). The assistance of Antonio R. Sampayo is also appreciated. The usual disclaimer applies. This work was carried out under Project INCITE09201042PR, financed by the Galician Regional Authority (Xunta de Galicia).

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1. Introduction

It is common for firms to have more information on their productive efficiency, market demand and capacity to pay taxes than a player like the government interested in taxing them adequately. One of the most difficult tasks for a government that lacks information on regulated firms is inducing voluntary information disclosure. This problem has attracted much attention in the literature (Loeb and Magat, 1979; Cech, 1991; Kim and Chang, 1993; Fjeldstad, 1996; Lederman, 2010) due to both its theoretical relevance and its practical consequences. Tax evasion is considered a problem in practically every country in the world. To illustrate, the tax fraud in the government-to-business (G2B) context is estimated to be 2-2.5% of GDP, amounting to about 200-250 billion euro per year in the EU (EU Commission, 2006). As for the USA, Lederman (2010) indicates that the federal tax gap, i.e. the annual gap between due and paid taxes, is enormous, with the Internal Revenue Service estimating it to be 345 billion dollars for 2001; this figure represents approximately three quarters of the 2008 federal budget deficit and exceeds deficits for the years 2005 to 2007.

Tax fraud can be understood as the outcome of an asymmetric information problem (Liu and Tan, 2008). In any examination of how tax incentives and disincentives impact taxpayers and tax collectors’ decisions as to whether or not to engage in fraudulent behaviour, imperfect information and uncertainty are widely recognised to lie at the core of the issue. Uncertainty opens the doors to strategic behaviour, particularly in the case of asymmetric information among agents (Fjeldstad, 1994). Asymmetric information is also a core problem for taxation law enforcement (Lederman, 2010). Taxpayers know the facts regarding the relevant transactions that they have engaged in during the tax year or can at least access this information readily, whereas the government is obliged to obtain this information a posteriori from either the taxpayers or third parties (Cords, 2005).

Value added tax (VAT) collection is a good example of informational asymmetries in the G2B context (Liu and Tan, 2008). The players include the firms that are obliged to declare and pay VAT and the corresponding auditing body controlling VAT collection. If perfect information prevailed, the government would be fully aware of the operations performed by firms, who would declare and pay the corresponding VAT honestly. However, it is not unusual for firms to have access to privileged information on their own operating details and actual transaction values; this information remains unknown or unknowable to other players, including the government, unless considerable spending is made in terms of monitoring effort and costs. In addition, optimizing firms have obvious incentives to exploit private information strategically in their own interest. In fact, these firms will do anything they can to reduce their tax burden by concealing certain private information so as to obtain tax advantages. Irrespective of whether this incentive is easily achieved without the government knowing or whether default penalties are severe enough, firms choose to cheat and commit tax fraud. To sum up, firms have private information on their potential to pay taxes and may report a tax position that implies lower payments than in reality. And, in an attempt to limit the embezzlement effect due to information asymmetry and, therefore, firm-extracted rents,
the government can adopt the necessary measures in terms of participation and incentive compatibility constraints to encourage honest tax payment.

Under these circumstances, what sort of strategy can effectively encourage firms to disclose their private information in a dynamic (two-period) setting? Should the government significantly alter its tax rule relative to a perfect information scenario to discourage dishonest behaviour? Does the government prefer to extract information in period 1 to better calibrate the per-unit taxes in period 2? Does information reporting turn out to be an overly costly strategy relative to the policy of not inducing information transmission? What are the social welfare implications of the two tax policies—the one that enables the government to induce firms to transmit information and the one that leaves the government more poorly informed in period 2 than the firms? These questions, which have profound implications for normative analysis, are examined in the current paper, where a two-period signalling tax model involving a government and a single firm that may be productively efficient or inefficient and, thus, may have a high or low potential to pay taxes, is considered. The government does not know what type the firm is, but, after observing the period-1 production of the firm, it can infer such type (in a separating equilibrium) or not (in a pooling equilibrium).

To induce the firm to reveal what it knows, the government has to reduce the per-unit output tax in period 1 relative to the (expected) per-unit tax that it would charge under perfect information. Since information transmission involves a cost in the form of a lower output in period 1, the government reduces the per-unit tax to alleviate this decrease. This has a negative impact on its revenue in period 1, leaving it worse off under asymmetric than under perfect information. The model then predicts an increasing time pattern on both expected per-unit taxes and tax revenue as information evolves from asymmetric to perfect. No information reporting is also a way of transmitting information. In this case, the period-1 and period-2 per-unit output taxes are equal, and the inefficient firm is forced to increase its period-1 output above the level it would produce if the efficient firm did not exist. This increase reaches the level at which there is no difference between keeping the same production level and camouflaging or deviating and being detected as an efficient firm, and therefore being imposed with a higher per-unit tax in period 2.

In sum, the government faces a dilemma upon forcing the firm to reveal information. Inducing the firm to disclose its information benefits the government in period 2 since it better calibrates the per-unit taxes in such period compared to when no information has been previously revealed. This increases the tax collection in period 2. However, for the firm to reveal what it knows, its period-1 production has to be lower compared to when no information is revealed. This is due to the fact that, in the former case, the inefficient firm is forced to reduce its production, whereas the efficient firm produces as if the inefficient firm did not exist. In contrast, if the government imposes a pooling equilibrium and leads the firm not to reveal its information, production in period 1 increases and is higher than when information is revealed. Bearing this production difference in mind and also the fact that the separating equilibrium involves a lower per-unit tax than the pooling equilibrium, the tax revenue in period 1 is generally lower if information is transmitted compared to when information is not
transmitted. Given the inefficiency level of the firm, it is only when the probability of having an inefficient firm is sufficiently low that the period-1 tax income is higher in the separating than in the pooling equilibrium.

Taking into account both periods, the government prefers the firm, for most of the region of admissible parameter values, not to reveal information, because the increase on tax revenue in period 2 will not outweigh signalling costs in period 1. Once more, given the inefficiency level of the firm, inducing information disclosure is better for the government than not inducing it only when the firm is quite likely to become a firm with a high potential to pay taxes.

From a social standpoint, both signalling-related distortions and the resulting government behaviour may cause welfare under asymmetric information to exceed welfare under perfect information. For instance, when the per-unit output tax in period 1 forces the firm to reveal its efficiency level, the firm’s production in this period is ‘reorganized’ relative to production under perfect information: the efficient firm increases its production and the inefficient firm reduces its production. The increase of firm’s expected net profits in period 1 relative to perfect information is especially substantial when the ratio of inefficient firms is very low and the efficiency gap is sufficiently high. In this case, welfare when information is revealed exceeds welfare under complete information. Things are rather different when the government does not force the firm to reveal its information. In this case, the period-1 production of the firm increases so much relative to perfect information that it leads to higher social welfare than under perfect information, except when both the firm is not likely to be inefficient and the inefficiency level is sufficiently high.

The analysis of the social implications of government behaviour allows us to determine whether its incentives are aligned or not with social incentives. In period 2 and for every admissible parameter values, welfare when the firm reveals no information is greater than when it does. Production in the former case is higher than in the latter, so consumers pay reduced prices for the good and obtain a greater surplus. Besides, when there is no revelation in period 1, the firm’s information advantage in period 2 relative to the government leads to greater net profits in this period relative to symmetric information. Finally, period-2 tax revenue under information disadvantage is lower than under symmetric information. As a whole, the consumer’s surplus and the firm’s net profits gains in pooling equilibrium relative to separating equilibrium outweigh by far the tax income losses, and period-2 welfare in pooling exceeds that in separating equilibrium. On the other hand, regarding period 1, the firm is undoubtedly worse off in the pooling than in the separating equilibrium because production distortions are more pronounced in the former case. Moreover, pooling equilibrium-related production distortions benefit consumers and the government more than separating equilibrium-related production distortions, except when the proportion of firms with low potential to pay taxes is sufficiently small.

Regarding both periods, a higher social welfare level arises when information is not revealed than when it is revealed in most of the region of parameters for which a pooling equi-
librium exists. This holds except when, given the inefficiency level, the probability of having an inefficient firm is sufficiently low, in which case inducing information transmission is socially better than not inducing information transmission. This is because either the government cannot group all types of firm or, even it can group them, this option is socially worse than inducing information disclosure.

The impact of private information and the outcome of information transmission have been examined in many contexts, for instance, regarding the role of prices and profits to signal information about (i) an incumbent’s cost to deter a potential entrant (Milgrom and Roberts, 1982; Masson and Shaan, 1982, 1987), (ii) market demand level (Mailath, 1991) or (iii) demand composition (Aguirre, 1999). The current paper’s contribution to the literature is the analysis of a government that is less informed than a taxed firm and which has to select the optimal tax policy.

The rest of the paper runs as follows. Section 2 outlines the model, Section 3 examines the behaviour of the government if it shared the information of the firm, Section 4 addresses the impact of asymmetric information on both the per-unit output tax size and fiscal revenue, Section 5 discusses the impact on social welfare and, finally, Section 6 concludes the paper.

2. The model

Consider a single industry comprising one firm that produces a good in two production periods, \( t = 1, 2 \). To get the explicit solution of the model, the firm is assumed to produce for a specific market with periodical linear inverse demand as

\[
p_t(q_t) = 1 - q_t,
\]

where \( p_t \) is the unit price in period \( t \) when \( q_t \) units of output are sold.

A government that imposes taxes exists and its goal is to maximize the per-period tax revenue. Although we could have considered a government whose aim in setting taxes is maximizing social welfare, we opted for a revenue-raising government for three reasons: tax sufficiency is one of the aims of the tax system; considering a revenue-raising government simplifies the achievement of close solutions; and outcomes in general terms do not seem to differ greatly from those that would be obtained by a welfare-maximizing government. Taxes are publicly known fixed amounts per output unit revised at the beginning of period 2. There is no government-firm negotiation, yet the former has all the bargaining power to set taxes.

The efficiency level of the firm (or its potential to pay taxes) is reflected in its marginal production cost or ‘type’. This cost is a random variable whose realization is constant across periods and initially known only to the firm. The government ex ante cannot identify this cost and, hence, ignores the potential of the firm to pay taxes; it only knows that the cost, \( \tilde{c} \),
can adopt either a low value (zero, for simplicity) or a high value (the strictly positive value $c$), each randomly assigned. Formally,

$$\tilde{c} = \begin{cases} 
0, & \text{with Probability } \mu \\
c, & \text{with Probability } 1-\mu 
\end{cases}$$

(2)

where $0 < \mu < 1$.

To guarantee well-behaved interior solutions, the following assumption regarding the inefficient firm is made.

**Assumption 1.** *If the firm is inefficient, its inefficiency level, $c$, is such that $0 < c < 1/2$.***

This range of values for parameter $c$ ensures that, given the demand and cost defined in (1) and (2), any type of firm always produces a positive level of output irrespective of the government’s beliefs about the type. Finally, for the sake of simplicity, no between-period discount factor is applied and all tax-game players are assumed to be risk-neutral.

In the signalling two-period tax game played by the government and the firm information is asymmetric in period 1. Complete information can be restored in period 2 or not according to the kind of pure-strategy perfect Bayesian equilibrium used to solve the game. In fact, to explore the impact of information transmission on the amount of the per-unit output tax, fiscal revenue and welfare level, we focus on the outcome of the separating perfect Bayesian equilibrium for the abovementioned game compared to that under perfect information. In addition, since no information transmission is also a form of transmitting information, the outcome of the pooling perfect Bayesian equilibrium is also examined relative to the outcome under perfect information. Finally, the results of both government strategies—forcing the firm to disclose its information or allowing the firm to keep informational advantage in period 2—are compared.

### 3. Perfect information

As a benchmark, we start by examining government behaviour in a context where the firm’s information on its efficiency or tax-paying potential is publicly known. In this case, the timing of the tax-game is as follows.

**Period 1.** Nature chooses the type of the firm (efficient or inefficient) and this type is publicly observed. The government charges the per-unit tax $\tau_{1k}$ for every unit sold by the firm in period 1 (the * superscript indicates perfect information) as a function of the firm’s type $k$, $k = L, H$, where $L$ denotes an efficient firm (i.e., a firm with high potential to pay taxes) and $H$ an inefficient firm (i.e., a firm with low potential to pay taxes). Finally, the firm produces the output level $q_{1k}^*$ according to its type and the per-unit tax to be paid.
Period 2. All that holds in period 1 applies to period 2. Thus, both the per-unit tax and the firm’s output level in period 1 remain valid for the second period, namely, \( \tau_{2k}^* = \tau_{1k}^* \) and \( q_{2k}^* = q_{1k}^* \).

In this context, the following result may be established.

**Lemma 1.** Under perfect information the government sets, in each period \( t = 1, 2 \), the per-unit tax \( \tau_{tL}^* = 1/2 \) if it faces an efficient firm and \( \tau_{tH}^* = (1 - c)/2 \) if it faces an inefficient firm; the tax revenue in each period is \( T_{tL}^* = 1/8 \) in the case of an efficient firm and \( T_{tH}^* = (1 - c)^2/8 \) otherwise.

**Proof.** See Appendix.

From Lemma 1, the output and profit of the firm in each period \( t \), \( q_t^*(\bar{c}) \) and \( \pi_t^*(\bar{c}) \), respectively, are as listed in Table 1.

<table>
<thead>
<tr>
<th>( \bar{c} )</th>
<th>( q_t^*(\bar{c}) )</th>
<th>( \pi_t^*(\bar{c}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/4</td>
<td>1/16</td>
</tr>
<tr>
<td>( c )</td>
<td>( (1 - c)/4 )</td>
<td>( (1 - c)^2/16 )</td>
</tr>
</tbody>
</table>

The equilibrium values in Table 1 allow us to obtain the per-period output for the firm calculated at the onset of the tax game, \( q_t^* \), the expected net profit, \( \pi_t^* \), and the expected revenue for the government, \( T_t^* \), as \( q_t^* = [1 - (1 - \mu)c]/4 \), \( \pi_t^* = [1 - 2(1 - \mu)c + (1 - \mu)c^2]/16 \), and \( T_t^* = [1 - 2(1 - \mu)c + (1 - \mu)c^2]/8 \), respectively, \( t = 1, 2 \). Lemma 1 also shows that the firm is expected to pay the per-unit tax \( \tau_t^* = [1 - (1 - \mu)c]/2 \) in each production period. In addition, since the tax chosen for an inefficient firm is lower than the tax chosen for an efficient firm, \( \tau_{tH}^* < \tau_{tL}^* \), the expected tax \( \tau_t^* \) decreases as the probability of the firm being efficient and/or the disparity between firms increases.

4. Asymmetric information

Things may change drastically if the government does not know the firm’s type at the same time than the firm itself. In a separating perfect Bayesian equilibrium, a firm’s type (efficient or inefficient) will dictate different outputs in period 1 and, on this basis, the government will be able to distinguish between the firms at the beginning of period 2. In contrast, the equilibrium is pooling if both types of firm produce the same output level in period 1; hence, the government will be unable to distinguish between them in period 2. Separating equilibrium is analysed in subsection 4.1 and pooling equilibrium in subsection 4.2. Finally, the tax-related decision of the government—to induce information disclosure or not—is considered in subsection 4.3.
4.1. Information transmission through the output produced in $t = 1$

The timing of the tax game when a separating equilibrium is possible is as follows.

**Period 1.** At the beginning of this period, nature chooses the firm’s type, and this type is only known to the firm. Before observing the firm’s output choice for period 1, the government, acting as a Stackelberg leader in setting taxes, fixes a per-unit output tax, $\tau^S_1$ (the $S$ superscript denotes signalling) for this period. The only common knowledge in period 1 is the distribution of the firm’s types given in (2). The firm acts as a Stackelberg follower and chooses its period-1 output level $q^S_{1k}$, $k = L, H$.

**Period 2.** Once the production for period 1 has been publicly observed, the government infers the firm’s type and thus updates its prior beliefs given in (2). Given that perfect information is restored in period 2, it publicly announces the period-2 per-unit taxes, $\tau^*_2k$, for each firm’s type, and firms choose their period-2 output, $q^*_2k$.

By means of backward induction we look for the separating pure-strategy perfect Bayesian equilibrium for the tax game. In period 2, the equilibrium profits for each type of firm may be compared to those obtained off the equilibrium path. Bearing in mind the firm and government’s behaviours in period 2 when the firm behaved honestly and misleadingly in period 1, the following result emerges.

**Lemma 2.** The taxed firm wishes to be understood in period 2 as one with a low potential to pay taxes.

**Proof.** See Appendix.

The explanation of Lemma 2 is quite simple. The per-unit tax that the firm pays in period 2, if understood by the government as inefficient with a probability of one, is smaller than if the government is sure that the firm is efficient. Hence, the output level and net profits in period 2 will be higher in the former case.

Now let us concentrate on the analysis of period 1. In this period the government is unable to distinguish one firm type from the other, so the per-unit output tax charged must be the same for both types of firm, that is, the firm with high potential to pay taxes and that one with low potential to pay taxes. Besides, a separating equilibrium implies that the production of each firm’s type differs in this period. Thus the government, after observing different output levels, can update its prior assessment and infer the firm’s type at the end of the period or, equivalently, at the beginning of period 2. Consequently, the government beliefs satisfy Bayes’ rule. In addition, given these beliefs, the government maximizes its expected revenue and, according to that strategy, the firm accepts to pay the tax if the strategy results in non-negative expected profits.

In these circumstances, the efficient firm cannot do better in period 1 than choosing its profit-maximizing output without considering period 2, namely,
\[ q_{1L}^S = \frac{1 - \tau_1^S}{2} = q_{1L}^m, \]  \hspace{1cm} (3)

where numeral and \( L \) subscripts denote that period-1 output corresponds to a low-cost (or efficient) firm, and superscripts \( S \) and \( m \) stand for signalling and monopoly regime, respectively. That is, the efficient firm produces as it would be if it were the only existing type.

On the other hand, the incentive compatibility (IC) conditions defining a separating equilibrium are given by

\[ \pi_{1H}(\tau_1^S, q_{1H}^S) + \pi_{1H}(\tau_2^S) \geq \pi_{1H}(\tau_1^S) + \pi_{2H}(\tau_2^S), \]  \hspace{1cm} (4)

and

\[ \pi_{1L}(\tau_1^S, q_{1H}^S) + \pi_{1L}(\tau_2^S) \leq \pi_{1L}(\tau_1^S) + \pi_{2L}(\tau_2^L). \]  \hspace{1cm} (5)

Condition (4) refers to the IC constraint for an inefficient firm. It states that its period-1 profit from producing the output \( q_{1H}^S \) and thus moving away from the output of the efficient firm, \( q_{1L}^S \), together with period-2 profit, once revealed as inefficient and pays the per-unit output tax \( \tau_2^H \), exceeds the sum of period-1 profit obtained from the production corresponding to an inefficient (myopic) monopolist, \( q_{1H}^m \), plus period-2 profit once it is misrepresented and the corresponding per-unit tax \( \tau_2^L \) is paid. Likewise, condition (5) denotes the IC condition for an efficient firm and shows that overall profit for this firm when misrepresented as inefficient (by producing the quantity of the inefficient firm, \( q_{1H}^S \) in period 1) does not exceed period-1 profit by producing the output stated in (3) plus period-2 profit, once the government infers it as efficient and charges it the (higher) per-unit output tax \( \tau_2^L \).

Particularizing (3)-(5) and solving them for the demand and cost structures considered in the model, the following result is obtained.

**Lemma 3.** A separating pure strategy perfect Bayesian equilibrium exists in the whole region of \((\mu, c)\)-parameters defined by \( 0 < \mu < 1 \) and Assumption 1. In the separating equilibrium of least cost, the following holds:

- **(i)** The government charges in period 1 the per-unit output tax

\[ \tau_1^S = \frac{1 - \mu}{2} \sqrt{\frac{(2 + c)c}{4}} \]

- **(ii)** to both firms; the firm produces the output

\[ q_{1H}^S = \frac{1}{4} - \frac{(1 + \mu)(2 + c)c}{8} \]

if it is inefficient, and the output
\[ q_{1L}^s = \frac{1}{4} + \frac{(1-\mu)\sqrt{(2+c)c}}{8} \]

if it is efficient; the tax due, irrespective of the type, amounts to

\[ T_i^s = \tau_i q_i^s = \frac{1}{8} \left[ 1 - \frac{(1-\mu)\sqrt{(2+c)c}}{2} \right] . \]

(ii) In period 2, both the government and the firm behave as they would under perfect information.

**Proof.** See Appendix.

The essential feature of Lemma 3 is that the efficient firm produces the same amount that it would produce if it were the only firm in existence. Given the single crossing property, the inefficient firm produces a lower amount than it would if it were the only agent. Indeed, to be perceived as inefficient and thus pay a small tax in period 2, the firm that is actually inefficient needs to produce, in period 1, a smaller output than it would in the case of perfect information, namely, \( q_{1H}^s < q_{1I}^m \). If there were no efficient firm, the government would want the inefficient firm to produce more in period 1 and this firm would also want to increase its production to approach an optimal level. However, the presence of an efficient firm implies that if the period-1 output of the inefficient firm were \( q_{1H}^s = q_{1I}^m \), the efficient firm would find it profitable to misrepresent by producing \( q_{1H}^m \) rather than \( q_{1I}^s \), thus breaking the separating equilibrium.

Comparison of per-unit taxes in both informational scenarios renders the following result.

**Proposition 1.** To induce the firm to reveal what it knows about its potential to pay taxes the government reduces the per-unit output tax in period 1 compared to that for the case of perfect information, namely, \( \tau_1^s < \tau_1^* \).

Under-taxation in period 1, which amounts to

\[ \tau_1^s - \tau_1^* = -\frac{(1-\mu)[\sqrt{(2+c)c}-2c]}{4}, \]

is the outcome of the cost to the firm of revealing its type. As stated by Lemma 3, to prevent imitation of the efficient firm, in period 1 the inefficient firm needs to produce an output below that of perfect information. In addition, the per-unit tax in period 1 exacerbates the lessening of the inefficient firm’s output and also decreases the output produced by the efficient firm. Thus, to minimize production cutbacks and the resulting reduction in fiscal revenue in period 1, the per-unit tax has to be smaller than it would be under perfect information. This difference only vanishes when \( \mu = 1 \) or \( c = 0 \). Otherwise, under-taxation in period 1 is more pronounced, as the firm is more likely to be inefficient. Regarding parameter \( c \),
under-taxation is not monotonic, as it increases with \( c \) for small values of \( c \), but decreases with \( c \) within the \( ([2\sqrt{3} - 3]/3, 1/3) \) interval.

Taking part (i) of Lemma 3 and Table 1 into account, asymmetric information leads the efficient firm to pay a smaller period-1 tax than under perfect information, \( \tau^*_1 < \tau^*_{L1} \). This is because this firm benefits from the inefficient firm’s incentive to be perceived as such (i.e. from the fact that the inefficient firm reduces its period-1 output) and, thus, forces the government to counteract this reduction by decreasing the tax imposed on both firms below that expected under perfect information. The inefficient firm also pays a smaller per-unit tax under asymmetric information, \( \tau^*_S < \tau^*_{H1} \), but only when parameters \( \mu \) and \( c \) satisfy \( \mu < 1 - 2c/\sqrt{2c} \), i.e. only when either the firm is very likely to be an inefficient firm or, if \( \mu \) is sufficiently large, when \( c \) is small. This is due to the large cost implied by information reporting, which in turn implies a highly significant tax reduction. Otherwise, the per-unit tax paid by the inefficient firm is higher than under perfect information, \( \tau^*_S > \tau^*_{H1} \). In sum, information transmission benefits the efficient firm more than the inefficient firm on the basis of per-unit output taxes.

The under-taxation of the efficient firm when it reveals what it knows leads it to over-produce as \( q^*_L - q^*_L = (1 - \mu)\sqrt{(2+c)c}/8. \) However, the inefficient firm, when it reveals what it knows, under-produces relative to perfect information, \( q^*_H - q^*_H = -[(1 + \mu) \sqrt{(2+c)c} - 2c]/8 \), due to the interaction of two effects: the cost of transmitting information (signalling effect) and over- or under-taxation relative to symmetric information. Over-taxation reinforces the signalling effect; under-taxation counteracts but does not outweigh the signalling effect. Overall, the period-1 output, as of onset of the tax game, decreases by

\[
q^*_i - q^*_i = \mu(q^*_L - q^*_L) + (1-\mu)(q^*_H - q^*_H) = -\frac{1}{8}(1-\mu)^2 \left[ \sqrt{(2+c)c} - 2c \right] \tag{6}
\]

relative to the perfect information context. This reduction hinges on the fact that signalling-related reduction in the output of the inefficient firm is a first-order effect, whereas the output increase related to under-taxation is a second-order effect. The former is thus dominant. In other words, even though the efficient firm produces more than in the perfect information context (taxes depend on output), the signalling-related reduction in the output of the inefficient firm reduces the expected production in period 1 below that under perfect information.

Finally, the period-1 fiscal revenue is always lower than \( T^*_1 \), the expected revenue in period 1 under perfect information. This is because \( \tau^*_1 < \tau^*_1 \), according to Proposition 1, and the expected production in this period, \( q^*_1 \), is also reduced relative to \( q^*_1 \) due to under-production by the inefficient firm. Bearing in mind that \( T^*_2 = T^*_2 \), the revenue loss across two periods due to asymmetric information and (costly) transmission of information amounts to

\[
T^*_2 - T^*_1 = \tau^*_S - \tau^*_1 = -\frac{1}{32}(1-\mu) \left[ 4\sqrt{(2+c)c} - 2(5-\mu)c + (3+\mu)c^2 \right]. \tag{7}
\]
4.2. No information transmission through the output produced in $t = 1$

We now turn to the case in which perfect Bayesian equilibrium is a pooling one where the per-unit output tax in period 1 leads both types of firm to produce the same output level in period 1. By doing so, the government cannot distinguish them because the firm does not reveal what it knows and the efficient firm will be treated somewhat more like the inefficient one and vice versa. It is fair to say that, in such equilibrium, the efficient firm mimics the inefficient firm. The timing of the tax-game is as follows.

**Period 1.** At the beginning of this period the government decides the period-1 per-unit tax $\tau^P_1$, where superscript $P$ denotes a pooling equilibrium. Hence, both firms produce the same output level $q^P_1$ in period 1, so the government learns nothing about their efficiency level through $q^P_1$. This means that, in equilibrium, the government’s subsequent beliefs are the same as its prior beliefs given in (2).

**Period 2.** In this period, the information structure of the game is similar to that of period 1. The government chooses the same per-unit tax for both types of firm, which is the tax that has been previously chosen for period 1, $\tau^P_2 = \tau^P_1$. Finally, the firm decides its output level for period 2, $q_{2k}(\tau^P_2)$ according to its type $k, k = L, H$, and the per-unit tax to be paid in this period.

As with the separating equilibrium calculation, we use backward induction to determine pooling equilibrium. It is clear that the optimal period-2 strategy for the government is to choose the per-unit tax $\tau^*_2 = [1 - (1 - \mu)c]/2$ for both types of firm, once it has observed the firm’s production in period 1, $q^P_1$, and, therefore, its posterior beliefs are $\Pr(\bar{c} = 0|q_1 = q^P_1) = \mu$. Once both of them face such tax, their respective production levels in period 2 are

\[ q^p_{2L} = \arg \max \pi_{2L} = \left(1 - \tau^*_2 - q^p_{2L}\right)q_{2L}, \]  

(8)

in the case of the efficient firm, and

\[ q^p_{2L} = \arg \max \pi_{2L} = \left(1 - c - \tau^*_2 - q^p_{2L}\right)q_{2L}, \]  

(9)

in the case of the inefficient firm.

Apart from the beliefs of the government in equilibrium, let us assume that it considers any deviation from the equilibrium output proposal in period 1 to come from an efficient firm. That is, if the production of any type of firm, $q_1$, differs from $q^P_1$, $q_1 \neq q^P_1$, the government believes for $q_1$ that $\Pr(\bar{c} = 0|q_1 \neq q^P_1) = 1$. Consequently, the per-unit output tax in period 2 for the firm that produced $q_1$ in period 1 would correspond to that of an efficient firm under perfect information, $\tau_2 = \tau^*_2$. On the other hand, when $q^P_1$, the equilibrium output in period 1, is produced, Bayes’ rule requires the government’s updated beliefs remain unchanged because this output is produced by both types of firm. Given that $\Pr(\bar{c} = 0|q_1 = q^P_1) = \mu$, its beliefs do
indeed satisfy Bayes’ rule. Finally, the fact that the information structure in \( t = 2 \) is the same as in \( t = 1 \) leads to \( \tau_i^p = \tau_i^p \).

Under these circumstances, the IC condition for an inefficient firm is given by

\[
\pi_H(\tau_1^p, q_1^p) + \pi_H(\tau_2^p, q_2^p) \geq \pi_H(\tau_1^p) + \pi_H(\tau_2^p),
\]

where \( q_2^p \) is its period-2 optimal production on facing the per-unit tax \( \tau_2^p \). The left-hand side of (10) represents the profits obtained by the inefficient firm in period 1 from producing, in this period, the pooling quantity plus the period-2 profits derived from producing the optimal quantity when the per-unit tax to be paid is \( \tau_2^p \). The right-hand side of (10) measures the profits that the inefficient firm would obtain by deviating from the pooling equilibrium strategy, in which case production is \( q_1^H = (1 - c - \tau_1^p)/2 \) in period 1 and \( q_2^H = (1 - 2c)/4 \) in period 2.

On the other hand, the IC condition to be met by the efficient firm is

\[
\pi_L(\tau_1^p, q_1^p) + \pi_L(\tau_2^p, q_2^p) \geq \pi_L(\tau_1^p) + \pi_L(\tau_2^p),
\]

where \( q_2^p \) is the period-2 optimal output produced upon facing the per-unit tax \( \tau_2^p \). As in (10), the right-hand side of (11) represents the profits obtained by an efficient firm in the pooling equilibrium and the left-hand side reflects the profits it would obtain by deviating from the pooling equilibrium path.

Solving (10) and (11) for the demand and cost structure assumed in the model yields the following result.

**Lemma 4.** Let

\[
\Delta = 2c - \sqrt{(1-\mu)(2+(1-\mu)c)}, \quad n^- = \frac{1+(1-\mu)c - \sqrt{(1-\mu)[2+(1-\mu)c]}}{4}c,
\]

and

\[
m^+ = \frac{1-(1-\mu)c + \sqrt{(1-\mu)[2+(3+\mu)c]}}{4}c.
\]

(i) In the region of \((\mu, c)\)-parameters defined by \( \Delta \leq 0 \), there is a continuum of pooling equilibria defined as follows:

(ii) In period 1, the government sets the per-unit output tax \( \tau_1^p = [1 - (1 - \mu)c]/2 \) for both types of firm; both firms produce an output level \( q_1^p \) such that \( q_1^p \in [n^-, m^+] \); and the government’s posterior beliefs are

\[
\Pr(\tilde{c} = 0) = \begin{cases} 
\mu, & \text{if } q_1 = q_1^p \\
1, & \text{if } q_1 \neq q_1^p.
\end{cases}
\]
(i2) In period 2, the government sets the per-unit tax $\tau_2 = \tau_1$ for both types of firm; the firm produces $q^P_{2L} = \frac{1 + (1 - \mu)c}{4}$ if it is efficient, and $q^P_{2H} = \frac{(1 - c - \mu c)}{4}$ if it is inefficient; and the government’s revenue amounts to $T_2 = [1 - (1 - \mu)c]^2/8$.

(ii) In the region of $(\mu, c)$-parameters for which $\Delta > 0$, no pooling equilibrium exists.

Proof. See Appendix.

The IC condition (10) for the inefficient firm is ‘automatically’ satisfied in the whole parameter range defined by $0 < \mu < 1$ and Assumption 1, since $q^P_1$ is its optimal production level in period 1. Indeed, if this firm produces this amount in period 1, the government keeps believing, in period 2, that it is efficient with probability $\mu$ and inefficient with probability $1 - \mu$, so the per-unit tax to be paid in this period, $\tau^P_2$, will remain the same as in period 1. On the other hand, if the government observes that production deviates from $q^P_1$, it infers that the firm is efficient and therefore sets the tax $\tau^*_{2L}$ instead of the tax $\tau^P_2$, $\tau^*_{2L} > \tau^P_2$. However, the fulfillment of the IC condition (11) for the efficient firm requires fulfillment of the requirement given in part (i) of Lemma 4, i.e. either the ratio of efficient firms is not very high or, if it is high, the gap between both types is sufficiently small, given the existing ratio of efficient firms.

Part (ii) of Lemma 4 states that if the proportion of efficient firms is sufficiently high and approaches 1 (and the per-unit tax $\tau^P_1$ then approaches $\tau^*_{1L}$) and $c$ is sufficiently high, given the value of $\mu$, then the output level candidate to be part of the pooling equilibrium would be rather small and would move far away from the optimal amount for the efficient firm. Therefore, this firm does not profit from camouflaging because the pooling amount is, in this case,
too small. This is illustrated in Figure 1, where the pooling equilibrium can be observed to exist only in the region of parameters for which, for each value of $c$, the ratio of efficient firms is below a specific value (which can be lower as $c$ grows).

Lemma 4 postulates the existence of a set of infinite pooling equilibria. A reasonable criterion to select a specific equilibrium is to choose the most beneficial one for the government. Bearing in mind that the tax imposed in period 1, $\tau^p = [1 - (1 - \mu)c]/2$, is positive and independent from the firm’s production in this period and, therefore, the tax income, $T^p = \tau^p q^p$, rises with $q^p$, it is clear that the optimal production level for the government is the highest possible level. The government thus ‘imposes’ the output level $q^p = m^*$ on both firms in the pooling equilibrium, i.e. that which leads the inefficient firm to be indifferent between maintaining or modifying its output level and thus be recognized in period 2 as an efficient firm. Since $q^p = m^*$ is higher than $(1 - c - \mu c)/4$,\(^6\) –which is the output that would be optimal for the inefficient firm given the tax due—the government punishes the inefficient firm in the pooling equilibrium by forcing its production to grow beyond the optimal amount, and rewards the efficient firm because its production level $q^p$ imposed as the pooling equilibrium is close to the optimal level for this firm, $[1 + (1 - \mu)c]/4$. This can be formally stated in the lemma below.

**Lemma 5.** The preferred pooling equilibrium for the government implies that, in period 1, the firm has to pay the tax

$$T^p_1 = \left[1 - (1 - \mu)c\right] \left[1 - (1 + \mu)c + \sqrt{\left(1 - \mu\right)\left[2 - (3 + \mu)c\right]}\right] / 8$$

irrespective of its type; both types of firm produce

$$q^p = \left\{1 - c - \mu c + \sqrt{\left(1 - \mu\right)\left[2 - (3 + \mu)c\right]}\right\} / 4;$$

finally, the description of the pooling equilibrium at output level $q^p$ is completed by Lemma 4.

The fact that, in the pooling equilibrium defined by Lemma 5, the same per-unit tax is imposed on both types of firm, which have the same production level in period 1, leads the efficient firm to obtain higher net profits,

$$\pi^p_{L} = \left[1 - 4c\left[\mu c - \sqrt{\left(1 - \mu\right)\left[2 - (3 + \mu)c\right]}\right]\right] / 16,$$

than the inefficient firm, $\pi^p_{H} = (1 - 2c)^2 / 16$, as the unit margin is higher in the former case. In period 2 the same per-unit tax is imposed on both firms, but their production level differs as, given the existing information structure, they are adjusted to their actual cost; therefore, the net profits of the efficient firm, $\pi^p_{L} = (1 + c - \mu c)^2 / 16$, are again higher than those of the inefficient firm, $\pi^p_{H} = (1 - c - \mu c)^2 / 16$. In short, when both periods are considered the efficient firm obtains greater profits than the inefficient firm in a pooling equilibrium situation.
Proposition 2. If the information is not transmitted, the government applies the same per-unit output tax to both types of firm; this tax is the same in each period \( t \) and equals the tax applied under symmetric information, \( \tau^p_1 = \tau^p_2 = \tau^*, \ t = 1, 2 \).

Thus, when the government intends that the firm does not reveal what it knows, it imposes the same per-unit tax as it would be expected to do if it had the information. This happens more often in period 2 than in period 1. However, in period 2 the production level when the firm has an informational advantage differs from the production level under perfect information, so the expected tax income in the pooling equilibrium also differs from that under perfect information; namely, \( T^p_2 < T^*_2 \). In short, a government less informed than the firm is in a worse situation than when it has the information, because –although the per-unit tax can be expected to be the same in both cases– the firm produces less than what it produces under perfect information when it does not reveal what it knows (and the government therefore has an information deficit in period 2). Furthermore, in period 1, the tax income in pooling equilibrium, \( T^p_1 \), can be either higher or lower than under perfect information, \( T^*_1 \). Particularly, \( T^p_1 > T^*_1 \) if parameters are such that

\[
-1 + (1 - \mu)(2 - c) + (1 - (1 - \mu)c) \left[ 1 - (1 + \mu)c + \sqrt{(1 - \mu)(2 - (3 + \mu)c)} \right] > 0,
\]

while \( T^p_1 < T^*_1 \) when (12) is negative. Bearing both periods in mind and comparing \( T^p = T^p_1 + T^p_2 \) to \( T^* = T^*_1 + T^*_2 \), the difference in government’s revenue between asymmetric and perfect information is given by

\[
T^p - T^* = \frac{1}{8} \left[ 1 - (1 - \mu)c \right] \sqrt{(1 - \mu)(2 - (3 + \mu)c)} - 2\mu c.
\]

This can be either positive or negative, as shown in Figure 2.

Figure 2. Tax revenue under pooling equilibrium relative to tax revenue under perfect information
4.3. Information transmission versus no information transmission

Comparison of the expected tax revenue for the government in the separating equilibrium, \( T^s = T^s_1 + T^s_2 \), and in the pooling equilibrium, \( T^p = T^p_1 + T^p_2 \), renders the proposition below.

**Proposition 3.** The government induces the firm to reveal what it knows only when parameters \( \mu \) and \( c \) satisfy the condition

\[
1 + \left[ 1 - \frac{1}{2} (1 - \mu) \sqrt{2 + (2 + c) c} \right]^2 - (1 - \mu) (2 + c) c - \left[ 1 - (1 - \mu) c \right] \\
\{ 2 (1 - c) + \sqrt{(1 - \mu) [2 + (3 + \mu) c c]} \} > 0.
\]

Otherwise, the government prefers not to induce information disclosure.

Regarding period 2, the government prefers the firm to transmit its information rather than not do so, since \( T^s_2 = T^p_2 > T^p_2 \). This is because the per-unit tax imposed in period 2 is the same in both cases but the expected output of the firm increases when both parties share the information, \( q^s_2 > q^p_2 \). On the other hand, the period-1 tax income when the firm is induced to disclose private information, \( T^s_1 \), is higher than when the firm is not induced to do so, \( T^p_1 \), if

\[
1 + \left[ 1 - \frac{1}{2} (1 - \mu) \sqrt{2 + (2 + c) c} \right]^2 + \left[ 1 - (1 - \mu) c \right] \\
\{ -1 + (1 - \mu) c - \sqrt{(1 - \mu) [2 + (3 + \mu) c c]} \} > 0,
\]

i.e., if the firm is not very likely to have high potential to pay taxes, \( \mu < 0.3808 \), irrespective of the value of \( c \), or whenever \( \mu > 0.3808 \), if \( c \) is sufficiently low. On the contrary, if \( \mu > 0.3808 \) and \( c \) is sufficiently high for each value of \( \mu \), then \( T^s_1 < T^p_1 \).

Bearing in mind the revenue obtained over two periods, we conclude that when the proportion of efficient firms is low enough as \( \mu < 0.3292 \), the government prefers to impose a pooling equilibrium over a separating equilibrium, irrespective of the value of parameter \( c \). This is because there is a great incentive for the efficient firm to pretend to be an inefficient firm. Therefore, forcing the inefficient firm to behave as such leads it to strongly reduce its production in period 1. Besides, the government is forced to make big reductions in the per-unit tax in period 1 to mitigate the incentive of the efficient firm to imitate the inefficient firm and thus enables the existence of separating equilibrium. Thus, the government prefers to pool rather than to separate. The same holds when \( \mu > 0.3808 \) and \( c \) is sufficiently low for each value of \( \mu \), so that separation is again too costly. Under these conditions, \( T^p > T^s \). On the contrary, if both the ratio of efficient firms is not very low, i.e., \( \mu > 0.3292 \), and parameter \( c \) is sufficiently high for each given value of \( \mu \), the signalling cost is reduced and \( T^s > T^p \). In this case, the erosion in peri-
fiscal revenue produced by separation is compensated for by the period-2 income increase. The result of Proposition 3 is illustrated in Figure 3.

Figure 3. The government’s choice between imposing a separating or a pooling equilibrium

Finally, the firm prefers to maintain its informational advantage in period 2, as its net profits are \( \pi^S_2 > \pi^P_2 \), while it prefers to signal itself and avoid pooling in period 1, \( \pi^S_1 > \pi^P_1 \), as the per-unit output tax imposed in the former case is lower and production distortions are also less pronounced. Bearing both periods in mind, it holds that \( \pi^S > \pi^P \) in the whole region of parameters, which means that the firm always prefers to reveal rather than conceal its information.

5. Welfare analysis

The social consequences of government behaviour is analysed in this section, both for when it decides to force the firm to disclose its information and for when it chooses not to do so. To this end, welfare is defined as the unweighted sum of the (expected) consumer surplus, the net profits of the firm and the tax revenue earned by the government.

5.1. Information transmission

In this case the welfare comparison between asymmetric and perfect information contexts can be restricted to period 1. In this period, the expected consumer surplus under perfect information amounts to \( CS^*_1 = [1 - 2(1 - \mu)c + (1 - \mu)c^2]/32 \), which, taking into ac-
count net profits of the firm and the fiscal revenue earned by the government (see Table 1), gives, as the period-1 expected welfare,

\[ W^*_1 = \frac{7}{32} \left[ 1 - 2(1 - \mu) c + (1 - \mu) c^2 \right]. \tag{15} \]

On the other hand, period-1 social welfare under separating equilibrium amounts to

\[ W^S_1 = \frac{1}{128} \left\{ 28 - 12(1 - \mu) \sqrt{(2 + c)} c - 2(1 - \mu) \left[ 17 + 3\mu - 8(1 + \mu) \sqrt{(2 + c)} c - (1 + 2\mu - 3\mu^2) c^2 \right] \right\}. \tag{16} \]

Comparison of (15) and (16) yields that, despite \( \tau^*_1 < \tau^*_1 \), the strategy of inducing the firm to disclose its information reduces social welfare relative to perfect information in almost the whole \((\mu, c)\)-region of parameters, \( W^S_1 < W^*_1 \). However, in the right-hand corner of the \((\mu, c)\)-space of parameters, there is a small region defined by condition

\[ 12\sqrt{(2 + c)} c - \left[ 22 + 16(1 + \mu) \sqrt{(2 + c)} c - 6\mu \right] c + (29 + 3\mu) c^2 < 0. \tag{17} \]

for which \( W^S_1 > W^*_1 \) holds or, equivalently, \( W^S > W^* \), given that \( W^S_2 = W^*_2 \). This can be expressed as follows.

**Proposition 4(a).** The behaviour of a less informed government to induce information transmission yields welfare over two periods as follows:

(i) \( W^S > W^* \), if both parameters \( \mu \) and \( c \) are sufficiently high and satisfy (17);

(ii) \( W^S < W^* \), otherwise.

**Proof.** See Appendix.

According to (6), the period-1 output when the firm reveals what it knows is lower than under perfect information. Therefore, consumer surplus and tax revenue are both reduced. However, the firm’s net profit is higher than it would under perfect information due to the reduction in production and the firm’s lower effective marginal cost as it pays a lower per-unit tax. This profit increase may outweigh the deterioration in the situation of consumers and the government. In particular, given that under asymmetric information the production of each firm is reordered relative to perfect information—the production of the efficient firm increases and the production of the inefficient firm decreases—the increase in the firm’s profit in the separating equilibrium is especially significant when there is a low ratio of firms with a low potential to pay taxes and the disparity between both types of firms is sufficiently high. In this case, \( W^S > W^* \), as indicated in part (i) of this proposition. Figure 4 illustrates the result of Proposition 4(a).
The line depicted in the upper right-hand corner of Figure 4 is the \((\mu, c)\)-locus defined by condition (17) as an equality; it separates the \((\mu, c)\)-region where signalling improves welfare relative to perfect information. If the firm is probably a firm with a low potential to pay taxes \((\mu \leq 0.139)\) and/or its inefficiency is not very high \((c \leq 0.3037)\), the social cost of signalling is substantial and welfare is lower than under perfect information. However, when \(\mu > 0.139\) and the gap between inefficient and efficient firms is sufficiently high, given the value of \(\mu\), signalling-related social welfare is higher than under symmetric information, \(W^s > W^*\).

5.2. No information transmission and welfare

In this case, we cannot restrict the welfare comparison under perfect and asymmetric information to period 1. Rather, the outcome in period 2 associated with a pooling equilibrium differs from that associated with a separating equilibrium. Bearing in mind the results of Lemmas 4 and 5, the consumer surplus can be obtained for each period, as well as the net profit of the firm and the revenue for the government. Once \(W^p\) has been calculated and compared to \(W^*\), the welfare difference between both contexts is given by

\[
W^p - W^* = \frac{1}{32} \left\{ 2 (3 - 3c + 5\mu c) \sqrt{(1 - \mu)[2 - (3 + \mu)]c} - 2(1 + 5\mu)c + (3 + 8\mu - 15\mu^2)c^2 \right\}
\]  

and evaluating the sign of (18) yields the following proposition.

**Proposition 4(b).** In the region of \((\mu, c)\)-parameters where a pooling equilibrium exists, welfare throughout both periods, compared with that under perfect information, is as follows:
(i) \( W^p > W^*, \) if parameters \( \mu \) and \( c \) meet the condition
\[
2 \left( 3 - 3c + 5\mu c \right) \sqrt{(1 - \mu) \left[ 2 - (3 + \mu) \right] c - 2(1 + 5\mu) c + (3 + 8\mu - 15\mu^2) c^2} > 0
\]

(ii) \( W^p < W^*, \) if parameters \( \mu \) and \( c \) are such that
\[
2 \left( 3 - 3c + 5\mu c \right) \sqrt{(1 - \mu) \left[ 2 - (3 + \mu) \right] c - 2(1 + 5\mu) c + (3 + 8\mu - 15\mu^2) c^2} < 0
\]

**Proof.** See Appendix.

In period 2 the production distortions arising in the pooling equilibrium with respect to perfect information lead consumers and the firm to be better off in the former context, and the government to be worse off. Overall, \( W^p_2 < W^*_2. \) In period 1, the firm is worse off in the pooling equilibrium than in perfect information, yet both consumers and the government may be in either a better or worse situation. Bearing both periods in mind, the result of Proposition 4b holds and is as illustrated in Figure 5.

**Figure 5. Welfare implication of undisclosed information**

5.3. Information transmission and welfare

In this subsection we examine the impact of the government’s optimal behaviour on social welfare. To this end, we know that \( W^p_2 > W^*_2, \) despite \( T^p_2 < T^*_2. \) Regarding period 1 the difference in the expected firm’s production is given by
\[
q^p_1 - q^*_1 = \frac{1}{8} \left\{ (1 - \mu) \sqrt{(2 + c) c - 2(1 + \mu) c + 2(1 - \mu) \left[ 2 - (3 + \mu) \right] c} \right\}, \quad (17)
\]
which is positive when \( \mu < 0.35, \) i.e. when the firm is very likely an efficient firm, or when \( \mu > 0.35 \) and the value of \( c \) is sufficiently small for each given value of \( \mu. \) In the
remaining case, i.e. if $\mu > 0.35$ and the value of $c$ is sufficiently high for each given value of $\mu$, the period-1 production of the firm is higher with than without information transmission.

Bearing (19) in mind, consumer surplus is higher under pooling than under separating equilibrium, $CS_i^p > CS_i^s$, when $\mu < 0.2467$ or when $\mu > 0.2467$ is accomplished with a sufficiently small value for parameter $c$. On the contrary, if $\mu > 0.2467$ and the value of $c$ is sufficiently high for each given value of $\mu$, consumers are worse in the pooling than in the separating equilibrium, $CS_i^p < CS_i^s$.

In turn, the government’s tax revenue in the pooling and separating equilibrium follows a similar pattern to that of the consumer surplus, yet the referential value of parameter $\mu$ is now $\mu = 0.3808$ rather than $\mu = 0.2467$. Finally, taking into account that $\pi_1^p < \pi_1^s$, we conclude that the period-1 social welfare is higher in the pooling than in the separating equilibrium, $W_1^p > W_1^s$, when $\mu < 0.2526$, for any value of $c$, or when $\mu > 0.2526$ and the value of $c$ is sufficiently small. In contrast, when $\mu > 0.2526$ and the value of $c$ is sufficiently high for each value of $\mu$ above 0.2526, then $W_1^p < W_1^s$. This may be recorded as follows.

**Proposition 5.** From a social viewpoint, the following occurs:

(i) If parameters $\mu$ and $c$ meet the condition given by

\[
4(1 - \mu) \left[ 4(1 + \mu)c - 3 \right] \sqrt{(2 + c)c - 8(3 - 3c + 5\mu c)} \sqrt{(1 - \mu)[2 - (3 + \mu)c]} c + 2\left(15 + 6\mu + 3\mu^2\right) c - \left(41 + 6\mu - 63\mu^2\right) c^2 > 0,
\]

not forcing the firm to reveal what it knows about its potential to pay taxes renders higher welfare than forcing it to reveal information.

(ii) If parameters $\mu$ and $c$ are such that

\[
4(1 - \mu) \left[ 4(1 + \mu)c - 3 \right] \sqrt{(2 + c)c - 8(3 - 3c + 5\mu c)} \sqrt{(1 - \mu)[2 - (3 + \mu)c]} c + 2\left(15 + 6\mu + 3\mu^2\right) c - \left(41 + 6\mu - 63\mu^2\right) c^2 \leq 0,
\]

not inducing the firm to reveal private information concerning its potential to pay taxes renders higher welfare than not inducing it to reveal information.

**Proof.** See Appendix.

Forcing information disclosure by the firm improves social welfare relative to allowing no information disclosure only when the ratio of efficient firms is sufficiently high, $\mu > 0.3292$, and the efficiency level is also sufficiently high for each value of $\mu$ above 0.3292. Otherwise giving up to inducing the firm to reveal the (private) information concerning its potential to pay taxes causes lower social welfare than inducing it to. Figure 6 illustrates this result.
6. Conclusions

Asymmetric information whereby one of the parties in a transaction has information that is not available to the other parties occurs in many situations and may induce opportunistic behaviour in informed players. A number of papers have addressed how less informed players can mitigate this problem in the context of an incumbent threatened by a potential entrant (Matthews and Mirman, 1983; Milgrom and Roberts, 1985; Mailath, 1991; Aguirre, 1999). This paper supplements the literature by exploring how a government may countervail opportunistic behaviour by adapting the amount of quantity-related taxes to firms privately informed on their efficiency level or potential to pay taxes. For this purpose we have considered a two-period signalling tax-game involving a firm and a revenue-raising government deciding the per-unit output tax in each period. The firm’s marginal production costs—whether low or high—are privately known and observation of period-1 output levels may allow such information to be inferred or not.

In the (asymmetric information) period 1, the per-unit output tax is the same for all types of firm because the government cannot distinguish between types. When the government taxes a firm and wants it to reveal information, it imposes a separating equilibrium and anticipates that an efficient firm will wish to misreport. This incentive is so strong that information transmission is always costly and leads to a reduction in the period-1 output of the inefficient firm. The government counteracts this incentive by reducing the per-unit tax and the amount of taxes the firm must pay in period 1 relative to that it would expect to charge with perfect information. On the other hand, when information is not revealed through the output of period 1, the per-unit output tax imposed in this period (and also in period 2) is the same as that it would expect to charge under perfect information. Overall, the model predicts that average per-unit output taxes under asymmetric information will not exceed those under perfect information.

Figure 6. Welfare implications of information transmission vs. no information transmission
Our main results depend on the value of the parameter measuring the probability of having an efficient firm (a firm with high potential to pay taxes) and that of the parameter measuring the inefficiency level of an inefficient firm (a firm with low potential to pay taxes). For example, on choosing between inducing the firm to reveal its information or not doing so, the government faces two opposite effects. Revealing information is always beneficial in period 2, yet not in period 1. In general, the increase in tax income in period 2 does not outweigh the signalling costs of period 1, and inducing revelation only benefits the government when, given the inefficiency level, the firm is quite likely to be efficient.

We also find that government behaviour to induce the firm to reveal what it knows generally decreases welfare relative to perfect information, except when the probability of having an efficient firm is sufficiently high and the disparity between the two types of firm is also sufficiently high. In this case, the increase on the firm’s net profits in period 1 with respect to symmetric information compensates greatly for the deterioration in this period of the situation of consumers and the government. Things are rather different when the government decides not to induce the firm to reveal what it knows. In this case, the firm’s period-1 production increases so much that, in general, it tends to lead to increased social welfare as compared to perfect information.

Finally, when the social implications of both options—to induce information disclosure or to induce no information disclosure—are compared, aggregate welfare in the former case is generally lower than in the latter in most of the parameter space. Inducing information revelation is socially preferable only when the proportion of efficient firms in the industry is not too small and the inefficiency level of those which are inefficient firms is very low.

Appendix

**Proof of Lemma 1.** The firm’s output in period \( t (t = 1, 2) \) is given by \( q^*_it = (1 - \tau_it)/2 \), if it is efficient, and \( q^*_it = (1 - c - \tau_it)/2 \), if it is inefficient. In the former case, the problem of the government, \( \max_{\tau_it} \tau_it(1 - \tau_it)/2 \), enables \( \tau^*_it = 1/2 \); in the latter, it is \( \max_{\tau_it} \tau_it(1 - c - \tau_it)/2 \), and leads to \( \tau^*_it = (1 - c)/2 \). The values in Table 1 for per-period output and profits of the firm hold straightforwardly. This completes the proof of the lemma.

**Proof of Lemma 2.** Let \( \pi(t|x|z) \) stand for period-2 firm’s profit if the firm represented itself in period 1 as having cost \( x \) when it actually had cost \( z \). Hence, \( \pi_2(0|c) = [(1 + c)/4]^2 \) and \( \pi_2(0|0) = [(1 - 2c)/4]^2 \). Comparison of these profits with those stated in Table 1 shows that \( \pi_2(0|c) > \pi_2(0|0) \) and \( \pi_2(c|c) > \pi_2(c|0) \). This proves the lemma.

**Proof of Lemma 3.** Spelling out IC conditions (4) and (5) yields

\[
(1-c-\tau_1-q^*_1it)(1-c-\tau_1) + \left(1-c-\tau_1\right)^2 - \left(\frac{1-c-\tau_1}{4}\right)^2 - \left(\frac{1-2c}{4}\right)^2 
\]

(A1)
respectively. Inequalities (A1) and (A2) are simultaneously satisfied by any output level \(q_{iH}^S \in [u^-, v^-]\), where \(u^-\) and \(v^-\) are, respectively, the smaller roots of the quadratic equations formed by taking (A1) and (A2) as equalities, that is,

\[
u^- = \frac{1 - \tau_i - c}{2} - \frac{\sqrt{(2 - 3c)c}}{4} \]  

(A3)

and

\[
u^- = \frac{1 - \tau_i}{2} - \frac{\sqrt{(2 + c)c}}{4} \]  

(A4)

Therefore, the first-period outputs within the interval \([u^-, v^-]\) form part of separating equilibria. The inefficient firm maximizes its period-1 profit by producing \(q_{iH}^m = (1 - c - \tau_i)/2\), yet this strategy does not satisfy conditions (A1) and (A2), because \((1 - c - \tau_i)/2 > v^-\) according to Assumption 1. Thus, the separating equilibria lead this firm to produce an output strictly below \(q_{iH}^m\) (the output that maximizes its period-1 profit) and within the interval \([u^-, v^-]\). Given that the per-unit tax grows with production (see Lemma 1), the government prefers the firm’s production to be as high as possible within the interval \([u^-, v^-]\). That is, it wants it to be \(q_{iH}^S = v^-\).

For the government, the period-1 expected output is given by

\[
q_i^S(\tau_i) = \mu \frac{1 - \tau_i}{2} + (1 - \mu) \frac{2(1 - \tau_i) - \sqrt{(2 + c)c}}{4} \]  

(A5)

and the per-unit tax imposed in this period is that which solves

\[
\max_{\tau_i} \tau_i \cdot q_i^S(\tau_i) = \tau_i \left[ \mu \left(\frac{1 - \tau_i}{2}\right) + (1 - \mu) \left(\frac{1 - \tau_i}{2} - \frac{\sqrt{(2 + c)c}}{4}\right) \right]. \]  

(A6)

Solving (A6) provides the level of tax chosen by the government, \(\tau_i^S\). Hence, substituting \(\tau_i^S\) in \(v^-\) and \(q_{iH}^S\) gives the optimal levels of output for each firm’s type. Finally, the fiscal revenue in period 1 is obtained as \(T_i^S = \tau_i^S q_i^S\), where \(q_i^S = \mu q_{iL}^S + (1 - \mu) q_{iH}^S\).

(ii) The proof of part (ii) is straightforward.

**Proof of Lemma 4.** (i) Let us begin analysing the IC condition (10) for the inefficient firm. In the (pooling) equilibrium, in period 2 this firm pays the per-unit tax \(\tau_i^p = [1 - (1 - \mu)c]/2\), so

\[
(1 - \tau_i - q_{iH}^S) q_{iH}^S + \left(1 + c\right)^2 - \left(\frac{1 - \tau_i}{4}\right)^2 - \frac{1}{16} \leq 0, \]  

(A2)
\[
\text{max} \left( 1 - c - \frac{1 - (1 - \mu)c}{2} - q_{2H} \right) q_{2H},
\]

(A7)

is the problem to be solved in order to determine the production level for this period. This yields \( q_{2H}^p = \frac{[1 - (1 + \mu)c]}{4} \). On the other hand, if this firm deviates from production level \( q_1^p \), the best it can do in period 1 is to produce \( q_{1H}^o = \frac{1 - c - \tau_1^p}{2} \), the monopolistic production corresponding to its type, given the per-unit tax, \( \tau_1^p \), where the ‘off’ superscript indicates that the firm deviates from the equilibrium path. Finally, this firm is considered an efficient firm in period 2, so the per-unit tax imposed is the one designed for an efficient firm, \( \tau_{2L}^* = 1/2 \), in this period. In short, the IC condition defined in (10) particularizes as

\[
\left( 1 - c - \tau_1^p - q_i^p \right) q_i^p + \left( \frac{1 - (1 + \mu)c}{4} \right)^2 \geq \left( \frac{1 - c - \tau_1^p}{2} \right)^2 + \left( \frac{1 - 2c}{4} \right)^2.
\]

(A8)

Analogously, the IC condition (11) that defines the pooling equilibrium for the efficient firm is

\[
\left( 1 - \tau_1^p - q_i^p \right) q_i^p + \left( \frac{1 - (1 + \mu)c}{4} \right)^2 \geq \left( \frac{1 - \tau_1^p}{2} \right)^2 + \frac{1}{16},
\]

(A9)

The solution of (A8) as an equality renders the roots

\[
m^- = \left\{ 1 - (1 + \mu)c - \sqrt{\left( 1 - (1 + \mu)c \right) \left[ 2 - (3 + \mu)c \right]} \right\} / 4
\]

and

\[
m^+ = \left\{ 1 - (1 + \mu)c + \sqrt{\left( 1 - (1 + \mu)c \right) \left[ 2 - (3 + \mu)c \right]} \right\} / 4.
\]

Therefore, any production level \( q_1 \in [m^-, m^+] \) meets the IC condition that defines pooling equilibrium for the inefficient firm. Likewise, solving (A9) as an equality yields the roots

\[
n^- = \left\{ 1 + (1 - \mu)c - \sqrt{\left( 1 - (1 - \mu)c \right) \left[ 2 - (1 - \mu)c \right]} \right\} / 4
\]

and

\[
n^+ = \left\{ 1 + (1 - \mu)c + \sqrt{\left( 1 - (1 - \mu)c \right) \left[ 2 - (1 - \mu)c \right]} \right\} / 4,
\]

by which any production level \( q_1 \in [n^-, m^+] \) meets the IC conditions that defines a pooling equilibrium. In short, both conditions (A8) and (A9) are fulfilled in the region of parameters in which condition \( n^- \leq m^+ \) is verified or, equivalently, where

\[
2c - \sqrt{\left( 1 - (1 - \mu)c \right) \left[ 2 + (1 - \mu)c \right]} - \sqrt{\left( 1 - (1 - \mu)c \right) \left[ 2 - (3 + \mu)c \right]} c \leq 0.
\]

(A10)
Under condition (A10) there is a continuum of output levels in period 1 that belong to the pooling equilibrium and that are defined by interval $q^p_1 \in [n^-, m^+]$. This completes the proof of part (i).

(ii) If the sign of (A10) is strictly positive, then $n^- > m^+$ and there is no output value that verifies (A8) and (A9) simultaneously.

Proof of Lemma 5. Out of all output levels in interval $q_1 \in [n^-, m^+]$, the government-selected level is $q^p_1 = m^+$, since period-1 tax revenue, $T^p_1 = \tau^p_1 q^p_1$, verifies $\partial T^p_1 / \partial q^p_1 = \tau^p_1 > 0$, i.e. the higher the firm’s production is, the higher the government’s tax revenue.

Proof of Proposition 4(a). (i) When there is information transmission, the expected consumer surplus in period 1 is

$$CS^*_1 = \frac{1}{128} \left\{ 4 \left[ 1 - (1 - \mu) \sqrt{(2 + c)c} \right] + 2 \left[ 1 + 2\mu - 3\mu^2 \right] c + \left[ 1 + 2\mu - 3\mu^2 \right] c^2 \right\}$$

(A11)

and the expected net profit for the firm

$$\pi^*_1 = \frac{1}{64} \left\{ 4 \left[ 1 + (1 - \mu) \sqrt{(2 + c)c} \right] - 2(1 - \mu) \left[ 11 + \mu - 4 \left( 1 + \mu \right) \sqrt{(2 + c)c} \right] c - \left( 3 - 2\mu - \mu^2 \right) c^2 \right\}$$

(A12)

Once (A11), (A12) and (7) are considered, the result given in part (i) follows directly.

(ii) Immediate.

Proof of Proposition 4(b). In case of a pooling equilibrium, consumer surplus in period 1 is

$$CS^p_1 = \frac{1}{32} \left\{ 1 - (1 + \mu) c + \sqrt{(1 - \mu)c \left[ 2 - (3 + \mu)c \right]} \right\}^2$$

(A13)

the net profit of the firm amounts to

$$\pi^p_1 = \mu \left( 1 - \tau^p_1 - q^p_1 \right) q^p_1 + (1 - \mu) \left( 1 - c - \tau^p_1 - q^p_1 \right) q^p_1 =

= \frac{1}{16} \left\{ 1 + 4c \left( -1 + \mu + (1 - \mu - \mu^2) c \right) + \mu \sqrt{(1 - \mu)c \left[ 2 - (3 + \mu)c \right]} \right\}$$

(A14)

and the government’s tax collection is

$$T^p_1 = \tau^p_1 q^p_1 = \frac{1}{8} \left\{ 1 - (1 - \mu)c \right\} \left\{ 1 - (1 + \mu) c + \sqrt{(1 - \mu)c \left[ 2 - (3 + \mu)c \right]} \right\}.$$
\[ W_1^p = CS_1^p + \pi_1^p + T_1^p = \frac{1}{32} \left\{ 7 + (9\mu - 7)c - \sqrt{(1-\mu)c(2-(2+3\mu)c)} \right\} \]
\[ \left\{ 1-(1+\mu)c + \sqrt{(1-\mu)c(2-(2+3\mu)c)} \right\} \]

(A16)

On the other hand, the payoffs for each player in period 2 are
\[ CS_2^p = \frac{1}{32} \left\{ 1-(1-\mu)[2-(3+\mu)c]c \right\}, \]

(A17)

\[ \pi_2^p = \mu \left( 1 - \tau_2^p - q_{2L}^p \right) q_{2L}^p + (1-\mu) \left( 1 - \tau_2^p - q_{2H}^p \right) q_{2H}^p = \]
\[ = \frac{1}{16} \left\{ 1-(1-\mu)[2-(3+\mu)c]c \right\} \]

(A18)

and
\[ T_2^p = \tau_2^p q_2^p = \frac{1}{8} \left[ 1-(1-\mu)c \right]^2, \]

(A19)

which allows us to obtain
\[ W_2^p = \frac{1}{32} \left\{ 1-(1-\mu)[14-(7+5\mu)c]c \right\}. \]

(A20)

Bearing (A16) and (A20) in mind, the social welfare throughout both periods is given by
\[ W^p = \frac{1}{32} \left\{ 14 + 3(2+5\mu-3c)\sqrt{(1-\mu)[2-(3+\mu)c]}c \right\} \]
\[ -6(5-3\mu)c + (17-6\mu-15\mu^2)c^2 \]

(A21)

and comparing (A21) and \( W^s \), which is given by \( W_1^s \) stated in (16) plus \( W_1^* \) defined in (15), yields the proposed result.

Notes

1. The decision to evade taxes made by a competitive firm is analysed by Virmani (1989), Yamada (1990) and Cremer and Gahvari (1993), among others.
2. Matthews and Mirman (1983), Harrington (1986), Bagwell and Ramey (1988), and von der Fehr (1992), among others, also examined the context in which the incumbent’s pre-entry behaviour transmits information on its type.
3. See Propositions 2 and 5 below.
4. With only two possible types of firms, a pure-strategy perfect Bayesian equilibrium is either a separating or pooling. Thus, it is enough to characterize the sets of separating and pooling equilibria.
5. The output reduction vanishes when \( c = 0 \) or when \( \mu = 1 \).
6. This is ensured by Assumption 1.
7. In more detail, the order is $T^p > T^* > T^S$ or $T^* > T^p > T^S$.

8. In this case, the order is $T^* > T^S$ when there is no pooling equilibrium, and $T^* > T^S > T^p$ when a pooling equilibrium exists.

9. Both consumers and the government are better in the pooling equilibrium than in the perfect information setting when the value of parameter $\mu$ is sufficiently low or the value of parameter $c$ is sufficiently high for each value of $\mu$ above a given threshold.

10. Specifically, $\mu < (4\sqrt{5} - 5)/11$.

References


Resumen

Este artículo analiza, mediante un juego de señalización de dos periodos en el que participan una empresa y un gobierno que trata de maximizar los ingresos fiscales en cada periodo, cómo se comporta éste a la hora de fijar un impuesto a la producción en un contexto en el que la información sobre la eficiencia de la empresa (o su capacidad para pagar impuestos) es privada. Para mitigar el oportunismo e inducir a la empresa a que transmita la información mediante su producción del periodo 1, el gobierno se ve obligado a fijar en dicho periodo un impuesto menor, en términos esperados, que si la información fuese perfecta, lo cual significa que el impuesto en el periodo 2 es mayor que en el periodo 1. Este comportamiento del gobierno generalmente reduce el bienestar. Por el contrario, cuando la información no es revelada, el impuesto esperado fijado por el gobierno es igual en los dos periodos y coincide con el que fijaría bajo condiciones de información perfecta. Además, si la empresa no revela la información sobre su potencial para pagar impuestos, el bienestar es generalmente (aunque no siempre) mayor que si la revela, porque la revelación reduce mucho los ingresos del gobierno en el periodo 1. Desde el punto de vista social, la no revelación de información aumenta el bienestar con respecto a la revelación, excepto cuando la probabilidad de que la empresa resulte eficiente y su nivel de eficiencia, —en caso de que resulte eficiente— son suficientemente elevados. En estas circunstancias, el nivel de bienestar social si el gobierno deriva la capacidad de pagar impuestos de la empresa una vez que observa el output que ésta produce en $t = 1$ es mayor que si no obtiene dicha información.

Palabras clave: Impuestos por unidad de producción, gobierno maximizador del ingreso fiscal, información asimétrica, monopolio, equilibrio separador y agrupador.

Clasificación JEL: H21, D82.