INEQUALITY FOUNDATIONS OF CONCENTRATION MEASURES: AN APPLICATION TO THE HANNAH-KAY INDICES

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ABSTRACT

In this paper we connect, in a consistent way, the Hannah-Kay concentration indices with some inequality measures coming from the income distribution literature. We isolate the inequality component underlying the concentration measures, providing an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms. Finally, our proposed decomposition is illustrated by means of an empirical example, which proves to be useful to identify the sources of a change in sectoral concentration along time.

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1. INTRODUCTION

Concentration indices are traditional instruments in industrial economics, which provide a synthetic measure of market structure, and allow to evaluate the degree of competition present in different industries (see, e.g., Waterson, 1984).

The aim of this paper is to relate some popular concentration indices, in particular those proposed by Hannah and Kay (1977), with some inequality measures coming from the income distribution literature. Since every concentration index possesses a specific inequality component, we will first provide a consistent relationship between both kinds of indices (concentration and inequality), and then we will show that the general entropy inequality indices are consistent with the Hannah and Kay concentration indices.

In addition, we will also provide an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms. Finally, we will present an application to real data in order to show the utility of the approach.

The rest of the paper is organized as follows. The relationship between concentration and inequality indices is derived in section 2, and the decomposition of the change in concentration, together with the empirical example, is shown in section 3. The main conclusions are summarized in section 4.

2. CONSISTENT RELATIONSHIPS BETWEEN CONCENTRATION AND INEQUALITY INDICES

Concentration indices are formally defined as a function $C:R^N \rightarrow R$ over a vector $s=(s_1,\ldots,s_N)$, where s_i is the relative market share of the ith firm:

$$s_i = \frac{X_i}{\sum_{i=1}^{N} X_i} \qquad (1)$$

being X_i an indicator of the size of the ith firm (usually sales or employment).

Assuming an axiomatic derivation as in Hannah and Kay (1977) or Encaoua and Jacquemin (1980), industry concentration indices can be expressed as a function of two variables [see, e. g., Waterson (1984)]:

$$C = f(N, I) \qquad f_N < 0, f_I > 0 \qquad (2)$$

where N denotes the number of firms in the industry, and I is a inequality index of firm size I: $R^N \rightarrow R$, defined over the vector $X=(X_1,\ldots,X_N)$. Under the classical "principle of transfers" (Dalton, 1920), I(·) must be strictly S-convex (Dasgupta, Sen and Starret, 1973).

More specifically, a new entrant into an industry would lead to an ambiguous effect on concentration. On the one hand, concentration directly falls due to the increased number of firms. But, on the other hand, the degree of inequality within the industry is also affected, so that concentration could actually rise in the case that the entrant is big enough.

Our aim in this paper will be to try to disentangle both effects by building a bridge between concentration indices and the classical inequality indices. To this end, in this section we will focus our attention on the consistent derivation of the Hannah and Kay concentration indices from the general entropy inequality indices, as defined by Cowell (1977):

$$I_{GE(c)} = \begin{cases} \frac{1}{N}\frac{1}{c(c-1)}\sum_{i=1}^{N}[(X_i/\bar{X})^c - 1], & \forall\, c \neq 0,1 \\ \frac{1}{N}\sum_{i=1}^{N}\ln(\bar{X}/X_i), & \text{if } c = 0 \\ \frac{1}{N}\sum_{i=1}^{N}[(X_i/\bar{X})\ln(X_i/\bar{X})], & \text{if } c = 1 \end{cases} \quad (3)$$

where, according to the income distribution literature, X_i denote the ith household income, $\bar{X}$ 1 is the mean income across households, and N is the number of households. Notice that, for our purposes, the concept of income will be extended to define the analogue concept for the firm, so that X_i would apply to any indicator of the firm's size[1].

Formally, we propose the following definition. A concentration index C is consistent with (i.e., can be consistently derived from) an inequality index I if, given N, for any two vectors s^1 and s^2 the following equivalence is satisfied:

$$C(s^1) \geq C(s^2) \;\Leftrightarrow\; I(s^1) \geq I(s^2) \quad (4)$$

which is equivalent to the restriction $f_I>0$ in equation (2). We will be concerned with the case of homogeneity of degree minus one in N on the concentration indices in equation (2)[2].

Next, we can write the Hannah and Kay class of concentration indices in the following way:

1 Moreover, it can be shown that, since the inequality indices defined throughout the paper are relative (i. e., zero-degree homogeneous in the X variable) inequality indices, they can be interpreted alternatively in terms of relative shares, i. e., I(X)=I(s).

2 Notice that inequality indices are also influenced by population changes; in particular, all the indices used in this paper satisfy the population replication axiom. More specifically, the Atkinson inequality indices satisfy the marginal population replication axiom (Salas, 1998), so they are good candidates to perform well under changes in population size.

$$C_{HK(\alpha)} = \begin{cases} [\sum_{i=1}^{N} s_i^{\alpha}]^{\frac{1}{(\alpha-1)}} & \text{if } \alpha > 0, \alpha \neq 1 \\ \exp[\sum_{i=1}^{N} s_i \ \ln s_i] & \text{if } \alpha = 1 \end{cases} \quad (5)$$

Notice that $C_{HK(1)}$ is defined as the limit of $C_{HK(\alpha)}$ when $\alpha \to 1$, which coincides with the antilogarithm of (minus) the first-order entropy concentration index; see also Waterson (1984). Now, from the previous definition, we can be derive in a consistent way the Hannah-Kay concentration indices from the general entropy inequality indices. In fact, equations (3) and (5) can be shown to be related through:

$$C_{HK(\alpha)} = \begin{cases} \frac{[1+\alpha(\alpha-1) I_{GE(\alpha)}]^{\frac{1}{\alpha-1}}}{N} & \text{if } \alpha = c > 0, \alpha = c \neq 1 \\ \frac{\exp[I_{GE(\alpha)}]}{N} & \text{if } \alpha = c = 1 \end{cases} \quad (6)$$

Two particular cases are of interest. First, since $C_{HK(2)}$ is the Herfindahl concentration index, C_H, we can write:

$$C_H = \frac{1+2\, I_{GE(2)}}{N} \quad (7)$$

Second, $C_{HK(1)}$ is consistent with $I_{GE(1)}$, i. e., the classical Theil 1 index:

$$C_{HK(1)} = \frac{\exp[I_{GE(1)}]}{N} \quad (8)$$

For the case $0 < \alpha < 1$, $C_{HK(\alpha)}$ indices are also consistent with the classical Atkinson indices $I_{A(\varepsilon)}$, defined for every $\varepsilon > 0$ in the following way (Atkinson, 1970):

$$I_{A(\varepsilon)} = \begin{cases} 1-[\frac{1}{N}\sum_{i=1}^{N}(\frac{X_i}{\bar{\bar{X}}})^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}, & \forall\ \varepsilon > 0, \varepsilon \neq 1 \\ 1-\exp[\frac{1}{N}\sum_{i=1}^{N}\ln(\frac{X_i}{\bar{\bar{X}}})], & \text{if } \varepsilon = 1 \end{cases} \quad (9)$$

so that, when $\varepsilon = 1-\alpha$, the following equivalence holds[3]:

3 Notice that a complete consistent equivalence between the Hannah-Kay and the general entropy (for all α, in equation 6) and Atkinson (for all $\alpha<1$, in equation 10) indices could be found by further generalizing the Hannah-Kay indices, if we extend the definition in equation (5) to

$$C_{HK(\alpha)} = -[\sum_{i=1}^{N} s_i^{\alpha}]^{\frac{1}{(\alpha-1)}} \quad \text{if } \alpha < 0$$

$$C_{HK(\alpha)} = \frac{[1 - I_{A(1-\alpha)}]^{\frac{\alpha}{\alpha-1}}}{N} \quad \text{if } 0 < \alpha < 1 \qquad (10)$$

3. DECOMPOSING THE CHANGE IN CONCENTRATION: AN EXAMPLE

In this section we provide a decomposition of the change in concentration from the two sources identified in equation (2), i.e., the number of firms N and the degree of inequality I. In particular, we can write equations (6) and (10) alternatively as:

$$C_{HK(\alpha)} = \frac{\varphi(I_{GE(\alpha)})}{N} \quad \text{if } \alpha > 0 \qquad (6')$$

$$C_{HK(\alpha)} = \frac{\psi(I_{A(1-\alpha)})}{N} \quad \text{if } 0 < \alpha < 1 \qquad (10')$$

where $\alpha\left(I_{GE(\alpha)}\right)$ and $\Psi\left(I_{A(1-\alpha)}\right)$ are the component of inequality in $C_{HK(\alpha)}$, which are increasing functions of the general entropy and Atkinson inequality indices, respectively. Now, from (6') and (10'), it is straightforward that:

$$\frac{\Delta C_{HK(\alpha)}}{C_{HK(\alpha)}} \simeq \frac{\Delta\varphi(I_{GE(\alpha)})}{\varphi(I_{GE(\alpha)})} - \frac{\Delta N}{N} \quad \text{if } \alpha > 0 \qquad (11)$$

$$\frac{\Delta C_{HK(\alpha)}}{C_{HK(\alpha)}} \simeq \frac{\Delta\psi(I_{A(1-\alpha)})}{\psi(I_{A(1-\alpha)})} - \frac{\Delta N}{N} \quad \text{if } 0 < \alpha < 1 \qquad (12)$$

We illustrate the former decomposition with an example taken from Bajo and Salas (1997). In that paper we computed a set of concentration indices for 68 sectors of the Spanish economy in 1993, using the Spanish Institute for Fiscal Studies' dataset coming from the Profit Tax reports by more than 300,000 firms (i.e., providing an almost exhaustive coverage of both firms and sectors). Then, our decomposition was applied to the change in concentration between 1992 and 1993, for the Hannah-Kay indices with α=0.5, 1, 1.5, 2, and 2.5.

Notice that, according to equation (11) —and analogously in the case of equation (12)— for any particular α, concentration would unambiguously increase when:

$$\frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

which, in turn, would occur in any of the following cases:

$$\text{(i)}\Delta N < 0 \text{ and } \Delta I > 0$$

$$\text{(ii)}\Delta N < 0,\ \Delta I < 0 \text{ and } \frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

$$\text{(iii)}\Delta N > 0,\ \Delta I > 0 \text{ and } \frac{\Delta\varphi(I)}{\varphi(I)} > \frac{\Delta N}{N}$$

On the other hand, for any particular α, concentration would unambiguously decrease when:

$$\frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

which would occur in any of the following cases:

$$\text{(iv)}\Delta N > 0 \text{ and } \Delta I < 0$$

$$\text{(v)}\Delta N > 0,\ \Delta I > 0 \text{ and } \frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

$$\text{(vi)}\Delta N < 0,\ \Delta I < 0 \text{ and } \frac{\Delta\varphi(I)}{\varphi(I)} < \frac{\Delta N}{N}$$

In table 1 we present an example of the decomposition shown in equations (11) and (12). As the last column of the table shows, we are able to explain reasonably well the change in concentration during the period. From the 68 sectors in our previous study, we have selected nine industries, which cover the six cases stated above.

In six of the sectors, concentration increases. In Food industry, Textiles, and Banking, concentration rises due to both a lower number of firms and a higher inequality —i.e., case (i) above—. In Basic chemicals, concentration rises due to a lower number of firms and despite a lower inequality for α=0.5, 1 and 1.5 —i.e., case (ii) above—; however, for α=2 and 2.5, higher inequality would also lead to higher concentration —i.e., case (i) above—. Finally, in Chemicals and Precision instruments, concentration rises due to a higher inequality and despite a higher number of firms —i.e., case (iii) above—.

In the three remaining sectors, concentration decreases. In Air and sea transportation, concentration falls due to both a higher number of firms and a lower inequality —i.e., case (iv) above—. In Computing services, concentration falls due to a higher number of firms and despite a higher inequality for α=1, 1.5, 2 and 2.5 —i.e., case (v) above—; however, for α=0.5 lower inequality would also lead to lower concentration —i.e., case (iv) above—. Finally, in House renting, concentration falls due to a lower inequality and despite a lower number of firms —i.e., case (vi) above—.

TABLE 1

DECOMPOSITION OF THE CHANGE IN CONCENTRATION, 1992-93

A) Index HK(0.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	2.00	-0.06	-2.02	1.96	97.98
Chemicals	2.81	2.94	0.12	2.82	100.12
Precision Instruments	0.41	3.57	3.14	0.43	103.14
Food Industry	6.15	5.19	-0.91	6.09	99.09
Textiles	9.83	1.33	-7.74	9.07	92.26
Air and Sea Transportation	-15.69	-12.63	3.63	-16.25	103.63
Banking	4.61	2.13	-2.36	4.50	97.64
Computing Services	-11.73	-2.20	10.80	-13.00	110.80
House Renting	-7.10	-9.20	-2.26	-6.94	97.75

B) Index HK(1)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	1.01	-1.02	-2.02	1.00	98.98
Chemicals	5.62	6.08	0.12	5.96	106.08
Precision Instruments	5.70	9.37	3.14	6.24	109.37
Food Industry	11.41	11.86	-0.91	12.76	111.86
Textiles	11.03	3.70	-7.74	11.44	103.70
Air and Sea Transportation	-52.50	-32.05	3.63	-35.67	67.95
Banking	5.14	2.93	-2.36	5.29	102.93
Computing Services	-10.63	0.15	10.80	-10.65	100.15
House Renting	-53.90	-36.49	-2.26	-34.23	63.51

C) Index HK(1.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	1.46	-0.50	-2.02	1.51	104.00
Chemicals	8.37	7.26	0.12	7.14	85.40
Precision Instruments	9.37	11.02	3.14	7.88	84.05
Food Industry	22.90	19.22	-0.91	20.13	87.89
Textiles	16.38	6.36	-7.74	14.10	86.06
Air and Sea Transportation	-31.73	-27.68	3.63	-31.31	98.66
Banking	4.52	1.72	-2.36	4.09	90.50
Computing Services	-6.83	2.88	10.80	-7.92	115.93
House Renting	-70.65	-69.46	-2.26	-67.21	95.12

D) Index H

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	2.53	0.46	-2.02	2.48	97.98
Chemicals	10.62	10.75	0.12	10.63	100.12
Precision Instruments	9.15	12.58	3.14	9.44	103.14
Food Industry	34.34	33.12	-0.91	34.02	99.09
Textiles	19.83	10.55	-7.74	18.29	92.26
Air and Sea Transportation	-26.98	-24.33	3.63	-27.96	103.63
Banking	3.96	1.51	-2.36	3.87	97.64
Computing Services	-5.63	4.57	10.80	-6.23	110.80
House Renting	-78.34	-78.83	-2.26	-76.57	97.74

E) Index HK(2.5)

SECTOR	Rate of change in the concentration index (1)	Rate of change in the inequality component (2)	Rate of change in the number of firms (3)	Explained rate of change (4)=(2)-(3)	Percentage of explanation (5)=(4)/(1)*100
Basic Chemicals	3.57	1.48	-2.02	3.49	97.98
Chemicals	12.52	12.66	0.12	12.54	100.12
Precision Instruments	8.15	11.55	3.14	8.41	103.14
Food Industry	44.91	43.59	-0.91	44.50	99.09
Textiles	21.64	12.23	-7.74	19.97	92.26
Air and Sea Transportation	-23.84	-21.08	3.63	-24.70	103.63
Banking	3.75	1.29	-2.36	3.66	97.64
Computing Services	-4.50	5.81	10.80	-4.99	110.80
House Renting	-79.89	-80.35	-2.26	-78.09	97.74

Source: Bajo and Salas (1997)

4. CONCLUSIONS

In this paper we have derived a consistent relationship between the Hannah-Kay concentration indices and some of the more popular inequality measures coming from the income distribution literature. We isolated the inequality component underlying the concentration measures, and then we provided an explicit additive decomposition of the change in concentration into the change in its two components: inequality and the number of firms.

This decomposition proved to be useful in empirical work since it could help to identify the sources of a change in sectoral concentration along time. We concluded by presenting an empirical application to the Spanish economy, which illustrated the procedure proposed in the paper.

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