EXTENDED BI-POLARIZATION AND INEQUALITY MEASURES (*)

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ABSTRACT

This paper aims to clarify the similarities and differences between the concepts of bi-polarization and inequality by proposing an extended measure of bi-polarization, which is consistent with the second polarization curve.

The standard decomposition property of population subgroups for the Gini coefficient can be generalized to the extended Gini coefficients. Then, it is explicitly shown that the Wolfson bi-polarization index can be obtained by subtracting the within-groups from the between-groups Gini coefficients, computed for groups separated by the median value.

Moreover, we demonstrate the existence of a critical interval of the sensitivity parameter values (v) of the extended Gini coefficient, within which the second polarization curve can be consistently expressed as the subtraction of the within-groups inequality component from the between-groups inequality component. This critical interval is defined by $v \in [2,3]$. This approach has the conceptual advantage of viewing inequality and polarization within the same framework.

Key Words: bi-polarization; extended Gini index; inequality decomposition.

JEL Classification: D39, D63.

1. INTRODUCTION

Recent papers agree on the conceptual difference between polarization and inequality; see, for instance, Love and Wolfson (1976), OECD (1993), Wolfson (1994, 1997), Esteban and Ray (1994), Esteban, Ray and Gradín (1999), and Gradín (1998). Polarization concentrates the income distribution on several focal or polar modes, and involves the disappearance of the middle class, whereas inequality relates to the overall dispersion of the distribution, and is inversely linked with equalizing mean-preserving spreads. On the one hand, polarization fails to satisfy the principle of transfers between polar groups, hence contrasting with inequality. On the other hand, as with inequality, polarization satisfies the principle of transfers within polar groups.

Approaches to statistical measurement also differ. The axiomatic inequality approach imposes classical S-convex inequality measures for consistency with the Lorenz domination criterion (see Atkinson, 1970; Dasgupta *et al*., 1973), whereas bi-polarization focus uses measures consistent with the second polarization curve devised by Foster and Wolfson (1992).[1] A particular geometrical measure is provided by Wolfson (1994, 1997), which clearly illustrates the differences between Lorenz curves and the concept of inequality.

This paper aims to clarify the similarities and differences between the concepts of bi-polarization and inequality by proposing an extended measure of bi-polarization. First, we explicitly show that the Wolfson bi-polarization index can be obtained by subtracting the within-groups from the between-groups Gini coefficients, computed for groups separated by the median value.[2] This insight helps to explain the identification and alienation concepts (which are negatively correlated with within-groups inequality, and positively correlated with between-groups inequality, respectively) represented in the polarization measure and characterized by Esteban and Ray (1994). This approach has the conceptual advantage of viewing inequality and polarization within the same framework, depending on whether the within-groups component is added or subtracted.

Second, we generalize the decomposition property for population subgroups to the extended Gini coefficients proposed by Donaldson and Weymark (1980, 1983) and Yitzhaki (1983). Finally, we demonstrate the existence of a critical interval $v \in [2,3]$ for values of the sensitivity parameter (v) of the extended Gini coefficient, within which the second polarization curve can be consistently expressed as the subtraction of the within-groups inequality component from the between-groups inequality component.

1 Another methodology can be found in Esteban and Ray (1994).

2 This idea can implicitly be found in Gradín (1998).

These general indices are consistent with the second polarization curve in the following sense: a progressive median-preserving transfer within (between) polar groups never reduces (increases) polarization. These measures enable a more robust sensitivity polarization curve-based analysis, and do not rely on a particular measure when polarization curves intersect.

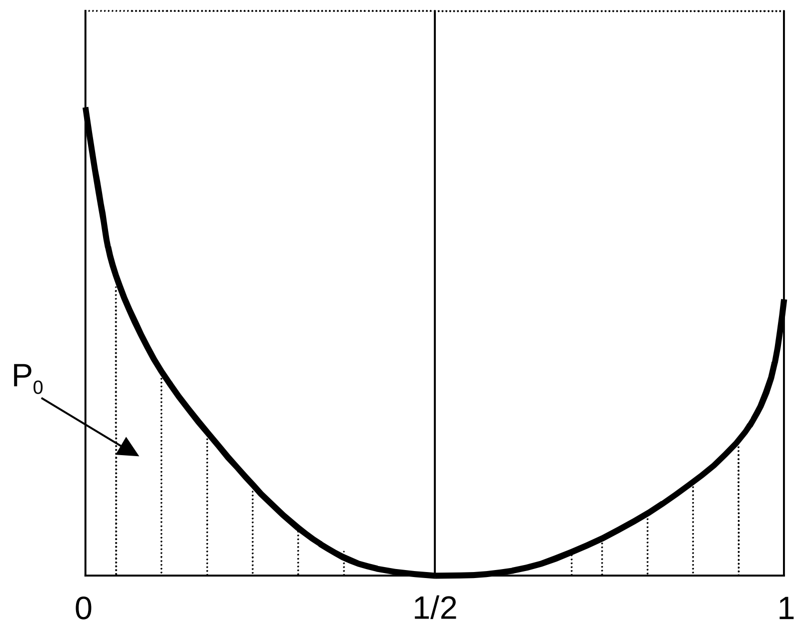
The structure of the paper is as follows. A relationship between the Wolfson bi-polarization measure and the population subgroups decomposition of the Gini index is given in section 2. In section 3, we generalize the subgroups decomposition property to the extended Gini coefficient. In section 4, a new bi-polarization measure, the extended Wolfson bi-polarization index, is provided. Section 5 presents conclusions.

2. WOLFSON BI-POLARIZATION INDEX AND INEQUALITY

Wolfson (1994, 1997) provided a formalization of the polarization concept in an analogous manner to the theoretical development of inequality measures. The conceptual foundation of inequality measures is associated with the Lorenz curve and is developed from the cumulative density function (cdf) for the distribution of income. The axes of the cdf are transposed so that population percentiles are on the horizontal axis and incomes are on the vertical axis. Having divided through by the mean income, this curve is then integrated from the right to the origin to obtain the usual Lorenz curve.

Figure 1

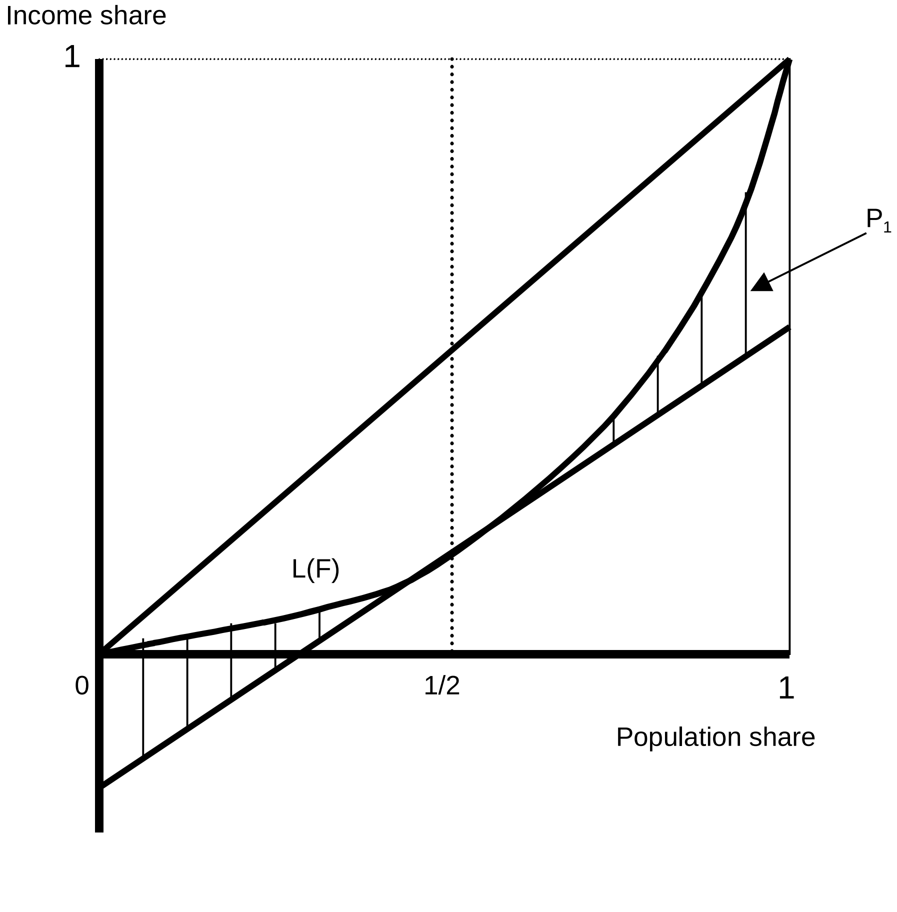
SECOND POLARIZATION CURVE



To formalize the concept of polarization, an analogous sequence of graphical transformations of the initial cdf is followed. First, the axes of the cdf are transposed. Then, the income levels are divided by the median rather than by the mean income. Next, the horizontal axis is shifted up to touch the resulting curve at the mid-point of the horizontal axis (the 50th population percentile). The part of the curve that relates to the 50 per cent of the population with income levels below the median is then flipped around the horizontal axis. Finally, the curve is integrated in both directions from the mid-point along the horizontal axis to generate the *second polarization curve* (see Figure 1). Hence, this polarization curve plays the role for the concept of polarization that the Lorenz curve plays for inequality.

Figure 2

BI-POLARIZATION MEASURE



However, as with Lorenz curves, it is possible to have intersecting polarization curves. As do Lorenz curves, polarization curves induce only a partial ordering over income distributions, while the shaded area under the polarization curve in Figure 1 (P_0) induces a complete ordering (like the Gini coefficient). Furthermore, Wolfson proposed a transformation of the polarization curve. If the curve is multiplied by the ratio of the median to the mean, and the horizontal axis is then tilted until its slope is the same as the tangent to the Lorenz curve at the 50th population percentile, the result is a transformed polarization curve that is identical to the Lorenz curve, which is shown in Figure 2. Thus, the shaded area P_0 under the polarization curve in Figure 1 is a simple transformation of the lightly shaded area in Figure 2 (P_1). To be specific, the lightly shaded area in Figure 1 (P_0) is:

$$P_0 = \frac{P_1}{m/\mu} = \frac{(T - G(F)/2)}{m/\mu}$$

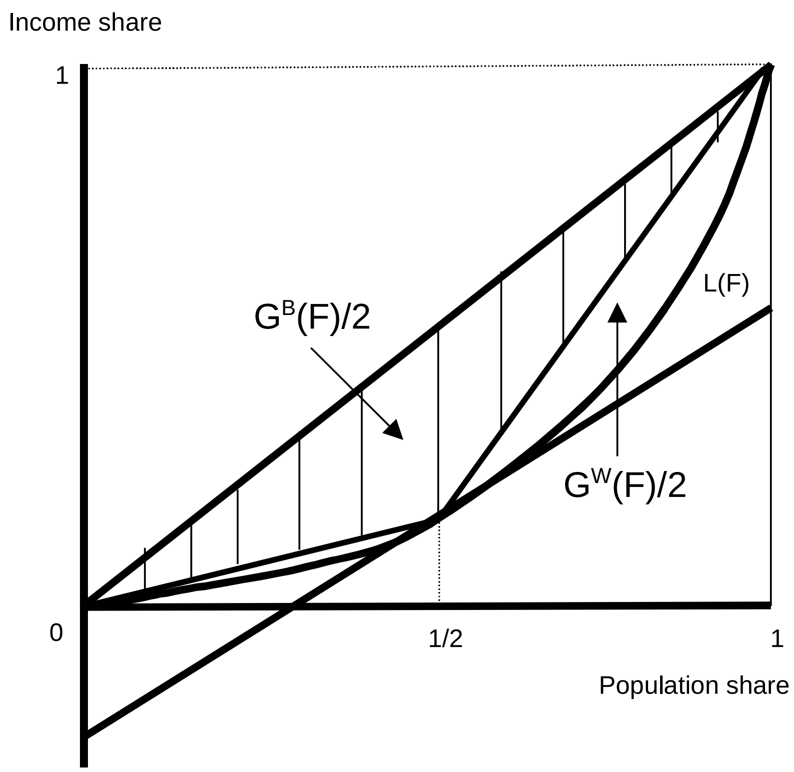
where m is the median, μ is the mean, m/μ is the slope of the tangent to the Lorenz curve at the 50th population percentile, G is the Gini index of inequality, and T is the area of the trapezoid defined by the 45-degree line and the median tangent (the vertical distance between the Lorenz curve and the 45-degree line at the 50th percentile). So that the range of the index is similar to that of the Gini, Wolfson arbitrarily quadrupled the area of P, so the formula becomes:

$$P_m = 4P_0 = \frac{\mu}{m} 2(2(0.5 - L(0.5)) - G(F)) \qquad (1)$$

where $L(0.5)$ is the value of the Lorenz curve at the 50th percentile.

Figure 3

BETWEEN –AND WITHIN– GROUPS COMPONENTS



Note that the vertical distance between the Lorenz curve and the 45-degree line at the 50th percentile is twice the area between the two-piece Lorenz curve (the one that would be obtained if every income in each subgroup were to be replaced by the corresponding subgroup mean) and the 45-degree line (the shaded area in Figure 3). That is, the difference between 50 per cent and the income share of the bottom half of the population, $0.5 - L(0.5)$, is equal to the between-groups Gini coefficient. Furthermore, since the subgroup income ranges do not overlap, there is an exact decomposition of the Gini coefficient into between-groups and within-groups contributions. Hence (1) can be expressed as:

$$P_m = \frac{2\mu}{m}\left[G^B(F) - G^W(F)\right] \tag{2}$$

where G^B is the between-groups Gini coefficient and G^W is the within-groups Gini coefficient, computed for groups separated by the median value.

This polarization expression prompts three interesting questions. First, the conceptual advantage of this approach is that inequality and polarization can be viewed within the same framework, with addition and subtraction of the within-groups component corresponding to inequality and polarization, respectively.

Secondly, a connection between Wolfson´s concept of polarization and the polarization model of Esteban and Ray (1994) has been established. On the one hand, the between-groups Gini coefficient relates to the accentuation of polarization by inter-group heterogeneity. That is, G^B represents feelings of alienation between dissimilar individuals. (Alienation is positively correlated with between-groups inequality.) On the other hand, the within-groups Gini coefficient relates to the accentuation of polarization by intra-group homogeneity. Hence, G^W represents feelings of identification between similar individuals. (Identification is negatively correlated with within-groups inequality.)

Thirdly, it can be shown that a similar result can be obtained if the polarization measure is computed for groups separated by the mean value rather than by the median value. In this case, the new expression of the Wolfson polarization index is:

$$P_\mu = 2\left[G^B(F) - G^W(F)\right]$$

When the single cut-off income of the distribution is the mean value instead of the median value, the area of the trapezoid (now defined by the 45-degree line and the mean tangent, which is parallel to the former) remains equal to the between-groups Gini coefficient; that is, $G^B(F) = 0.5 - L(0.5)$. Note, though, that this result does not apply to cut-off incomes other than those implied by the median and mean values. This is because $G^B(F) = p_z - L(p_z)$, where z is the chosen critical income and $p_z = F(z)$.

In the next section, we generalize the population subgroups decomposition property to the extended Gini coefficients, which will allow an extension of measures that are consistent with the second polarization curve.

3. POPULATION SUBGROUPS DECOMPOSITION OF THE EXTENDED GINI COEFFICIENT

We apply the geometric decomposition approach developed for the standard Gini coefficient by Lambert and Aronson (1993) to decompose the extended Gini index coefficient, $G(v)$, proposed by Donaldson and Weymark (1980, 1983) and Yitzhaki (1983) into the following three terms: the between-groups extended Gini coefficient, $G^B(v)$; the within-groups extended Gini coefficient, $G^W(v)$; and the residual extended Gini component, $R(v)$.[3]

Let the ordered income distribution $X = (x_1, x_2, ..., x_N)$, in a population Π whose mean is μ, be partitioned into n subgroups Π_k, where $k = 1, 2, ..., n$, with means μ_k. Total inequality for this income distribution, as measured by the extended Gini coefficient, is:

$$G(v) = 1 - v(v-1)\int_0^1 (1-q)^{v-2} L(q)\,dq$$

where the inequality aversion parameter v is such that $v > 1$. Note that only $L(q)$ depends on the specific distribution since $(1 - q)$ reflects the rank in the income distribution. Integrating the last expression by parts yields:

$$G(v) = v(v-1)\int_0^1 (1-q)^{v-2} [q - L(q)]\,dq$$

Hypothetically, suppose that, starting from a situation of equality (everybody receives μ), inequality is introduced in three stages (rather than all at once). First, to introduce between-groups inequality, total income ($N\mu$) is redistributed between subgroups so that each person in every subgroup Π_k receives the mean income of that subgroup (μ_k). With the Lorenz curve for this notional income distribution denoted by $L_B(q)$, we can write the transformation (from a situation of perfect equality to one in which there is between-groups inequality) in terms of the extended Gini index as follows:

$$G^B(v) = v(v-1)\int_0^1 (1-q)^{v-2} [q - L_B(q)]\,dq$$

Next, inequality within groups is introduced. We assign to individuals their own actual incomes and retain the allocation of individuals to subgroups from before. Within each subgroup, individuals are assigned in ascending order based on their incomes (from low-income to high-income individuals). Denoting by $C(q)$ the concentration curve for this lexicographic distribution of incomes, the implied within-groups extended Gini coefficient is:

$$G^W(v) = v(v-1)\int_0^1 (1-q)^{v-2} [L_B(q) - C(q)]\,dq$$

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[3] Duclos (2000) provides a decomposition by income source. We provide a different population-subgroups decomposition.

Then, having accounted for the overlapping effect (which occurs between subgroups) by re-ranking individuals from the poorest overall to the richest overall, we obtain the true income distribution, so the residual term is:

$$R(v) = v(v-1)\int_0^1 (1-q)^{v-2}\left[C(q) - L(q)\right]dq$$

This three-stage procedure decomposes the extended Gini index into three contributions: between-groups; within-groups; and residual:[4]

$$G(v) = G^B(v) + G^W(v) + R(v)$$

Were there no overlapping between subgroups, the concentration curve and the Lorenz curve for the true distribution of income would coincide, in which case, the within-groups component would be:

$$G^W(v) = v(v-1)\int_0^1 (1-q)^{v-2}\left[L_B(q) - L(q)\right]dq$$

In this case, the residual term would be zero.

In the next section, we use this decomposition of the extended Gini coefficient to generalize the Wolfson index of polarization, noting that the polar subgroups do not overlap.

4. EXTENDED WOLFSON BI-POLARIZATION MEASURE

We define the *extended Wolfson bi-polarization measure* as $P(v) = G^B(v) - G^W(v)$, in which the polarization measure depends on a sensitivity parameter *v*. The parameter *v* determines the weights assigned to the identification and alienation terms. Specifically, the higher is *v*, the higher the weight given to the identification and the alienation terms (although with different relative effects).

Definition: A polarization index is consistent with the second polarization curve if a progressive median-preserving transfer [in the sense used by Wolfson (1997)] within (between) polar subgroups never reduces (increases) polarization.

The following proposition can be established.

Proposition: *Given a particular income distribution, X, the extended Wolfson bi-polarization measure,* $P(v) = G^B(v) - G^W(v)$ *is consistent with the second polarization curve if* $v \in [2, 3]$.

Proof: Suppose that we implement a progressive median-preserving transfer that does not cross the median; that is, it is a transfer between two indi-

4 This decomposition approach can also be applied to the more general Mehran family of rank-dependent measures of inequality.

viduals in the same polar subgroup. In this case, $G^B(v)$ does not change, whereas $G^W(v)$ decreases as the Lorenz curve approaches the 45-degree line. Therefore, in this case, polarization as measured by the extended Wolfson bi-polarization index increases, while S-convex inequality is reduced.

Now, consider a progressive transfer that does cross the median. Let us write the within-groups and the between-groups Gini coefficient variations in discrete terms. The change in the between-groups Gini coefficient is always negative:

$$\Delta G^B(v) = v(v-1)\sum_{i=1}^{N}(1-\frac{i}{N})^{v-2}\left[(\frac{i}{N}-L_B^1(\frac{i}{N}))-(\frac{i}{N}-L_B^0(\frac{i}{N}))\right]=$$

$$=-v(v-1)\sum_{i=1}^{N}(1-\frac{i}{N})^{v-2}\left[L_B^1(\frac{i}{N})-L_B^0(\frac{i}{N})\right]$$

The superscripts 0 and 1 denote the periods before and after implementation of the progressive transfer, respectively. Graphically, L_B approaches the 45-degree line as the mean income of the first subgroup increases and the mean income of the second subgroup decreases. Therefore, $\Delta G^B(v)$ is negative. However, we wish to find the exact change in the between-groups coefficient. Transferring one unit of income between subgroups, noting that the increase (decrease) in the mean of the first (second) subgroup is equal to $\frac{1}{N/2}=\frac{2}{N}$, we obtain:

$$\Delta G^B(v) = -v(v-1)\frac{2/N}{N\mu}\left[\sum_{i=1}^{N/2}(1-\frac{i}{N})^{v-2}i+\sum_{i=\frac{N}{2}+1}^{N}(1-\frac{i}{N})^{v-2}(N-i)\right]<0$$

The change in the within-groups coefficient is given by:

$$\Delta G^W(v) = v(v-1)\sum_{i=1}^{N}(1-\frac{i}{N})^{v-2}\left[(L_B^1(\frac{i}{N})-L^1(\frac{i}{N}))-(L_B^0(\frac{i}{N})-L^0(\frac{i}{N}))\right]=$$

$$=-\Delta G^B(v)-v(v-1)\sum_{i=1}^{N}(1-\frac{i}{N})^{v-2}\left[L^1(\frac{i}{N})-L^0(\frac{i}{N})\right]$$

Consider the least favourable case, in which there is a progressive transfer between the richest individual and the poorest, and there is no re-ranking between them. In this case, the decrease in $G^W(v)$ is the largest possible. Polarization may increase as a result, which is inconsistent with the second polarization curve.[5] In this extreme case:

[5] If $G^W(v)$ increases, or decreases by less than $G^B(v)$ decreases, the extended Wolfson bi-polarization measure is consistent with the second polarization curve for every sensitivity parameter value, *v*.

$$L^1\left(\frac{i}{N}\right)-L^0\left(\frac{i}{N}\right)=\frac{1}{N\mu} \qquad \forall i=1,\ldots,N-1.$$

This result implies that the within-groups coefficient does not increase:

$$\Delta G^W(v)=v(v-1)\frac{2/N}{N\mu}\left[\sum_{i=1}^{N/2}\left(1-\frac{i}{N}\right)^{v-2}i+\sum_{i=\frac{N}{2}+1}^{N}\left(1-\frac{i}{N}\right)^{v-2}(N-i)\right]-v(v-1)\frac{1}{N\mu}\sum_{i=1}^{N}\left(1-\frac{i}{N}\right)^{v-2}=$$

$$=v(v-1)\frac{1}{N\mu}\left[\sum_{i=1}^{N/2}\left(1-\frac{i}{N}\right)^{v-2}\left(\frac{2i}{N}-1\right)+\sum_{i=\frac{N}{2}+1}^{N}\left(1-\frac{i}{N}\right)^{v-2}\left(\frac{2(N-i)}{N}-1\right)\right]\leq 0$$

Hence, alienation between the two groups does not increase while identification does not decrease. Consequently, polarization will increase or decrease depending on the weights attached to each term. In fact:

$$\Delta P(v)=\Delta G^B(v)-\Delta G^W(v)=2\Delta G^B(v)+v(v-1)\frac{1}{N\mu}\sum_{i=1}^{N}\left(1-\frac{i}{N}\right)^{v-2}=$$

$$=-2v(v-1)\frac{2/N}{N\mu}\left[\sum_{i=1}^{N/2}\left(1-\frac{i}{N}\right)^{v-2}i+\sum_{i=\frac{N}{2}+1}^{N}\left(1-\frac{i}{N}\right)^{v-2}(N-i)\right]+v(v-1)\frac{1}{N\mu}\sum_{i=1}^{N}\left(1-\frac{i}{N}\right)^{v-2}= \quad (3)$$

$$=v(v-1)\frac{1}{N\mu}\left[\sum_{i=1}^{N/2}\left(1-\frac{i}{N}\right)^{v-2}\left(1-\frac{2i}{N/2}\right)+\sum_{i=\frac{N}{2}+1}^{N}\left(1-\frac{i}{N}\right)^{v-2}\left(1-\frac{2(N-i)}{N/2}\right)\right], \quad v>1$$

From (3), it is clear that the sign of $\Delta P(v)$ depends on the sign of the term in the square brackets, which sums positive and negative weights. To be specific:

if $1\leq i\leq\frac{N}{4}$ or $\frac{3N}{4}\leq i\leq N$, the term in square brackets is positive;

and if $\frac{N}{4}<i<\frac{3N}{4}$, the term in square brackets is negative.

Note that the term in square brackets is zero if $i = N$.

Therefore, the sign of $\Delta P(v)$ depends on the sensitivity parameter, v. If v is sufficiently low (sufficiently close to unity), the weights assigned to the high incomes are sufficiently larger than the weights assigned to lower incomes for polarization to increase. Polarization also increases if v is sufficiently high for the weights assigned to the low incomes to be sufficiently larger than the weights on higher incomes. To go further, it is necessary to calculate the second derivative of the weight function $f(x)$:

$$f(x)=\left(1-\frac{x}{N}\right)^{v-2}$$

Clearly, this function is strictly convex for any $v \in \{(1, 2)$ or $(3, \infty)\}$ and strictly concave for any $v \in (2, 3)$. In the remaining cases, $v = \{2, 3\}$ and the weight function is linear. Hence, there is a critical interval of sensitivity parameter values, $[v_1, v_2] = [2, 3]$, within which polarization is reduced for the set of transfers under consideration.[6,7] In other words, the extended Wolfson bi-polarization measure, $P(v) = G^B(v) - G^W(v)$, is always consistent with the second polarization curve in this case, and as long as the weights are not too low or too high, the identification and alienation terms behave consistently with the second polarization curve.

Remark:

Given a particular income distribution X, there always exists $v \in \{v_1, v_2\}$ where $v_1 \in (1,2]$ and $v_2 \in [3, \infty)$ such that the extended Wolfson bi-polarization measure, $P(v) = G^B(v) - G^W(v)$, is always consistent with the second polarization curve. Notice, however, that it is only if $v \in [2, 3]$ that the previous consistency property is guaranteed, regardless the initial income distribution.

Example:

Let us give the following numerical example to explain how it works. Suppose the initial income distribution Y_0=(100, 125, 150, 200, 225, 250). Assume the clear-cut mean- and median-preserving transfers similar to the ones in Wolfson (1994) and Gradín (1998) to end up with Y_1=(125, 125, 125, 225, 225, 225). It produces an unambiguous increase in polarization and unambiguous decrease in inequality (according to the second polarization and Lorenz curve dominance criterion). Moreover, as only the within-groups inequality changes (in this case, it reduces) and no change in the between-groups component is produced, we obtain a reduction in polarization for every v parameter.

Suppose now a second transfer of 99 units that crosses the median, from the poorest individual to the richest one to end with Y_2=(1, 125, 150, 200, 225, 349). It increases both the within- and the between-groups inequality, what is consistent with an unambiguous dominance of the second polarization in favour of the initial distribution Y_0. Then we obtain the predicted result of an increase in the polarization indices for $v \in [2,3]$. However, an eventual reduction in polarization for $v > 6$ is produced in this particular case.

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[6] If $v = 2$ (Wolfson polarization measure) polarization decreases because the weights are independent of the rank. When $v = 3$, a simple but tedious development of equation (3) proves the assertion above.

[7] If this procedure is applied to the Mehran family of rank-dependent measures of inequality (instead of the extended Gini index), the extended Wolfson bi-polarization measure is consistent with the second polarization curve in those intervals in which the weight function is concave or linear.

Finally, if we compare the extended polarization indices for Y_1 and Y_2, we observe in table 1 that for *v*=2, polarization is higher in Y_1 while, for *v*=2.5, it is higher in Y_2. This implies that there is a cross in the second polarization curve as, in fact, Figure 4 shows. In short, this is very much analogous to the relationship between the Lorenz curve and the extended Gini coefficient.

Table 1

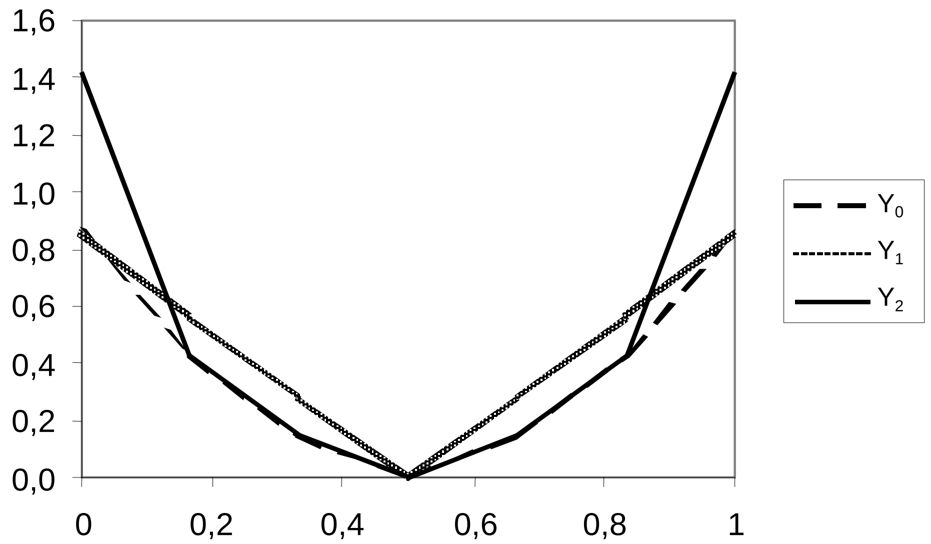
EXTENDED BI-POLARIZATION

$P(v) = G^B(v) - G^W(v)$			
v	**Y_0**	**Y_1**	**Y_2**
1,5	0,05808	0,07842	0,07175
2	0,10600	0,13793	0,13761
2,5	0,13961	0,18001	0,18299
3	0,16188	0,20977	0,20929
5	0,18826	0,26365	0,20363
6	0,18539	0,27263	0,17444

Figures for the example in the main text.

Figure 4

SECOND POLARIZATION CURVES FOR THE EXAMPLE



5. CONCLUSIONS

In this paper, we propose a class of bi-polarization indices, which is consistent with the second polarization curve. We demonstrate the existence of a critical interval for the sensitivity parameter values (v) of the extended Gini coefficient, within which the second polarization curve can be consistently expressed as the subtraction of the within-groups inequality component from the between-groups inequality component. This critical interval is defined by $v \in [2,3]$.

This result is due to the standard decomposition property of population subgroups for the Gini coefficient having been generalized to the extended Gini coefficients. This makes it possible to show explicitly that the Wolfson bi-polarization index can be obtained by subtracting the within-groups from the between-groups Gini coefficients, computed for groups separated by the median value.

This approach has the conceptual advantage of setting inequality and polarization within the same framework, with inequality and polarization corresponding to addition and subtraction of the within-groups component, respectively. Moreover, these measures enable a more robust sensitivity polarization curve-based analysis, and do not rely on a particular measure when polarization curves intersect. For instance, inconclusive polarisation change results can be drawn for the Spanish economy using ECHP for 1994 and 1995, where results are consistent with a cross in the second polarisation curve.

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