A Microanalytic Full Model for the Laffer Curve in Personal Income Taxation

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The standard approach to evaluate the Laffer curve of personal income taxation normally focuses attention on the impact on income tax revenue only. However, this is an incomplete depiction of reality as changes in income tax rates affect revenue collection from other taxes as well -i.e. consumption taxes and social security contributions. In addition, to the extent that administration and compliance costs correlate with the magnitude of the marginal rates, the Laffer curve should also take this correlation into account. This paper develops a complete microeconomic model for the Laffer curve of the personal income tax taking into account all these omissions.

Keywords: Laffer curve; social security contributions; tax revenue; personal income tax; administration costs; compliance costs.

JEL classification: D1; D61; H2; H21; H24; H31; H55
1. INTRODUCTION

Tax revenue adequacy has always been one of the defining features of a healthy budgetary policy. The need to achieve sufficient levels of tax collection has been defended from a broad spectrum of arguments. Arguments ranging from the candid search by a benevolent and generous State for the "common good" to less naïve ones such as the one defended by the School of Public Choice, in which the State’s voracious appetite for tax revenue reflects not only the search for the general interest but also the desire to satisfy the private preferences of politicians and bureaucrats. Whether for one reason or another, the study of the revenue capacity of tax systems has been, is and will be one of the primary concerns of fiscal policy. In this context, this paper analyzes the controversial Laffer curve in the Personal Income Tax (PIT). The Laffer curve generates controversy because it appeals to the existence of limits to taxation, even when the intention is to maximize tax revenue. Its invocation, therefore, is usually interpreted as a call for budgetary restraint, which means that certain political options tend to classify it more as an instrument at the service of budgetary austerity than as a rigorous tool for economic analysis. This perception is, to a large extent, responsible for the fact that the valuation of the Laffer curve is made more from the ideological sphere than from the neutrality of economic analysis.

Since Arthur B. Laffer drew his famous curve on the napkin of a Washington restaurant in the mid-1970s, the existence of an inverse relationship between tax rates and revenue has occupied a significant part of the tax debate. Far from the anecdote of the napkin, the Laffer curve is one of the most studied and well-founded concepts of contemporary economic thought. However, surprisingly, the Laffer curve debate has been fought essentially in the political arena, feeding more on proclamations and opinions than on rigorous and restful analysis. This simplistic way of framing the discussion of the Laffer curve has divided the interested audience into two irreconcilable groups: those who defend the existence of the Laffer curve and those who deny it. The former group is the group of the credulous or the “gullible” group, the latter the “negationists” group. Although, economic theory and most of the empirical evidence give the former as victors – i.e. the Laffer curve is an indubitable reality. However, the victory of the gullible is not complete because the gullible mistake of identifying the existence of the Laffer curve with the taxpayer’s forced location in the descending leg of that curve. That is to say, although the negationists are categorically mistaken in denying the existence of the Laffer curve, the postulates of the gullible are not entirely correct and need to be nuanced.

As Arthur Laffer himself acknowledges, and as authors such as Wanniski (1978), Fullerton (1982) or Lindsey (1985) demonstrate, the idea of the existence of an inverse relationship between tax collection and rates does not originate from Arthur Laffer. This relationship was noted much earlier by authors such as David Hume (1756), Adam Smith (1776) or Jules Dupuit (1844), among others. However, it must be recognized that after Arthur Laffer’s intervention in the debate, his ideas came to the fore and became a preferential place in tax design1.

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1 To a large extent, the ideas underlying the Laffer curve helped to generate the paradigm shift that occurred in the tax systems of the 1980s. In contrast to the previous tax systems, which were supposed to seek greater (formal) equity, in the 1980s efficiency became the preferred tax principle. Ronald Reagan’s 1981 and 1986 reforms and Margaret
Since the 1970s, much research has been undertaken to show not only the existence of the Laffer curve but also the exact location of the different national economies on this curve. The 1980s began with an explosion of more or less complex economic models that sought to quantify the "Laffer effect" of taxation. These initial models include Stuart (1981) and Feige and McGee (1985), for Swedish reality, and those of Fullerton (1982), Canto et al (1983), Bender (1984) and Lindsey (1985), for the American case. With the emergence of behavioral economics, analysis of the Laffer phenomenon soon began to appear from this newly launched approach. Outstanding examples are, for example, the study by Swenson (1988) or the subsequent study by Sutter and Weck-Hannemann (2003). The first carries out an experiment to test whether taxes generate disincentives on labour supply, confirming the existence of the Laffer curve. The second, reports a more sophisticated experiment, in which the tax rate becomes endogenous from an interactive game of two players, in which one acts as taxpayer (subject A) and another as tax authority (subject B). Sutter and Weck-Hannemann also confirm experimentally the existence of the Laffer curve\(^2\).

More recently, through a set of controlled experiments, Lévy-Garboua et al (2009) postulate the existence of a new kind of Laffer curve, different from the conventional one, which they baptize with the name of behavioural Laffer curve. This new curve arises as a punitive reaction to the perception of injustice of taxes and has the peculiarity of reaching the forbidden zone significantly earlier than the conventional Laffer curve. In this same tradition, although with a different approach, Ortona et al (2008) draw attention to an interesting fact: the use given to tax collection and the existence of excessive administration and compliance costs have a decisive impact on taxpayers' valuation of taxes. Therefore, States must be cautious in the way they spend what they collect, since waste seems to induce a fall in the taxpayer's labor effort and a lower willingness to pay taxes, with the consequent decrease in collection and the strengthening of the Laffer effect.

Other papers, mostly theoretically grounded, have analyzed the factors that reinforce or weaken this Laffer effect of taxation. Administrative bureaucracy, corruption or the black economy are some of the elements that seem to have the most influence. Forte (1987), for example, states that the existence of rent-seeking bureaucracy and administrative and compliance costs require to distinguish between two different Laffer curves: one linked to gross revenue and the other associated with the net revenue of those costs. Forte concludes that the Laffer curve of net receipts is a left-hand translation of the Laffer curve of gross receipts for two reasons: because conceptually gross receipts and net receipts differ and because the indirect costs of taxation - administration and compliance costs - cause the gross receipts associated with any tax rate to be lower.

Thatcher's 1984 reforms were the first to be decisively inspired by this new pattern. These reforms were later imitated by other countries.

\(^2\) In addition, Sutter and Weck-Hannemann analyze the impact of making decisions about work effort and tax rate under “the veil of ignorance”. To this end, the results of two experiments are contrasted: one where the relative position of the players - taxpayer or tax authority - is known from the beginning; another, under uncertainty, where the players do not know their respective roles - taxpayer or tax authority - until the end of the experiment. In this state of uncertainty (veil of ignorance), both players must decide on the tax rate and the level of effort they would be willing to exert. The results show that under the veil of ignorance taxpayers tend to work harder and set lower tax rates - around 20% lower.
In the same line of reasoning, Sanyal et al (2000) assess the effect of a corrupt Tax Administration. These authors confirm that, in a corrupt environment, stepping up the tax effort required of taxpayers, through an increase in tax rates or through a tightening of tax audits, may result in a loss of revenue. A loss of revenue that is more likely if sanctions and tax rates affect positively the level of corruption in the Tax Administration. Similarly, Panadés (2003) finds that, when the costs of evasion are low, the rise in tax rates or the tightening of anti-fraud policies, in addition to not being effective to increase tax collection, are regressive. For its part, Vogel (2012) confirms that, for the PIT and the corporation tax (CT), the existence of the black economy reinforces the characteristic inverted U-shape of the Laffer curve. However, Vogel (2012) does not detect such an impact on excise taxes.

Endogenous growth models have also led to a fruitful generation of Laffer curve studies. From this family of studies, of macroeconomic nature, tax revenue is analyzed from a multi-temporal perspective where the fiscal policy actions carried out in the present have consequences on the welfare and budgetary sustainability of the future. Studies of this tradition have coined the notion of the dynamic Laffer curve, which postulates that the expansionary effects associated with a reduction in tax rates in the present are, in general, strong enough to ensure an increase in social welfare and future tax bases, thus guaranteeing the financial sustainability of the Budget in the long term. Ireland (1994), Pecorino (1995), Novales and Ruiz (2002), Nutahara (2015), Oudheusden (2016) and Bosca et al. (2017) are outstanding examples of this type of studies. Under different assumptions, all these studies corroborate the existence of the dynamic Laffer curve not only in the tax levied on wages but also in the taxes levied on capital and consumption.

Finally, other authors have opted to model the Laffer curve analytically. This approach has the advantage that the Laffer curve can be mathematically analyzed and, if adequate microdata is available, it can be empirically computed for every individual taxpayer in the population. To the extent that tax microdata reliably represents the true distribution of taxable incomes, this analytical approach is arguably the most reliable and robust method for deriving the Laffer curve, especially in the case of personal income taxation. References using this analytical approach include Giertz (2009), Saez et al (2012), Creedy (2015), Creedy and Gemmell (2013, 2015) and Sanz (2016a, 2016b). However, although this analytical approach is possibly the most robust of all the existing approaches for analyzing the Laffer curve, its implementation in the existing literature suffers from an important limitation: analysis is restricted exclusively to the collection consequences on the PIT. In other words, the existing analytical models ignore the fact that marginal tax rate changes also alter the revenue from other taxes and levies such as those from taxes on consumption or Social Security contributions. Moreover, current analytical models also overlook the fact that taxes impose additional burdens in the form of administrative and compliance costs that widen the gap between gross and net Laffer curves. This paper aims to fill all these gaps and to develop a full-fledged model of the PIT Laffer curve.

3 Recently, the Laffer curve of excise taxes has been the subject of monographic analysis in Aasness and Nygard (2014) and Guedes de Oliveira and Costa (2015). The first of these studies evaluates for Norway the Laffer curve of taxes levied on tobacco and alcoholic beverages. The second paper calculates, from the estimation of a quadratic collection function, the Laffer curve of VAT for 27 of the 28 countries that make up the current European Union (EU).
The paper is structured as follows. Section 2 presents the set-up in which the full-fledged model of the Laffer curve is developed, distinguishing between gross and net Laffer curves. In this second section we obtain the functions of the Laffer curve for the PIT according to the revenue impacts taken into account. Specifically, the functions of the Laffer curve are obtained under different revenue settings: when only the revenue impact on personal income taxation is recorded, when the revenue from consumption taxes is added and when social security contributions are incorporated into the analysis. Finally, the Laffer curve net of administration and compliance costs is also derived. Based on these functions, Section 3 analyzes the changes in the bill of an individual taxpayer, obtaining the so-called mechanical and behavioral effects, from which revenue-maximizing rates and revenue-maximizing elasticities are calculated. With these mathematical developments, in Section 4 the aggregate revenue from a population of taxpayers is modeled whereas Section 5 offers a simulation exercise that illustrates the consequences on the Laffer curve of missing out commodity taxation, social security contributions, and administration and compliance costs. Section 6 concludes.

2. AN EXTENDED ANALYTICAL MODEL FOR THE LAFFER CURVE OF THE PIT

As mention at the outset, the existing microeconomic models on the Laffer curve are incomplete because they ignore many of the revenue effects resulting from the alteration of the marginal PIT rates. For example, they fail to consider the fact that changes in the marginal rate also modify the average rates of the taxpayer population and, therefore, they omit the changes in the collection of taxes levied on consumption. Likewise, it should also be borne in mind that social security contributions are deductible from the taxpayer’s tax base and, consequently, the modification of the marginal tax rates in the PIT also alters the actual collection from social contributions. Finally, following Forte (1987), it should not be overlooked that taxes also impose burdens in terms of administration and compliance costs, and to the extent that these "hidden costs" of taxation correlate with the magnitude of the tax rates, the difference between gross and net Laffer curves will widen. In summary, the existing analytical models in the literature on the Laffer curve of the PIT limit their analysis to the PIT’s own collection, ignoring the revenue implications on other taxes and levies as well as on the administration and compliance costs. To have a more realistic picture of the Laffer effect of taxation all these omissions should have to be taken into account. For this purpose, the following sub-section defines an appropriate analytical framework for inserting all these ignored effects related to the modification of marginal rates.

2.1. An extended set-up

Following Sanz (2016b), we assume an economy that levies taxes on personal income as well as on consumption. In addition, labour income is levied with Social Security contributions to cover contingencies such as retirement or sickness. Moreover, all of these levies are costly in terms of administration and compliance. All these items operate according to the following scheme:
(1) Taxable income, $y$, is taxed by applying a tax schedule, $\zeta = \zeta(\tilde{\tau}, \tilde{A})$, characterized by a vector of increasing marginal tax rates $\tilde{\tau} = (\tau_0, \tau_1, \ldots, \tau_k)$ and a set of sequential income thresholds $\tilde{A} = (a_0, a_1, \ldots, a_k)$ – i.e. the relevant income tax function is $T_Y = T_Y(y, \zeta)$.

(2) $T_C = T_C(V, \zeta)$ is the consumption tax function, defined on disposable income, $V = y - T_Y$, according to a set of tax-inclusive rates, $\zeta = (t_1, t_2, \ldots, t_q)$, levied on the $Q$ categories of goods and services exchanged in the economy. Vector $\zeta$ includes VAT rates and excises. So that if the taxpayer’s total consumption, $C$, is distributed among the $Q$ goods and services according to weights $w_{qi} = \frac{C}{Q}$, where $\sum_{q=1}^{Q} w_{qi} = 1$, the taxpayer’s effective tax rate on consumption is $\alpha_i = \chi_i \cdot \sum_{q=1}^{Q} w_{qi} \cdot t_q$, where $\chi_i$ represents the (marginal) taxpayer’s propensity to consume.

(3) As part of total taxable income, any working taxpayer, $i$, will have gross taxable earnings $y_{wi}$ equal to $y_{wi} = w_i \cdot h_i$, where $w_i$ is the gross hourly wage of the individual and $h_i$ his hours of work. These gross earnings and their uses, apart from being taxed by income and consumption taxes, will also face the social security contributions (SSC) on the part of the employee. As a consequence, this new levy will generate an additional Social Security (net) revenue equals to

$$T_{ss_i}^w = \bar{t}_{wi}^{ss} \cdot (1 - \sigma \cdot \tau_{ki}) \cdot y_{wi}$$

where $\bar{t}_{wi}^{ss}$ represents the average Social Security tax rate of the employee and $\sigma$ the proportion of the employee contribution that is deductible from his taxable income.

(4) Paying taxes is a costly activity in itself, causing compliance and administration costs to emerge. A key difference between these two types of costs is that while the former are privately absorbed by the taxpayer bearing them, the latter are charged to total collected revenue. Although compliance costs are borne privately, notwithstanding, they can affect tax revenue by asserting a negative impact on the supply of taxable income. Therefore, to the extent that $y$ is affected by the magnitude of the compliance costs, these costs will reduce not only the net tax revenue but also the gross tax revenue associated to any tax rate. Although there is little evidence about the actual shape of the compliance and administration cost functions, it seems sensible to regard both of them as an increasing and convex function of the marginal tax rates -i.e. $\frac{dc}{dt} > 0$ and $\frac{dc}{dt^2} > 0$.

From this analytical framework, collected in (1)-(4), we can define the gross Laffer curve of an individual taxpayer, $T_{gi}$, as the sum of his PIT, consumption and Social Security tax functions:

$$T_{gi} = T_{yi} + T_{ci} + T_{ss_i}^w$$

---

4 We assume no tax shifting in SSC – i.e. statutory incidence equals economic incidence.

5 As Forte (1987) indicates, it can be argued that with positive administration costs at a rate close to zero, marginal administration costs may grow less than proportionally at subsequent mild rates whereas at higher rates marginal administration costs may grow more than proportionally.
By subtracting from this gross Laffer curve the administration and compliance costs of the individual taxpayer, \( T_{yi} \), we obtain his net Laffer curve:

\[
T_{ni} = T_{gi} - T_{yi}
\]

### 2.2. The gross Laffer curve of an individual taxpayer in this extended framework

Assuming a global income tax based on the notion of extensive income, the income tax burden associated to income \( y_i \) of an individual taxpayer \( i \) will be as follows:

\[
T(y_i) = \left\{
\begin{array}{ll}
\tau_1 \cdot (y_i - a_1) & \text{if } a_1 < y_i \leq a_2 \\
\tau_1 \cdot (a_2 - a_1) + \tau_2 \cdot (y_i - a_2) & \text{if } a_2 < y_i \leq a_3 \\
\tau_1 \cdot (a_2 - a_1) + \tau_2 \cdot (a_3 - a_2) + \tau_3 \cdot (y_i - a_3) & \text{if } a_3 < y_i \leq a_4 \\
\vdots & \vdots \\
\tau_1 \cdot (a_2 - a_1) + \cdots \tau_{K-1} \cdot (a_{K-2} - a_{K-1}) + \tau_K \cdot (y_i - a_K) & \text{if } y_i > a_K
\end{array}
\right.
\]

that, replicating Creedy and Gemmell (2006), it can be explicitly rewritten in a compact format as:

\[
T_{yi} = \tau_{ki} \cdot [y_i - a'_{ki}]
\]

where \( \tau_{ki} \) is the top marginal rate associated with \( y_i \) and \( a'_{ki} \) denotes the effective threshold defined as

\[
a'_{ki} = \frac{1}{\tau_{ki}} \cdot \sum_{j=1}^{K} a_j \cdot (\tau_j - \tau_{j-1}).
\]

In relation to consumption tax revenue, as consumption emerges from disposable income (after PIT), \( T_{ci} \) can be written as:

\[
T_{ci} = \chi_i \cdot [y_i - T_{yi}] \cdot \sum_{q=1}^{Q} w_q \cdot t_q
\]

that after substituting [4] into [5] and rearranging terms it becomes:

\[
T_{ci} = \alpha_i \cdot [y_i \cdot (1 - \tau_{ki}) + \tau_{ki} \cdot a'_{ki}]
\]

Therefore, under this setting, an individual taxpayer -with incomes \( y_i \) and \( y_{wi} \), such that \( y_{wi} \leq y_i \), and total consumption \( C_i \)- will generate gross total tax revenue, \( T_{gi} \), equals to:

\[
T_{gi} = T_{yi} + T_{ci} + T_{ssi}
\]

that can be written more compactly, by taking into account Equations [1], [4] and [6], as:

\[
T_{gi} = \tau_{ki} \cdot (y_i - a'_{ki}) \cdot (1 - \alpha_i) + \left[ \alpha_i + \frac{\xi_{wi}}{y_i} \cdot (1 - \sigma \cdot \tau_{ki}) \cdot \theta_i \right] \cdot y_i
\]

where \( \theta_i = \frac{y_{wi}}{y_i} \).

This (extended) tax function captures the overall revenue consequences of a change in PIT marginal rates. Namely, it includes the revenue impact of a rate change not only on the personal income tax but also on consumption taxes and social security contributions. Therefore, as far as
consumption arises out of disposable income (after PIT) and SSC qualify as deductible from earned income in the PIT. Equation [8] is the precise tax function to consider in order to assess the (total) gross revenue implications derived from changes in marginal tax rates. However, welfare and revenue implications of income tax rate changes are normally analyzed focusing attention only on income taxation – i.e. using \( T_{yi} \). As a result, most of the existing empirical and theoretical work on the economic impact of income tax rate changes derives from the “abridged” version of reality represented by the partial tax function \( T_{yi} \), instead of the more accurate picture depicted by [8].

Table 1 exhibits, in an easy-comparable format, the three alternative Laffer curve functions depending on the taxes considered in modelling. The first tax function is the one used in standard analysis where only the PIT is considered, the second incorporates consumption tax revenue into consideration (see Sanz (2016b)) and, finally, the third is the extended tax function [8] which, by adding SSC into the analysis, embraces the whole revenue implications of PIT tax rates changes in standard tax systems.

From the equations exhibited in Table 1, it can be inferred that, for an individual taxpayer, the gross Laffer curve can be generalized as:

\[
T_{gi} = \tau_{ki} \cdot (A \cdot y_i - B \cdot a'_{ki})
\]  

where:

\[
a'_{ki} = \sum_{j=1}^{k} a_j \cdot \left( \frac{\tau_j - \tau_{j-1}}{\tau_{ki}} \right)
\]

This is a generalization of the PIT function reported by Creedy and Gemmell (2006) which, depending on the values of the coefficients \( A \) and \( B \), opens the possibility of taking into account, together with the PIT, other taxes and levies that are also affected by the magnitude of PIT rates. To be specific, if only income taxation is considered, then \( A = B = 1 \), and [9] reproduces Creedy and Gemmell’s formula. When income and consumption taxes are taken together into account, then \( A = \left( 1 + \frac{(1-\tau_{ki}) \cdot a_i}{\tau_{ki}} \right) \) and \( B = (1 - a_i) \). Finally, by making allowance for Social Security Contributions together with income and consumption taxes, it turns out that \( A = \left( 1 + \frac{(1-\tau_{ki}) \cdot a_i + \tilde{t}_{ws} \cdot \theta_i \cdot \frac{1 - a_i - \tau_{ki}}{\tau_{ki}}}{\tau_{ki}} \right) \) whereas \( B \) remains as \( B = (1 - a_i) \).
2.3. The net Laffer curve: the relevant tax function in the presence of administration and compliance costs

Equation [8] depicts the extended (individual) gross Laffer curve. This Laffer curve, however, overlooks the existence of administration and compliance costs. Therefore, the net Laffer curve can be inferred by discounting the individual revenue consequences of the administration and compliance costs, $T_{\psi i}$:

$$T_{ni} = T_{yi} + T_{ci} + T_{wni} - T_{\psi i}$$

Quantifying and ensuring payment of a tax generates administration and compliance costs to emerge. However, most of the studies on the Laffer curve, either theoretical or empirical, have ignored this fact. To the best of my knowledge, the only exception to this omission is Forte (1987), who distinguishes between gross and net Laffer curves by including the operating costs of taxation into the analysis. In the context of the Laffer curve, a key conceptual difference between administration and compliance costs is that whereas the former is privately absorbed by the taxpayers the latter uses the revenue raised from taxpayers and consequently reduces the disposable tax revenue for public use. As a result, as Forte posited, this loss of tax revenue due to administration and compliance costs should be made explicit into the analysis of the Laffer curve. As highlighted by Shaw et al. (2010), the structure of tax rates and the ease with which the tax base can be disguised are particularly relevant for administration costs. In this respect, as far as the personal income tax is concerned, high administration costs can be inferred. Likewise, to the extent that high tax rates may make taxpayers more prone to tax evasion and tax avoidance, administration costs can be expected to be highly correlated with the size of marginal tax rates. Although there is little evidence about the actual shape of the administration cost function, for the reasons mentioned above, it seems sensible to regard the administration costs of taxation as an increasing and convex function of the marginal tax rates -i.e. $\frac{dAC_a}{dT} > 0$ and $\frac{d^2 AC_a}{dT^2} > 0$. In addition to the administration costs, the compliance costs associated with the payment of taxes should not be forgotten when defining the Laffer curve. As mentioned above, this other concealed burden of taxation levies on taxpayers privately. However, as noted by Forte (1987), it can affect tax collection as it asserts a negative impact on the supply of taxable income. Therefore, to the extent that $y_i$ is affected by the magnitude of the compliance costs, these costs will reduce not only the net tax revenue but also the gross tax revenue associated to any tax rate.

Although the generic requirement of being increasing and convex with respect to marginal tax rates is compatible with alternative functional forms, we will assume an exponential cost function for both. For administration costs (AC):

$$AC_i (\bar{\xi}_i) = A_0^{\bar{\xi}_i} \cdot e^{a \bar{\xi}_i}$$

---

6 As Forte (1987) indicates, it can be argued that with positive administration costs at a rate close to zero, marginal administration costs may grow less than proportionally at subsequent mild rates whereas at higher rates marginal administration costs may grow more than proportionally.
where $A^i_0$ identifies the initial *per capita* administration cost necessary to start the operation of the tax system, while $\alpha$ is the factor at which the individual administration cost vary in the face of changes in the (average) marginal rate of the taxpayer, $\bar{\tau}_i$, when one or more of the marginal rates of the tax schedule are changed. Specifically, the average marginal rate of a given individual taxpayer will be determined by:

$$\bar{\tau}_i = \sum_{j=0}^{k-1} \frac{a^i_{j+1} - a^i_j}{y_i} \cdot \tau_j + \frac{y_i - a^i_k}{y_i} \cdot \tau_k$$ \hspace{1cm} \text{[12]}$$

As in the case of administration costs, the compliance costs borne individually by each taxpayer, $C^i_i$, will describe an exponential trajectory of magnitude equal to:

$$C^i_i(\bar{\tau}_i) = C^i_0 \cdot e^{b \cdot \bar{\tau}_i}$$ \hspace{1cm} \text{[13]}$$

where $C^i_0$ represents the initial compliance cost of taxpayer $i$ and $b$ captures the expansion factor in the face of changes in his or her (average) marginal tax rate, $\bar{\tau}_i$. This private compliance cost, $C^i_i(\bar{\tau}_i)$, generates additionally a collective compliance cost in the form of a revenue loss associated with the reduction in the reported taxable income due to the compliance costs privately borne. In this way, the complete compliance cost of an individual taxpayer will be given by the sum of two components:

$$C(C^i_i(\bar{\tau}_i)) = C^i_i(\bar{\tau}_i) + T_i(y_i(C^i_i(\bar{\tau}_i)))$$ \hspace{1cm} \text{[14]}$$

the first right-hand term represents the private compliance cost borne by the taxpayer to meet his or her tax obligations while the second quantifies the reduction in the individual tax bill associated with the reduction in the reported taxable income due to those compliance costs. Whereas the former component has no impact on the Laffer curve, as it is privately paid by the taxpayer, the latter component has an impact on the profile of the Laffer curve as it erodes the revenue power of the tax. The mechanism to reduce tax collection is produced through the effect of compliance costs on the magnitude of the reported taxable income. Therefore, the revenue effect of the administration and compliance costs will be:

$$T(y_i(\bar{\tau}_i)) = A \cdot y_i - B \cdot \alpha^i_1 - A_0 \cdot e^{a \cdot \bar{\tau}_i} - T_i(y_i(C^i_i(\bar{\tau}_i)))$$ \hspace{1cm} \text{[15]}$$

Consequently, taking into account [10] and [11], the net Laffer curve will be given by:

$$T(y_i(\bar{\tau}_i)) = A \cdot y_i - B \cdot \alpha^i_1 - A_0 \cdot e^{a \cdot \bar{\tau}_i} - T_i(y_i(C^i_i(\bar{\tau}_i)))$$ \hspace{1cm} \text{[16]}$$

3. THE INDIVIDUAL (GROSS) TAX BILL AND THE CHANGE IN THE MARGINAL TAX RATES

3.1. Rate changes and the gross tax bill of an individual taxpayer

Given the general tax function [9], an alteration in any tax rate $\tau_h: \tau_h \in \zeta = \zeta(\bar{\tau}, \bar{A})$, will induce a change in the tax bill of the individual taxpayer, $T_g^h$, equals to:

$$\frac{dT^h}{d\tau_h} = \frac{\tau^h}{\tau_h} \cdot A \cdot y_i \cdot (\eta_{\tau_h} + \eta_{\alpha^h} + \eta_{\alpha^i_{\tau_h}}) + \frac{\tau^h}{\tau_h} \cdot B \cdot \alpha^i_{\tau_h} \cdot (\eta_{\tau_h} + \eta_{\alpha^i_{\tau_h}} + \eta_{\alpha^i_{\tau_h}})$$ \hspace{1cm} \text{[17]}$$
that depending on the relative position of $\tau_h$ with respect to the taxpayer’s marginal rate, $\tau_{k_i}$, will turn out to be:

\[
\frac{dT_{gl}}{d\tau_h} = \begin{cases} 
\frac{\tau_{k_i}}{\tau_h} \cdot A \cdot y_{l_i} \cdot (\eta_{A,\tau_{k_i}} + \eta_{y_{l_i},\tau_{k_i}}) - \frac{\tau_{k_i}}{\tau_h} \cdot B \cdot a_{k_i}' \cdot (\eta_{B,\tau_h} + \eta_{a_{k_i}',\tau_{k_i}}) & \text{if } \tau_h < \tau_{k_i} \\
A \cdot y_{l_i} \cdot \left(1 + \eta_{A,\tau_{k_i}} + \eta_{y_{l_i},\tau_{k_i}}\right) - B \cdot a_{k_i}' \cdot \left(1 + \eta_{B,\tau_{k_i}} + \eta_{a_{k_i}',\tau_{k_i}}\right) & \text{if } \tau_h = \tau_{k_i} \\
0 & \text{if } \tau_h > \tau_{k_i}
\end{cases} [18]
\]

The equations in [18] capture the general form of the marginal effect on the gross revenue caused by a change in the rates of a multi-rate tax schedule. As can be seen, this marginal effect depends on $A$ and $B$ – i.e. on the type and number of taxes incorporated in the modelling of the Laffer curve. Depending on $A$ and $B$ this marginal effect can be particularized as follows:

- $A = B = 1$ (income tax only)

\[
\frac{dT_{gl}}{d\tau_h} = \begin{cases} 
(a_{h+1} - a_h) & \text{if } \tau_h < \tau_{k_i} \\
y_{l_i} \cdot \left(1 + \eta_{y_{l_i},\tau_{k_i}}\right) - a_k & \text{if } \tau_h = \tau_{k_i} \\
0 & \text{if } \tau_h > \tau_{k_i}
\end{cases} [19]
\]

- $A = \left(1 + \frac{1-\tau_{k_i}}{\tau_{k_i}} \cdot a_i\right); B = (1 - a_i)$ (income tax + taxes on consumption)

\[
\frac{dT_{gl}}{d\tau_h} = \begin{cases} 
\frac{\tau_{k_i}}{\tau_h} \cdot \eta_{a,\tau_{k_i}} \cdot a_i \cdot \left(y_{l_i} \cdot \frac{1-\tau_{k_i}}{\tau_{k_i}} + a_{k_i}'\right) + (1 - a_i) \cdot (a_{h+1} - a_h) & \text{if } \tau_h < \tau_{k_i} \\
y_{l_i} \cdot \left(1 + \eta_{y_{l_i},\tau_{k_i}}\right) - a_k + \frac{da_{k_i}}{d\tau_{k_i}} \cdot \left[y_{l_i} - \tau_{k_i} \cdot \left(y_{l_i} - a_{k_i}'\right)\right] & \text{if } \tau_h = \tau_{k_i} \\
-\alpha_i \cdot \left[y_{l_i} \cdot \left(1 - \eta_{y_{l_i},\tau_{k_i}} \cdot \frac{1-\tau_{k_i}}{\tau_{k_i}}\right) - a_k\right] & \text{if } \tau_h > \tau_{k_i}
\end{cases} [20]
\]
As suggested by the existing literature, in the face of a change in marginal rates the tax bill of an individual taxpayer will be altered through two alternative channels: the mechanical effect (ME) and the behavioural effect (BE). The former captures the revenue change with no behavioural reaction of the taxpayer -i.e. the pure arithmetical revenue change. The latter, conversely, provides a measure of the variation in the tax payment resulting from the taxpayer's behavioural response. Both effects move in opposite directions and together dictate the actual change in the tax bill of the taxpayer. As highlighted in Sanz (2016a), the computation of ME and BE at an individual level allows to characterize the Laffer curve individually and to place each taxpayer on his/her individual Laffer curve. To be specific, if the taxpayer is located in the rising section of his Laffer curve whereas if the contrary applies, the taxpayer would be placed in the decreasing or “prohibited” section of his Laffer curve. Needless to say, that the knowledge of the precise location of the taxpayers within their own personal Laffer curves is vital to infer the actual revenue impact of a given rate change. In addition, as the first-order condition of the revenue maximization problem, \( \tau^L \), may be also computed by solving for \( \tau \) in the equalization of ME and BE. The analytical relevance of ME and BE is further reinforced if we bear in mind that they are first-order approximations to a number of money metrics. As highlighted by Gierzts (2009), the Marginal Excess Burden is approximated by BE whereas ME roughly quantifies the Hicksian Equivalent Variation.

As can be inferred from the equations reported in [19]-[21], the magnitude of ME and BE will differ depending on A and B; affecting the whole profile of the Laffer curve, including the revenue-
maximizing tax rates as well as the welfare implications of the rate change under consideration. This is why it is so important to model the Laffer curve correctly and fully. If we leave out of the modelling the impact of a rate change on any of the affected tax structures – income tax, consumption taxes or the Social Security contributions –, we will be ignoring relevant effects of the rate change under scrutiny and, therefore, we will probably be prescribing incorrect policy actions. In order to show this issue, in what follows mechanical and behavioural effects are computed depending on the tax structures incorporated in the modelling of the Laffer curve.

Starting from [19]-[21], rearranging terms and taking into account that $\eta_{y_i(1-\tau_{ki})}$ it is possible to isolate the mechanical and behavioural effects associated with any given rate change. Focusing the analysis when $\tau_h = \tau_{ki}$, if we only account for the effects on the revenue of the personal income tax, a modification in $\tau_{ki}$ will result in a revenue change equals to:

$$\frac{dT_i}{d\tau_{ki}} = (y_i - a_k) - y_i \cdot \tau_{ki} \cdot \eta_{y_i(1-\tau_{ki})}$$

where $\eta_{y_i(1-\tau_{ki})}$ is the so-called taxable income elasticity popularized by Feldstein (1995, 1999) and $a_k$ represents the nominal threshold of bracket $k$ in the income tax schedule. As shown, the first bracketed term in the right-hand side of [22] captures ME whereas the second term is BE. Notwithstanding this, when income and consumption taxes are taken together into account, the revenue change becomes:

$$\frac{dT_i}{d\tau_{ki}} = (y_i - a_k) \cdot (1 - \alpha_i) - y_i \cdot \frac{\tau_{ki}}{1 - \tau_{ki}} \cdot \eta_{y_i(1-\tau_{ki})} - \alpha_i \cdot \left[ \eta_{y_i(1-\tau_{ki})} \cdot y_i + \eta a_i(1-\tau_{ki}) \cdot \left( y_i + \frac{\tau_{ki} \cdot a'_k}{1 - \tau_{ki}} \right) \right]$$

where $\alpha_i$ is, as mentioned at the outset, the average tax rate on taxpayer’s consumption and $\eta a_i(1-\tau_{ki})$ denotes the elasticity of $\alpha_i$ with respect to the net-of-income-tax rate. As can be seen, when consumption taxes appear on the scene both, ME and BE, are different from the ones reported in [22]. Additionally, if we account for SSC, the revenue change as a whole is:
A Microanalytic Full Model for the Laffer Curve in Personal Income Taxation

\[
\frac{dT_i}{d\tau_{ki}} = (y_i - a_k) \cdot (1 - \alpha_i) - \bar{\tau}_{wi} \cdot \theta_i \cdot y_i
\]

\[
-\bar{\tau}_{wi} \cdot \theta_i \cdot y_i \cdot \frac{\tau_{ki}}{1 - \tau_{ki}} \cdot \eta_{y_i,(1-\tau_{ki})} - \alpha_i \left[ \eta_{y_i,(1-\tau_{ki})} \cdot y_i + \eta_{\alpha_i,(1-\tau_{ki})} \cdot \left( y_i + \frac{\tau_{ki} \cdot a_{ki}}{1 - \tau_{ki}} \right) \right]
\]

\[
-\bar{\epsilon}_{wi} \cdot \theta_i \cdot y_i \cdot \left( \eta_{\epsilon_{wi}^c,(1-\tau_{ki})} + \eta_{y_{wi}^c,(1-\tau_{ki})} \right)
\]

where behavioural reactions affecting collection of SSC are encapsulated in \( \eta_{\epsilon_{wi}^c,(1-\tau_{ki})} \) and \( \eta_{y_{wi}^c,(1-\tau_{ki})} \). The former is the elasticity of the average SSC tax rate of the employee to the net-of-income-tax rate. The latter is the elasticity of the employee’s taxable labour income.

Table 2 reports the biases caused in the mechanical and behavioural effects of a change in \( \tau_h \) when \( \tau_h = \tau_{ki} \), due to not considering consumption taxes and SSC in the modelling. Obviously, these biases caused by neglecting the consequences of income tax rates on revenue from consumption and SSC are the source of miscalculation of the actual Laffer curve for personal income taxation. This miscalculation does have significant implications on the revenue and welfare effects of the rate change.

**Table 2**

**OVERESTIMATION IN ME AND BE CAUSED BY OMITTING THE IMPACT OF PIT MARGINAL TAX RATES ON CONSUMPTION TAX REVENUE AND SSC – FOR CHANGES IN \( \tau_h \) WHEN \( \tau_h = \tau_{ki} \).**

<table>
<thead>
<tr>
<th>Omission</th>
<th>Excess in ME</th>
<th>Excess in BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption tax Revenue</td>
<td>( \alpha_i \cdot (y_i - a_k) )</td>
<td>( \alpha_i \cdot \left[ \eta_{y_i,(1-\tau_{ki})} \cdot y_i + \eta_{\alpha_i,(1-\tau_{ki})} \cdot \left( y_i + \frac{\tau_{ki} \cdot a_{ki}}{1 - \tau_{ki}} \right) \right] )</td>
</tr>
<tr>
<td>SSC</td>
<td>( \alpha_i \cdot (y_i - a_k) + \bar{\epsilon}_{wi} \cdot \theta_i \cdot y_i )</td>
<td>( \alpha_i \cdot \left[ \eta_{y_i,(1-\tau_{ki})} \cdot y_i + \eta_{\alpha_i,(1-\tau_{ki})} \cdot \left( y_i + \frac{\tau_{ki} \cdot a_{ki}}{1 - \tau_{ki}} \right) \right] + \bar{\epsilon}<em>{wi} \cdot \theta_i \cdot y_i \cdot \left( \eta</em>{\epsilon_{wi}^c,(1-\tau_{ki})} + \eta_{y_{wi}^c,(1-\tau_{ki})} \right) )</td>
</tr>
</tbody>
</table>
Together with this, if the rate change occurs in a bracket lower than the one in which the taxpayer’s taxable income falls—i.e. \( \tau_h < \tau_{k_i} \)—taxpayer’s average rate will be altered even though his marginal rate remains intact. As reported in equations [19]-[21], this change in the average tax rate will cause a mechanical effect in the PIT as well as a behavioural effect on commodity taxation. Table 3 shows the bias in the calculation of these mechanical and behavioural effects due to the omission of commodity taxation and social security contributions in the modelling of the Laffer curve of the PIT when the rate change is in inner tax brackets.

<table>
<thead>
<tr>
<th>Omission</th>
<th>Excess in ME</th>
<th>Excess in BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption tax Revenue</td>
<td>( a_i \cdot (a_{h+1} - a_h) )</td>
<td>( \frac{\tau_{k_i}}{\tau_h} \cdot \eta a_i \tau_h \cdot \alpha_i \cdot \left( y_i \cdot \frac{1 - \tau_{k_i} + a'<em>{k_i}}{\tau</em>{k_i}} \right) )</td>
</tr>
<tr>
<td>Consumption tax Revenue +</td>
<td>( a_i \cdot (a_{h+1} - a_h) )</td>
<td>( \frac{\tau_{k_i}}{\tau_h} \cdot \eta a_i \tau_h \cdot \alpha_i \cdot \left( y_i \cdot \frac{1 - \tau_{k_i} + a'<em>{k_i}}{\tau</em>{k_i}} \right) )</td>
</tr>
<tr>
<td>+</td>
<td>Consumption tax Revenue + SSC</td>
<td></td>
</tr>
</tbody>
</table>

3.1.2. The revenue-maximizing rates

As we have just seen, the failure to take into account the effect of the marginal rates of personal income tax on the revenue of consumption taxes or social security contributions has consequences on the magnitude of the mechanical and behavioural effects produced. One consequence of these biases is the one that occurs on the magnitude of the revenue-maximizing rates. If the Laffer curve is modelled taking into account only the revenue impact on the personal income tax itself, the revenue-maximizing marginal rate, \( \tau^L \), or Laffer rate as it is known in the literature, will be given by the following expression:

\[
\tau^L = \frac{(y_i - a_k)}{y_i \eta y_{i(1-\tau_{k_i})} + (y_i - a_k)}
\]  

[25]

whereas if consumption taxes are incorporated, \( \tau^L \) reaches:

\[
\tau^L_{t+c} = \frac{(y_i - a_k)(1 - a_i) - a_i y_i \left( \eta y_{i(1-\tau_{k_i})} + \eta a_i (1 - \tau_{k_i}) \right)}{(y_i - a_k)(1 - a_i) - a_i y_i \left( \eta y_{i(1-\tau_{k_i})} + \eta a_i (1 - \tau_{k_i}) + \frac{\text{me}}{\tau_{k_i}} \eta a_i (1 - \tau_{k_i}) \right)}
\]  

[26]

and if the Social Security contributions are also added to the model, the revenue-maximizing rate will be determined by:

\[
\tau^L_{t+c+SSC} = \frac{(y_i - a_k)(1 - a_i) - a_i y_i \left( \eta y_{i(1-\tau_{k_i})} + \eta a_i (1 - \tau_{k_i}) \right) - \varphi}{(y_i - a_k)(1 - a_i) - a_i y_i \left( \eta y_{i(1-\tau_{k_i})} + \eta a_i (1 - \tau_{k_i}) + \frac{\text{me}}{\tau_{k_i}} \eta a_i (1 - \tau_{k_i}) \right) - \varphi}
\]

[27]

donde \( \varphi = \frac{\eta y_{w+SSC}}{\tau_{k_i}} \cdot \theta_i \cdot y_{i} \cdot \left( 1 + \eta y_{w+SSC(1-\tau_{k_i})} + \eta y_{w+SSC(1-\tau_{k_i})} \right) \)
3.1.3. The revenue-maximizing elasticities

As pointed out by Fullerton (1982), the two fundamental parameters in the debate on the Laffer curve are the marginal rates and the taxable income elasticity. The emphasis on the disincentive effects associated with marginal rates is explained only in terms of the magnitude of the taxable income elasticity. If this elasticity is high enough, even low marginal rates could cause the taxpayer to be in the prohibitive zone of his Laffer curve. Conversely, if the elasticity is low enough, even with high marginal rates, the taxpayer could be in the normal zone of his Laffer curve. That is to say, the location of the taxpayer in the revenue-rate space -i.e. Laffer curve- depends decisively on the magnitude of the taxable income elasticity. If this elasticity is low enough, the maximum revenue will be reached with a high marginal tax rate. And vice-versa, if the elasticity is high enough, the revenue will be maximized with a reduced marginal tax rate. Following this reasoning, Fullerton (1982) suggests a new curve where the emphasis is placed on the taxable income elasticity. Specifically, Fullerton proposes drawing a "modified Laffer curve" that, instead of relating marginal rates and revenue, delimits the combination of rates and elasticities that ensure revenue maximization. This new curve, which we call Fullerton curve after its impeller, identifies the marginal rate that maximizes revenue for a given elasticity. Thus, as outlined in Figure 1, the combinations of rates and elasticities to the southeast of the curve signifies the "normal zone" while the points to the northwest identify the combinations of rates and elasticities falling into the prohibitive area of the Laffer curve.

![Diagram of the Fullerton curve](image)

The Fullerton curve therefore identifies the boundary value of the elasticity separating the normal zone from the prohibitive zone on the Laffer curve. Fullerton curves are also affected by the number and type of taxes incorporated in the modelling. In particular, if only the effect on PIT revenue is considered, the functional form of the Fullerton curve will be:
if, on the other hand, we incorporate consumption taxes, the Fullerton curve becomes the following function:

\[
\eta_{i+C}^F = \frac{(y_{i-a_k}) \cdot (1-a_l) \cdot \alpha_{i} \cdot \eta_{a_{i}} \cdot (1-a_{k}) \cdot \left[ y_{i} \cdot \frac{r_{ki}}{1-t_{ki}} \cdot a_{k} \right]}{y_{i} \cdot \frac{r_{ki}}{1-t_{ki}} + \alpha_{i}}
\]

which will take the form of function [30] if we also consider the impact on social security contributions.

\[
\eta_{i+C+SSC}^F = \frac{(y_{i-a_k}) \cdot (1-a_l) \cdot \alpha_{i} \cdot \eta_{a_{i}} \cdot (1-a_{k}) \cdot \left[ y_{i} \cdot \frac{r_{ki}}{1-t_{ki}} \cdot a_{k} \right] - \varphi}{y_{i} \cdot \frac{r_{ki}}{1-t_{ki}} + \alpha_{i}}
\]

### 3.2. The effect of the administration and compliance costs: The net Laffer curve

Does the existence of compliance and administration costs affect the collection capacity of marginal tax rates in personal income taxation? As already mentioned in subsection 2.1, the answer is affirmative and it does so in two ways. First, to the extent that marginal rates modify the per capita administration costs of the tax system, \( AC (\bar{t}_i) \). Second, to the extent that the marginal rate affects the magnitude of the taxable income declared by the taxpayer through the effect of the marginal rate on the taxpayer’s compliance costs, \( T_i(y_i(C_i(\bar{t}_i))) \). Namely, administration and compliance costs affect the profile of the Laffer curve to the extent that marginal rate changes affect function [14], replicated here below:

\[ T_{\psi_i} = AC (\bar{t}_i) + T_i(y_i(C_i(\bar{t}_i))) \]

This implies that the expressions presented in the preceding section should incorporate the effects of the change in \( \tau_h \) on \( T_{\psi_i} \), summarized as follows – see Appendix –:

\[
\frac{dT_{\psi_i}}{d\tau_h} = \begin{cases} 
A_0' \cdot a \cdot e^{b \cdot \tau_i} \cdot \frac{y_{i-a_k}}{y_i} + \tau_k \cdot \eta_{y_i \cdot C_i} \cdot b \cdot (a_{h+1} - a_h) & \text{if} \quad \tau_h < \tau_{ki} \\
A_0' \cdot a \cdot e^{b \cdot \tau_i} \cdot \frac{y_{i-a_k}}{y_i} + \tau_k \cdot \eta_{y_i \cdot C_i} \cdot b \cdot (y_i - a_k) & \text{if} \quad \tau_h = \tau_{ki}
\end{cases}
\]

where \( \eta_{y_i \cdot C_i} \) quantifies, in the form of elasticity, the sensitivity of the taxpayer’s reported taxable income to his compliance costs. Although we have insisted on this in other parts of this research, it is important to note that when modelling the Laffer curve compliance costs are only considered to the extent that they affect the magnitude of the reported taxable income, not the compliance cost itself. Note that while the first summands of the expressions in [31] quantify the mechanical effect associated with the change in the per capita administrative costs, the second summands calculate something like a behavioral effect of those compliance costs that translates into a lower tax bill due to the reduction in the reported taxable income. The illustrative examples presented below will simulate both, gross and net Laffer curves.
4. CHANGES IN AGGREGATE REVENUE

From the perspective of tax policy, the revenue impact of changing the marginal rates of personal income tax is more relevant if analyzed for a whole population. Consequently, in order to determine the aggregate revenue impact in a population, we will make a mathematical aggregation based on the preceding individual microeconomic model. As we shall see, many of the results obtained for the individual taxpayer are naturally carried over to the case of many taxpayers. In this context of many heterogeneous taxpayers, the aggregate of individual results will require to have a relatively detailed knowledge of the distribution in the population not only of the tax units but also of the taxable incomes. Specifically, a set of analytical expressions is provided below to determine the aggregate revenue change to be expected in a population of N taxpayers. In addition, the characterization of the aggregate Laffer curve is also offered, identifying the maximizing marginal rate of revenue taking into account the total taxpaying population.

4.1. Aggregate revenue change and marginal tax rates

We start from the extended individual tax function where all tax structures involved – income taxes, taxes on consumption and social contributions – and administration and compliance costs are included:

$$T_{ni} = \tau_{ki} \cdot (y_i - a_{ki}) \cdot (1 - \alpha_i) + \left[ a_i + 1 - \sigma \cdot \tau_{ki} \cdot \theta_i \right] \cdot y_i - \left[ A_i \cdot e^{ax_i} + T_i(y_i(C_i(\bar{r}_i))) \right]$$

[32]

From this function and taking into account the analytical expressions [22], [23], [24] y [31], an individual taxpayer would be subject to the mechanical and behavioural effects summarized in Table 4. As can be seen, before a change in \( \tau_h \), the magnitude of these mechanical and behavioural effects depends on the relative position of \( \tau_{ki} \) with respect to the modified marginal tax rate, \( \tau_h \).

4.2. Aggregation over the whole population

If our interest is to quantify the aggregate revenue impact of a change in \( \tau_h \) on a finite population of \( N \) filers, the expressions in Table 4 indicate that it is necessary to discriminate the total population of taxpayers according to their location in the multi-step income tax function with respect to the rate change. Therefore, in the analytical developments that follow, \( N_h \) will identify the number of taxpayers for whom \( \tau_{ki} = \tau_h \) whereas \( N^+_h \) will collect the number of taxpayers to whom it happens that \( \tau_{ki} > \tau_h \). In addition, \( N^0_h \) denotes the number of taxpayers with \( \tau_{ki} < \tau_h \), so they are not affected in any way by the rate change. Therefore, \( N = N^0_h + N_h + N^+_h \).

Due to the large number of effects to consider, in the calculation of the aggregate revenue change, we will compute the aggregate mechanical and behavioral effects separately.
4.2.1. The aggregate mechanical effects

In what follows, all the mechanical effects contained in Table 4 are aggregated for the total population of taxpayers, $N$. This aggregate mechanical effect across the whole population, $ME_N$, is computed as follows:

$$ME_N = \sum_{N_h} (1 - \alpha_t) \cdot (a_{h+1} - a_h) - \sum_{N_h} A_1^i \cdot a \cdot e^{b \cdot \tau_i} \cdot \frac{(a_{h+1} - a_h)}{y_i} + \sum_{N_h} (y_i - a_k) \cdot (1 - \alpha_t)$$

$$- \sum_{N_h} \tau_{wi} \cdot \theta_i \cdot y_i - \sum_{N_h} A_2 \cdot a \cdot e^{b \cdot \tau_i} \cdot \frac{(y_i - a_k)}{y_i}$$

that gives:

$$ME_N = N_h^+ \cdot (a_{h+1} - a_h) \cdot \left[ (1 - \bar{\alpha}_h^+) - A_1^i \cdot a \cdot \left( \frac{e^{b \cdot \tau_i}}{\bar{y}_h} \right)_h \right]$$

$$+ N_h \cdot \left[ (\bar{y}_h - a_h) \cdot (1 - \bar{\alpha}_h) - (\tau_{wi})_h \cdot \delta_h \cdot \bar{y}_h - A_2 \cdot a \cdot \left( \frac{e^{b \cdot \tau_i}}{\bar{y}_h} \right)_h \cdot \frac{(\bar{y}_h - a_h)}{\bar{y}_h} \right]$$

where the bar indicates the mean value of the corresponding parameter. A barred parameter combined simultaneously with a subscript $h$ and a superscript + indicates that the mean should be calculated over the taxpayers for whom $y_i < a_{h+1}$, while if the barred parameter has only a subscript $h$, the mean should be computed for taxpayers whose income falls into bracket $h$ – i.e. for whom $\tau_{ki} = \tau_h$. For example, $\bar{\alpha}_h^+$ indicates the mean effective tax rate on consumption for taxpayers falling into tax brackets above $h$ whereas $\alpha_h$ denotes the average of the same tax rate but computed over taxpayers falling in bracket $h$.

It is worth noting that the first addend in [34] collects the outside mechanical effects whereas the second addend captures the within mechanical effects.
Table 4

IDENTIFICATION OF THE MECHANICAL AND BEHAVIOURAL EFFECTS ON AN INDIVIDUAL TAXPAYER OF A RATE CHANGE IN PIT, DEPENDING ON THE RELATIVE POSITION OF $\tau_h$ AND $\tau_{k_i}$ AND ON THE TAX REVENUE IMPACTS TAKEN INTO ACCOUNT*

<table>
<thead>
<tr>
<th>Taxes and levies considered</th>
<th>MECHANICAL EFFECTS</th>
<th>BEHAVIOURAL EFFECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_h &lt; \tau_{k_i}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(a_{h+1} - a_h)$</td>
<td></td>
</tr>
<tr>
<td>PIT only</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 - $\alpha_i$) $\cdot$ $(a_{h+1} - a_h)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PIT + CT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 - $\alpha_i$) $\cdot$ $(a_{h+1} - a_h)$</td>
<td>$\frac{\tau_{k_i}}{\tau_{h}} \cdot \frac{1}{\eta_{a_i}} \cdot \alpha_i$ $\cdot$ $\left( \frac{y_i \cdot (1 - \tau_{k_i}) + a_{k_i}}{\tau_{k_i} + a_{k_i}} \right)$</td>
<td></td>
</tr>
<tr>
<td>PIT + CT + SS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 - $\alpha_i$) $\cdot$ $(a_{h+1} - a_h)$</td>
<td>$\frac{\tau_{k_i}}{\tau_{h}} \cdot \frac{1}{\eta_{a_i}} \cdot \alpha_i$ $\cdot$ $\left( \frac{y_i \cdot (1 - \tau_{k_i}) + a_{k_i}}{\tau_{k_i} + a_{k_i}} \right)$</td>
<td></td>
</tr>
<tr>
<td>PIT + CT + SS + ACC</td>
<td></td>
<td>(1 - $\alpha_i$) $\cdot$ $(a_{h+1} - a_h)$</td>
</tr>
</tbody>
</table>

*Meaning of the acronyms: PIT = Personal Income Tax; TC = Taxes on consumption; SS = Social Security Contributions; ACC = Administration and Compliance Costs.
4.2.2. The aggregate behavioural effects

Here below the aggregate behavioural effect across the whole population, $BE_N$, is computed:

\[
BE_N = \sum_{n=1}^{N} \left( \frac{\tau_{ki}}{\tau_h} \cdot \eta_{a_i,\tau_k} \cdot \alpha_i \cdot \left( y_i \cdot \frac{1 - \tau_{ki}}{\tau_k} + a_i^r \right) - \sum_{n=1}^{N} \tau_k \cdot \eta_{y_i,\tau_i} \cdot b \cdot \left( a_{h+1} - a_h \right) 
- \sum_{n=1}^{N} y_i \cdot \frac{\tau_{ki}}{1 - \tau_{ki}} \cdot \eta_{y_i,(1-\tau_k)} \cdot \sum_{n=1}^{N} \tau_{ki} \cdot \eta_{a_i,\tau_k} \cdot \left( y_i + \frac{\tau_{ki}}{1 - \tau_k} a_i^r \right) \right) 
- \sum_{n=1}^{N} \tilde{y}_{wi} \cdot \theta_i \cdot \eta_{y_i,\tau_i} \cdot \left( \tilde{y}_{wi}(1-\tau_k) + \eta_{y_{wi},(1-\tau_k)} \right) - \sum_{n=1}^{N} \tau_{ki} \cdot \eta_{y_i,\tau_i} \cdot b \cdot (y_i - a_k) \right] \]  

which gives:

\[
BE_N = N_h \cdot \left( \frac{(\bar{y}_h)_{h}}{\tau_h} \cdot \left( \eta_{a_i,\tau_k} \right)_{h} \cdot \tilde{y}_h \cdot \frac{1 - (\bar{y}_h)_{h}}{(\bar{y}_h)_{h}} + (\bar{a}_h)_{h} \cdot (\tilde{y}_h)_{h} \cdot (\bar{y}_h)_{h} \cdot (\tilde{y}_h)_{h} \cdot \left( \eta_{y_i,\tau_k} \right)_{h} \right) 
- N_h \cdot \left( \bar{y}_h \cdot (\tilde{y}_h)_{h} \cdot \left( \eta_{y_i,\tau_k} \right)_{h} \right) 
- N_h \cdot \tilde{y}_h \cdot \left( \eta_{a_i,\tau_k} \right)_{h} \cdot \left( \tilde{y}_h \cdot (\tilde{y}_h)_{h} \cdot \left( \eta_{y_i,\tau_k} \right)_{h} \right) 
- \sum_{n=1}^{N} \left[ \left( \tilde{y}_{wi} \right)_{h} \cdot \theta_i \cdot \eta_{y_i,\tau_i} \cdot \left( \tilde{y}_{wi}(1-\tau_k) + \eta_{y_{wi},(1-\tau_k)} \right) \right] 
- N_h \cdot \left( \tilde{y}_h \cdot \left( \eta_{y_i,\tau_i} \right)_{h} \cdot b \cdot (\tilde{y}_h - a_h) \right) \right)
\]

According to the full-fledged model, given a change in $\tau_h$, the expected shift in tax collection for a population of $N$ taxpayers will be given by:

\[
dT = (ME_N + BE_N) \cdot d\tau_h \]  

This revenue change, however, will not be computed correctly if the modelling of the tax function overlooks the effect of personal income tax rates on consumption taxes, social security contributions or on administration and compliance costs. Table 5 summarizes the explicit forms of function [36] depending on the taxes and costs considered in the analysis. The corresponding aggregate revenue-maximizing rates and revenue-maximizing elasticities are summarized in Table 6. As can be seen, in assessing the revenue impact of PIT rates, the modelling of consumption taxes, social security contributions and administration and compliance costs are a must. Otherwise, catastrophic consequences in revenue projections will occur and the actual aggregate profile of the Laffer curve will not be captured.
Table 5
CHANGE IN AGGREGATE TAX REVENUE FOR A POPULATION OF SIZE N DEPENDING ON THE TAXES CONSIDERED

<table>
<thead>
<tr>
<th>PIT only</th>
<th>[dT = \left{ \left( (a_{h+1} - a_h) \cdot N_h^+ + (\bar{y}<em>h - a_h) \cdot N_h \right) - \frac{\tau_h}{1 - \tau_h} \cdot \tilde{\eta}</em>{y,1(1-\tau_h)} \cdot N_h \cdot \bar{y}_h \right} \cdot d\tau_h ]</th>
</tr>
</thead>
</table>
| PIT + CT | \[dT = \left\{ \left( (a_{h+1} - a_h) \cdot (1 - \bar{a}_h) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot (1 - \bar{a}_h) \cdot N_h \right) 
- \left[ \left( \frac{\tau_h}{1 - \tau_h} + \bar{a}_h \right) \cdot \tilde{\eta}_{y,1(1-\tau_h)} \cdot \left( \frac{1 - \tilde{m} e_h}{1 - \tau_h} \cdot \bar{a}_h \cdot \tilde{\eta}_{a,1(1-\tau_h)} \right) \right] \cdot N_h \cdot \bar{y}_h \right\} \cdot d\tau_h \] |
| PIT + CT + SS | \[dT = \left\{ \left( (a_{h+1} - a_h) \cdot (1 - \bar{a}_h) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot (1 - \bar{a}_h) \cdot N_h - \tilde{\eta}_{\bar{y},1(1-\tau_h)} \cdot \bar{a}_h \cdot \tilde{\eta}_{a,1(1-\tau_h)} \right) \right\} \cdot N_h \cdot \bar{y}_h \right\} \cdot d\tau_h \] |
| PIT + CT + SS + ACC | \[dT = \left\{ \left( (a_{h+1} - a_h) \cdot (1 - \bar{a}_h) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot (1 - \bar{a}_h) \cdot N_h - \tilde{\eta}_{\bar{y},1(1-\tau_h)} \cdot \bar{a}_h \cdot \tilde{\eta}_{a,1(1-\tau_h)} \right) \right\} \cdot N_h \cdot \bar{y}_h \right\} \cdot d\tau_h \] |

*Meaning of the acronyms: PIT= Personal Income Tax; TC = Taxes on consumption; SS = Social Security Contributions; ACC= Administration and Compliance Costs.
Table 6
REVENUE-MAXIMIZING RATES AND REVENUE-MAXIMIZING ELASTICITIES IN AGGREGATE DEPENDING ON THE TAXES CONSIDERED IN THE MODELLING*

\[
\eta^F = \frac{(a_{h+1} - a_h) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot N_h}{N_h \cdot \bar{y}_h \cdot \tau_h} \cdot \left(\frac{1 - \tau_h}{(1 - \tau_h)} \cdot \left[(a_{h+1} - a_h) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot N_h \right] \right)
\]

\[
\tau^F = \frac{(a_{h+1} - a_h) \cdot (1 - \bar{a}_h^*) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot (1 - \bar{a}_h) \cdot N_h - N_h \cdot \bar{a}_h \cdot \bar{y}_h \cdot \eta^F_{\bar{y}_h(1-\tau_h)} + (1 - \bar{m}^e_{\bar{a}_h}) \cdot \eta^F_{\bar{a}_h(1-\tau_h)}}{(a_{h+1} - a_h) \cdot (1 - \bar{a}_h^*) \cdot N_h^+ + (\bar{y}_h - a_h) \cdot (1 - \bar{a}_h) \cdot N_h + \eta^F_{\bar{y}_h(1-\tau_h)} \cdot N_h \cdot \bar{y}_h \cdot (1 - \bar{a}_h)}
\]

where:

\[
\chi = N_h \cdot \bar{F}_{\bar{a}_h}^{SS} \cdot \bar{a}_h \cdot \bar{y}_h \cdot \left[1 + \eta^F_{\bar{y}_h(1-\tau_h)} + \eta^F_{\bar{a}_h(1-\tau_h)} \right]
\]

*Meaning of the acronyms: PIT= Personal Income Tax; CT= Taxes on consumption; SS = Social Security Contributions; ACC= Administration and Compliance Costs.

5. AN ILLUSTRATIVE SIMULATION

This final section illustrates the way in which the Laffer curve of an individual taxpayer varies depending on whether consumptions taxes, social security contributions and administration and compliance costs are considered or not when defining the profile of the Laffer curve. Ideally, the optimal exercise would be to apply the models presented in this research on a real microdata base of the current Spanish fiscal system. However, this exercise would require individualized information in relation to personal income taxation, social contributions and taxes on consumption. Moreover, the administration costs per capita and the individualized private compliance costs for each taxpayer should also be available. Unfortunately, in Spain we do not have a microdata base pooling together such amount of precise individualized information, so we carry out an illustrative exercise applying the functions presented above to a virtual taxpayer. This exercise will be sufficient to illustrate the consequences that can be expected from disregarding
the impact that income tax rates exert on the revenue of consumption taxes and social contributions, as well as on administration and compliance costs.

Table 7 reports the parameter values used in the simulation, which roughly replicate those of the average Spanish taxpayer. With these parameter values we will simulate an infinitesimal increase of the marginal rate \( (dr = 0.001) \) under the assumption that the tax base is taxed entirely by a single marginal rate – i.e. \( a_h = 0 \) and \( a_{h+1} = \infty \). This clarification is important because most of the papers on the Laffer curve assume this same premise, although they usually do not make it explicit despite being an assumption that, as we will see later, is critical to describe the profile of the Laffer curve as well as for determining the magnitude of the tax rate that maximizes revenue. However, despite this highly restrictive assumption of taxing the entire tax base at a single rate, it helps us to understand the need of a fully-fledged Laffer curve.

<p>| Table 7 |</p>
<table>
<thead>
<tr>
<th>PARAMETERs USED IN THE SIMULATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Behavioural parameters</td>
</tr>
<tr>
<td>( \eta_{y_{k1}(1-\tau_{k1})} = 0.6 )</td>
</tr>
<tr>
<td>( \bar{\eta}<em>{\tau</em>{\omega_{k1}(1-\tau_{k1})}} = 0.45 )</td>
</tr>
<tr>
<td>( \eta_{y_{k1}\omega_{k1}} = -0.2 )</td>
</tr>
</tbody>
</table>

As Figure 2 shows, not to account for consumption taxes, social security contributions and administration and compliance costs in assessing the effects of PIT rates do have significant implications on the ability to capturing the actual Laffer curve. The consequences can be summarized as follows:

a.- Any marginal rate of PIT systematically generates a lower revenue level when the impact on taxes other than personal income taxation is obviated. This misrepresentation of the total revenue achievable is especially evident in in the intermediate range of tax rates, being less evident for very low or very high rates.

b.- The underestimation of revenue is especially relevant in the case of not considering taxes on consumption and, to a lesser extent, social contributions and administration and compliance costs.

c.- Not counting on the effects on the other tax structures is equivalent to a shift to the left of the actual Laffer curve. That is, the normal zone of the actual Laffer curve is shrunk whereas the prohibitive zone is expanded. This implies that the true maximizing rates of collection are really lower than those prescribed by the revenue of personal income tax alone.
Figure 3, which depicts the Fullerton curves simulated with the same data, ratifies the same conclusions noted in the three preceding points. Not considering the effect of the marginal rates of personal income taxation on the collection of other taxes generates the illusion of a prohibitive zone less extensive than it really is. Namely, if we only pay attention to the collection of income tax, it could happen that we are apparently operating in the normal zone of the Laffer curve when in reality we would be located in the prohibitive zone of the actual Laffer curve.

However, despite this illuminating analysis presented above, we should not forget that the traditional income taxes in most countries tend to have a step-wise structure. In other words, the total tax base is segmented into income brackets that bear different increasing marginal rates. This fact has not been taken into account in the figures and numbers above, nor is it taken into
account in most of the existing literature. Therefore, an interesting question that should be
answered is whether this type of tax design, applied extensively in most tax systems, can affect
the profile of the Laffer curve. In other words, may a tax design based on a step-wise structure
alter the profile of the Laffer curve? To answer this question, we will replicate the previous
simulations under a step-wise income tax. To do so, we will use the general tax schedule applied
in Spain in 2018, shown in Table 8.

<table>
<thead>
<tr>
<th>K</th>
<th>$a_K$</th>
<th>$\tau_K$</th>
<th>$a'_K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>12,450</td>
<td>0.24</td>
<td>2,593.75</td>
</tr>
<tr>
<td>3</td>
<td>20,200</td>
<td>0.3</td>
<td>6,115</td>
</tr>
<tr>
<td>4</td>
<td>35,200</td>
<td>0.37</td>
<td>11,617.568</td>
</tr>
<tr>
<td>5</td>
<td>60,000</td>
<td>0.45</td>
<td>20,218.889</td>
</tr>
</tbody>
</table>

*This tax schedule assumes that all the Autonomous Communities of the Common Regime replicate the
state tax band.

In the new simulations we will assume the same marginal rate increase as before – \(d\tau = 0.001\) –
but on this occasion when the increased marginal rate exceeds the marginal rate separating each
tax-bracket we will take into account that a new threshold applies. This means that the new
marginal rate will only affect the amount of the tax base that is above this new threshold. The rest
of the taxable income will be levied by previous tax rates. This fact will necessarily have an
important effect on the magnitude of the collection gain associated with the marginal rate
increases as well as on the profile of the Laffer curve.

Figure 4 illustrates the Laffer curves derived from the new simulations run on this more realistic
assumption of multi-rate tax schedules. As shown in the right-hand panel of figure 4, the
existence of multi-rate tax schedules significantly limits the collection capacity of marginal rate
increases since they do not apply to the taxpayer's entire income, but to a fraction of it.
Consequently, any rate increase will yield less revenue than under the alternative assumption of a
single tax rate, which is the “hidden” assumption of most of the empirical work in the existing
literature.

On the other hand, the left-hand panel of Figure 4, apart from confirming that accounting for all
the taxes under scrutiny is a must to capture the actual Laffer curve, indicates that the Laffer
curves under a multi-rate tax schedule are much narrower than suggested by the (usually hidden)
assumption of a single rate, reaching the prohibitive zone earlier and, therefore, at much lower
marginal rates. This result is confirmed by the revenue-maximizing rates and revenue-maximizing
elasticities reported in Tables 9 and 10.
6. CONCLUSIONS

In this research, the Laffer curve of personal income taxation has been explored analytically in detail and an extended model has been proposed. The common denominator of the existing literature on this topic is to infer the Laffer curve paying attention exclusively to the effects of PIT rates on the revenue of the personal income tax itself. This analytical approach constitutes an important drawback that affects the profile of the Laffer curve. The reason for this is that PIT
marginal rates, in addition to affecting the collection of personal income taxation, also affect the revenue collection of other taxes and levies. Specifically, we identify two levies that should be incorporated into the discussion of the Laffer curve of the PIT: consumption taxes and social security contributions. The former omission overlooks the fact that PIT marginal rates also alter the taxpayer’s average tax rate and consequently, the revenue from consumption taxes. The latter omission, insofar as social security contributions are deductible from the tax base, overlooks the effective revenue collected from social security contributions. Along with these two omissions, we must not forget that taxing is a costly activity in itself and it generates at least two additional costs that are often overlooked: administration costs and compliance costs. The existence of these costs allows us to distinguish two distinct notions of the Laffer curve, a gross Laffer curve, which captures the relationship of marginal rates to the collection of all taxes involved, and a net Laffer curve, once administration and compliance costs are discounted. To the extent that administration and compliance costs are correlated with PIT marginal tax rates, so will be the net Laffer curve.

Based on the above arguments, this paper develops a fully-fledged analytical model for the Laffer curve of PIT. This extended model includes consumption taxes, social security contributions as well as administration and compliance costs. This modelling has been developed for both, the individual taxpayer and the aggregate. The simulations carried out confirm that not taking into account these omitted taxes and levies has important effects on the shape of the Laffer curve. Specifically, the most important impact occurs with the omission of consumption taxes, followed by social security contributions and, finally, by not taking into account the administration and compliance costs. If we consider all the omissions that should be taken into account in the modelling of the Laffer curve, the revenue-maximizing rates drop dramatically from 62.5% to 28.20%. In other words, omitting the impact of PIT rates on the revenue collection from consumption taxes, social security contributions and administration and compliance costs overestimate the normal zone of the Laffer curve as well as the potential revenue power of the tax system.
APPENDIX

As highlighted in the main text, tax compliance costs borne by an individual taxpayer has two components:

\[ CC_i(\bar{\tau}_i) = C_i(\bar{\tau}_i) + T_i(y_i(C_i(\bar{\tau}_i))) \]  \[ A.1 \]

therefore, in the event of a change in \( \tau_h \), the costs of compliance borne by an individual taxpayer will be modified in the following terms:

\[ \frac{dCC_i(\bar{\tau}_i)}{d\tau_h} = \frac{dC_i(\bar{\tau}_i)}{d\tau_h} + \frac{dT_i(y_i(C_i(\bar{\tau}_i)))}{d\tau_h} \]  \[ A.2 \]

that taking into account [13], the change in \( C_i(\bar{\tau}_i) \) will be given by:

\[ \frac{dC_i(\bar{\tau}_i)}{d\tau_h} = \begin{cases} 
C_0^i \cdot e^{b \cdot \bar{\tau}_i} \cdot b \cdot \frac{(a_{h+1} - a_h)}{y_i} & \text{if } \tau_h < \tau_k \\
C_0^i \cdot e^{b \cdot \bar{\tau}_i} \cdot b \cdot \frac{(y_i - a_k)}{y_i} & \text{if } \tau_h = \tau_k 
\end{cases} \]  \[ A.3 \]

whereas the change in \( T_i(y_i(C_i(\bar{\tau}_i))) \) will be \( \frac{dT_i(y_i(C_i(\bar{\tau}_i)))}{d\tau_h} = \frac{dT_i}{d\tau_h} \cdot \frac{dy_i}{d\tau_h} \cdot \frac{dC_i}{d\tau_h} \cdot \frac{dT_i}{d\tau_h} \) that after some mathematical arrangements and taken into account that \( \eta_{T_i,y_i} = \frac{\tau_k}{\text{me}} \) where \( \tau_k \) is the marginal tax rate and \( \text{me} \) is the average tax rate in the PIT of the taxpayer –it can be rewritten as:

\[ \frac{dT_i(y_i(C_i(\bar{\tau}_i)))}{d\tau_h} = \begin{cases} 
\tau_k \cdot \eta_{y_i,C_i} \cdot b \cdot (a_{h+1} - a_h) & \text{if } \tau_h < \tau_k \\
\tau_k \cdot \eta_{y_i,C_i} \cdot b \cdot (y_i - a_k) & \text{if } \tau_h = \tau_k 
\end{cases} \]  \[ A.4 \]

It is worth noting that whereas \( \frac{dC_i(\bar{\tau}_i)}{d\tau_h} \) is privately borne by the taxpayer, \( \frac{dT_i(y_i(C_i(\bar{\tau}_i)))}{d\tau_h} \) is the cost to society in the form of a tax collection cut in the PIT. Therefore, of these two elements, the only relevant factor to characterize the Laffer curve is the second one. Therefore, the tax revenue variation in the total population associated with the increase in compliance costs caused by \( d\tau_h \) will be determined by:

\[ \frac{dT[Y(C(\tau))]}{d\tau_h} = \sum_{N_h} \tau_h \cdot b \cdot (y_h - a_h) \cdot \eta_{y_h,C} + \sum_{N_h^*} \tau_k \cdot b \cdot (a_{h+1} - a_h) \cdot \eta_{y_h,C} \]

\[ = N_h \cdot (\bar{\tau}_{k_i})_h \cdot (\bar{\eta}_{y_i,C_i})_h \cdot b \cdot (y_h - a_h) + N_h^* \cdot (\bar{\tau}_{k_i})_h^* \cdot (\bar{\eta}_{y_i,C_i})_h^* \cdot b \cdot (a_{h+1} - a_h) \]
References


