In a two-period model with agent heterogeneity we analyze a pension reform toward a stronger link between contributions and benefits (as recently observed in several countries) in a pension system with a Bismarckian and a Beveridgean component. We show that such a policy change may discourage human capital investment at the margin and thus reduce the average educational level in an economy. The life expectancy differential between skilled and unskilled individuals drives this result. Furthermore, we investigate the consequences on the intragenerational redistribution characteristics of the pension system—in the sense of the number of net-recipients relative to net-payers—as well as welfare effects.

Keywords: Social Security, Education, Life Expectancy, Pension Reform, Redistribution.


1. I

This paper studies the effects of a pension reform toward a stronger link between individual contributions and benefits on the average educational level of the economy. In particular...
lar, we are interested in the consequences of the interplay of different forms of redistribution between skilled and unskilled workers for human capital investment when a (public) pension system becomes more ‘Bismarckian’, a direction into which many countries headed since the 1990s. In a two-period model with agent heterogeneity we argue that a pension reform of the described type may discourage human capital investment at the margin. Furthermore, we show that the reform will change the ratio of net-payers to net-recipients of the pension system. Governments considering a pension reform should therefore keep in mind that (exogenously) given life expectancy and productivity differences between skilled and unskilled individuals may induce an undesired outcome.

By now, it is widely accepted that output is the key variable for solving the demographic challenge in the Western world (cf. Barr 2004, p. 207). According to standard growth theory human capital accumulation, i.e., education in a broader sense, leads to a higher growth path and thus to more output. When more output is available due to higher education-induced growth, shifting resources from young to old becomes easier in aging countries, regardless of whether their pension systems are pay-as-you-go (PAYG) or funded. However, not only may education have an impact on pension systems but also the design of pension systems on human capital accumulation and therefore growth.

When individuals maximize their lifetime utility, they will take redistributive taxation via the pension system into account. PAYG pension systems introduce an implicit tax on income (cf., e.g., Sinn, 2000). While this tax inevitably follows from intergenerational redistribution (from young to old), tax rates differ depending on the level of intragenerational redistribution between individuals of the same generation (usually from rich to poor). Following the convention by Cremer and Pestieau (1998), a pension system with zero or little intragenerational redistribution may be called ‘Bismarckian’ while a system with flat-rate benefits is called ‘Beveridgean’, assuming that in both systems contributions are collected by means of a payroll tax. The more Beveridgean a pension system is, the higher is implicit taxation (Sinn, 2000). Hence, activities creating additional income will become less attractive under these circumstances. Since education is positively correlated with income, a pension system with a high level of Beveridgean redistribution may discourage human capital investment (cf. au and Poutvaara, 2006).

In our analysis –unlike most of the existing literature– we consider that there exist not only one, but two intragenerationally redistributive channels within a pension system which could influence the incentive to invest in human capital. The first channel is the previously mentioned, ‘traditional’ intragenerational rich-to-poor redistribution which follows from the fact that rich persons contribute relatively more than poor persons, but receive (almost) the same benefit under a Beveridgean system. Only under a pure Bismarckian system no intragenerational redistribution takes place. As in Cremer and Pestieau (1998) and Krieger and Traub (2008, 2011) we use the so called ‘Bismarckian factor’ to describe a mixture of Bismarckian and Beveridgean elements when pension benefits are calculated. The higher the Bismarckian factor, i.e., the tighter the link between one’s own contributions and one’s own benefits, the more attractive is a pension system for a high income earner. However, this rich-to-poor redis-
tribution is softened by a poor-to-rich redistributive effect when higher incomes correlate with higher life expectancy (cf. Borck, 2007). If high income earners receive pension benefits for a longer time span than low income earners, this potentially leads to redistribution from the poor to the rich when the effect is sufficiently strong to compensate for lower absolute contributions. This, however, is true only when at least some Beveridgean redistribution takes place 1.

In the literature, the distinction between Beveridgean and Bismarckian pension systems has attracted some attention. Parts of the literature ask whether there is a negative relation between the level of intragenerational redistribution and the size of PAYG pension systems (cf., e.g., Casamatta, Cremer and Pestieau, 2000a; Köthenbürger, Poutvaara and Profeta, 2008). Other models explain why real world pension systems usually contain both Beveridgean and Bismarckian elements (Conde-Ruiz and Profeta, 2007; Casamatta, Cremer and Pestieau, 2000b; Cremer and Pestieau, 1998, 2003; Kolmar, 2007). Recently, Hachon (2008, 2009, 2010a, 2010b) turned to the effects of a pension system’s progressivity (or regressivity) on economic growth and the income distribution, thereby distinguishing between Beveridgean and Bismarckian pension systems 2.

A further strand of the literature which is closely connected to our analysis discusses the effect of the design of pension systems on the educational decisions of individuals. Individuals decide whether to invest in (costly) education when they expect a positive return on their investment. When redistributive taxation of high incomes via the pension system is sufficiently strong, individuals may prefer to invest less into their human capital as their return on investment is reduced. This argument can be found in au and Poutvaara (2001, 2006) and Poutvaara (2007). However, this literature ignores the life expectancy channel.

It is important to note that our analysis of the effects of a Bismarckian pension reform on an economy’s educational level is based on the observation that recent pension reforms in European countries share two common characteristics: we observe a trend toward higher funding and toward a stronger link between individual contributions and pension benefits, as shown by Werding (2003), Fenge et al. (2003) or Krieger and Traub (2008, 2011) for ECD countries since the mid-1980s. More funding means less intergenerational redistribution in the first place, while a stronger link between individual contributions and pension benefits reduces intragenerational redistribution in the pension system. We do not consider policy changes toward more funded systems but focus on the second phenomenon and analyze how such a policy reform affects the level of education in an economy in terms of the response of the marginally educated individuals.

Based on the previous discussion, our model then analyzes the effect of a change in the pension system on the educational level in an economy and considers—thereby closing a gap in the literature—in particular a positive correlation between individual education and longevity. Intuitively, there are good reasons to believe that life expectancy depends on the educational background of an individual. The fact that skilled people usually face a more stable social situation, have higher incomes and have a way of living which more agrees with health compared to unskilled people 3, justifies this. Furthermore, Bopp and inder (2003) for example
found substantial mortality gradients by education in German-speaking Switzerland in the 1990s for ages between 25 and 90 in a longitudinal data set of the Swiss national Cohort.

While—at first glance— it may seem a little far-reaching to assume, for instance, a 20-year old person seriously considering educational effects on retirement income, there are at least two arguments in support of this. First, according to Sinn (2005), behavioral changes take place as a reaction to changes in social systems, however, they often take a long time to become widely anticipated. For instance, by the way of observation and imitation, generation after generation adapted to the new institutional circumstances after the first introduction of public pension systems, until finally, fertility rates slumped to today’s historically low levels. Therefore, when a pension reform takes place today people will not immediately start to invest more or less into their human capital. Some years or decades from now, however, it may be a common wisdom that ‘you have to go to university in order to be able to finance retirement’. A second argument follows from the observation that changes of subjective variables may suffice to induce certain behavioral changes. In our model, we argue that life expectancy differences play an important role when it comes to educational decisions. Psychologists find that each additional year of education increases subjective life expectancy (irowsky and Ross, 2000). If individuals also (ex ante) believe in higher life expectancy due to education and if life expectancy increases lifetime utility via the pension system, investing into human capital may appear to be a reasonable strategy, even if—at the end of the day—life expectancy turns out to be falsely predicted.

The main result of our analysis is that a pension reform toward a more Bismarckian pension system may discourage human capital investment at the margin and thus reduce the average educational level in an economy if individuals differ in life expectancy. This is because educated individuals have a long retirement period during which they benefit relatively more from flat (Beveridean) pension benefits. Reducing these benefits in a reform towards a Bismarckian system makes investments into education attractive for fewer individuals. Furthermore, we show that the reform not only changes individual benefits and welfare but may also change the composition of net-recipients and net-payers in the system, together probably leading to political consequences in a democracy.

The paper is structured as follows. After introducing the model in Section 2, we analyze the effects of a pension reform and present our results in Section 3 before Section 4 briefly discusses the results and concludes.

2. T

Each individual in our model lives for two periods. While the individual’s time endowment in the first period (‘working life’) can be used for either higher education and supply of skilled labor or exclusively for unskilled work, the second period (‘retirement’) represents the evening of life where individuals no longer work but receive pension benefits. The individual time endowment in both periods is normalized to one.
Whether an individual goes for education or not, depends—to a large extent—on her ability. We assume heterogeneous agents who differ in their ability to acquire skills in the sense that a more able individual needs less time to do so than a less able one. Specifically, \( h \in [0, 1] \) denotes the time fraction in period one needed to acquire skills (e.g., the time span until graduating from a university) and reflects an individual’s ability. While an \( h \) close to zero indicates very high abilities, an \( h \) close to one means very low abilities. Abilities are distributed among individuals according to a cumulative distribution function \( F(h) \) with \( f(h) \) representing the corresponding density function.

Each worker’s totally inelastic labor supply is normalized to one. A skilled worker earns net labor income \((1 - t)(1 - h)w\), where \( w \) is the wage rate per unit of effective labor; \( t \in [0, 1] \) is the payroll income tax rate or contribution rate to the pension system; and \((1 - h)\) reflects working time, i.e., time endowment net of time spent on education. An unskilled worker is assumed to provide fewer units of effective labor per unit of working time compared to a skilled worker. Her net income amounts to \( qw(1 - t) \), where \( q \in [0, 1] \) reflects the overall difference in productivity across worker types (i.e., skilled vs. unskilled workers). Following Razin and Sadka (1999), there will thus be a cutoff level of \( h \), denoted by \( h^* \), in terms of the education decision of individuals, which is given by

\[
(1 - h^*)w(1 - t) = qw(1 - t) \quad \Leftrightarrow \quad h^* = 1 - q,
\]

so that every individual with an ability below \( h^* \) will acquire education and become a skilled worker, while all individuals with an ability above \( h^* \) will not acquire education and remain unskilled. Condition (1) implies that for a relatively more able person to be indifferent between acquiring education or not, the alternative wage as an unskilled must be higher, i.e., a low \( h^* \) requires a high alternative wage income \( qw \). However, if a relatively more able person is indifferent, there will be only few skilled workers in total, i.e., for a given skill distribution a high alternative income leads to a high supply of unskilled labor.

In our framework, condition (1) will serve as the starting point for further analysis and will now be complemented by incorporating education costs and the pension system. This approach follows Razin and Sadka (1999, 2000) and Razin, Sadka and Swagel (2002a, 2002b, 2002c). Let us first consider the pension system in more detail.

The pension payout for a retiree consists of a flat-rate benefit \( b \) and a component that is contingent on first-period labor income. The latter is given by \( \alpha(1 - h)w \) for skilled workers, and \( aqw \) for unskilled workers, where \( \alpha \in [0, 1] \) is the ‘Bismarckian factor’ (Cremer and Pestieau, 1998) which will be discussed in more detail below. Individuals receive pension benefits for the time they are alive in the second period. We assume that skilled pensioners have a life expectancy of one (i.e., they live for the entire retirement period) while the life expectancy of an unskilled pensioner is only a fraction \((1 - \sigma)\) of period two with \( \sigma \in [0, 1] \). Figure 1 summarizes the time structure of the model.
Before continuing with our discussion of pension benefits, some clarifications regarding the use of ‘life expectancy’ in our model may be helpful. Although we use the term life expectancy here, there is apparently no uncertainty in our model. In fact, we could implicitly assume that each individual knows only the average life expectancy of the skill group she belongs to. That is, at the individual level there would be uncertainty about the time of death, and $\sigma$ would become a random variable with known mean (and variance). But then, it would be possible to condition one’s own death on the individual education decision, which is not very realistic. Keeping this in mind, we assume for simplicity that individuals know their life expectancy.

We can now return to our model’s definition of pension benefits. Let the pension benefits for a skilled individual with ability $h$ denote by $P_h$, and for an unskilled pensioner by $P_q$. Then,

$$P_h = b + \alpha(1 - h)w,$$

$$P_q = (1 - \sigma) [b + \alpha qw].$$

The two extremes of the pension system would be the pure Beveridgean system ($b > 0, \alpha = 0$) and the pure Bismarckian system ($b = 0, \alpha > 0$).

Note that, as our focus is on intragenerational redistribution, we use a simplified framework which, in principle, accommodates overlapping generations. We assume, however, that each generation reproduces itself exactly. The pension benefits should in fact be denoted by $(1 + n)P_h$ and $(1 + n)P_q$, where $n$ reflects the return on contributions to the PAYG system. For simplicity, we assume $n = 0$ which will, however, not change our results qualitatively. Another standard assumption is that ability is inherited. Furthermore, contribution rates are fixed and decision making lasts for individual lifetime. This allows us to consider only one generation at each point in time. This is because all generations are identical with respect to size and skill distribution (cf. Krieger, 2004).

Finally, we assume two kinds of education costs: (i) direct costs $hg$, where $g$ is some per unit cost of education, and (ii) opportunity costs in the sense of foregone earnings from un-
skilled labor supply in period one while going for education. This provides us with all necessary ingredients to set up the extended indifference condition determining the education decision of individuals (cf. Razin and Sadka, 1999, 2000; Razin, Sadka and Swagel, 2002a, 2002b, 2002c).

When deciding on education any individual considers her abilities, the net pension benefit (accounting for life expectancy) and the cost of education. Comparing net lifetime incomes, i.e., labor income and pension benefits net of pension contributions and education costs, an individual only goes for education if her ability exceeds a certain cutoff level \((1 - h^*)\) which is implicitly given by

\[
(1-t)(1-h^*)w - h^*g + \frac{1}{1+r}P_{h^*} = (1-t)qw + \frac{1}{1+r}P_q
\]

(4)

with \(P_{h^*} = b + \alpha(1 - h^*)w\) and \(P_q\) from (3). With \(r\) as the exogenously given interest rate, \((1/1+r)P_j (j \in \{h^*, q\})\) represents the present value of the pension payment. An individual of ability \(h\) is exactly indifferent between acquiring skills and working as an unskilled worker. Remember that low values of \(h\) indicate high abilities, i.e., only individuals with \(h \leq h^*\) go for education.

Indifference condition (4) yields some interesting insights. The individual, who is indifferent between going for education or not, now has to take a lifetime perspective. That is, education decisions affect both the present and the future (i.e., retirement). For instance, consider the costs of education. When going for education, there will be an opportunity cost in the sense of foregone earnings today. However, when not going for education, future pension benefits will be lower. The latter implies that we have to compare the (direct) cost of education today, \(h^*g\), with the future (opportunity) cost of being unskilled. In present value terms, this cost is given by \((1/1+r)\alpha(b + aqw)\), where \(\alpha\) indicates the percentage of the pension benefit lost due to dying early. At the same time, the pension benefit is lower here (compared to a skilled person’s benefit) due to \(q\). Assume for the moment an exogenous decrease of \(g\), such that \(h^*g < (1/1+r)\alpha(b + aqw)\). Then, the previously indifferent individual will (ceteris paribus) go for education as its cost now falls short of the cost of being unskilled during retirement.

This is, however, only one aspect to be considered when deciding about education. The previously discussed cost differential could be compensated when today’s income differential is not too large. That is, Razin and Sadka’s (1999) condition from (1) needs to be taken into account, too. This leads us to the following observation:

\[
h^*g \leq \frac{1}{1+r}\alpha db + qw \iff (1-h^*) \leq q.
\]

(5)

which implies that the indifference condition holds whenever costs and income flows resulting from the educational decision balance appropriately. In order to avoid tedious case dis-
tinctions, we will restrict ourselves to the case of equality in the following. That is, we will assume that (5) holds, i.e., \( h^g = \frac{1}{1 + r} \alpha (b + c q w) \) and \( 1 - h^* = q^6 \).

Let us now turn to the role of the government in our model. The government is confined to collecting the payroll income tax and to distributing pension benefits. We assume that this governmental task is costless, i.e., we ignore administrative costs. The budget constraint in a PAYG pension system then reads

\[
N w \int_0^{\bar{h}^i} (1 - h) f(h) \, dh + q \int_0^{\bar{h}^i} f(h) \, dh = 0
\]

(6)

\[
N \int_0^{\bar{h}^i} P_h f(h) \, dh + P_q \int_0^{\bar{h}^i} f(h) \, dh,
\]

i.e., the aggregate pension benefits of generation \( -i \) are financed by the contributions of the currently working generation \( i \). The size of a generation is denoted by \( N \). With our assumption of a non-growing population, \( N \) is the same across all generations.

Since we assume a pension system with a fixed contribution rate \( t \), any change of the structure of benefits, i.e., higher or lower Beveridgean/Bismarckian benefits, needs to be compensated within the benefits sphere as total contributions are given. Given the basic structure of our model, we expect differences in life expectancy to play a role here as well. For further reference, we will briefly elaborate on this in the following. In a first step, we reformulate budget constraint (6) – as shown in Appendix A – as follows:

\[
t \bar{w} = b + \alpha \bar{w},
\]

(7)

or, equivalently, rather as

\[
b = b(\alpha) = t \bar{w} - \alpha \bar{w},
\]

(8)

where

\[
\bar{w} := \frac{N w \int_0^{\bar{h}^i} (1 - h) f(h) \, dh + q \int_0^{\bar{h}^i} f(h) \, dh}{N \int_0^{\bar{h}^i} f(h) \, dh + (1 - \sigma) \int_0^{\bar{h}^i} f(h) \, dh}
\]

(9)

is the ratio of the aggregate wage income of the currently young generation relative to the life expectancy weighted size of the currently old generation. Furthermore,

\[
\bar{\tilde{w}} := \frac{N w \int_0^{\bar{h}^i} (1 - h) f(h) \, dh + q (1 - \sigma) \int_0^{\bar{h}^i} f(h) \, dh}{N \int_0^{\bar{h}^i} f(h) \, dh + (1 - \sigma) \int_0^{\bar{h}^i} f(h) \, dh}
\]

(10)
is the life expectancy weighted average wage income level of the currently old generation 7. See that only for \( \sigma = 0 \) (i.e., in case of no life expectancy differential), we have \( \hat{w} = \hat{\hat{w}} = \bar{w} \), with \( \bar{w} \) as the average labor income level and

\[
tw = b + \alpha \bar{w},
\]

i.e., the contribution of (only) an average income individual exactly supplies the retirement benefit of a retiree who was an average income individual himself. Then, inspection of equation (7) indicates that this equality does no longer hold if retirees differ in life expectancy.

3. T B

This section analyzes the effects of a pension reform on the educational level, on intra-generational redistribution via the pension system and on welfare. As already argued before, our simplifying assumptions regarding reproduction behavior guarantee that individual decision making will not change over time. If a generation decides on the educational level and, thus, implicitly retirement income, this income – although received in the subsequent period and covered from next period’s workers’ contributions – will be just as expected today.

3.1. A

The change of the Bismarckian parameter in a fixed contribution rate system requires an adjustment of the flat-rate benefit in order to keep the system’s budget balanced. Let us take a closer look at this adjustment before moving on to our analysis of pension reforms’ interplay with educational decisions and life expectancy in the remainder of this section. We begin by stating the following emma.

1 In a fixed contribution rate system, a change in the Bismarckian parameter \( \alpha \) requires an adjustment of the flat-rate benefit \( b \). This adjustment consists of a direct budget effect and an indirect effect due to the reform’s effect on the economy’s educational level.

Recall equation (8) which captures the relationship between contributions and benefits in the pension system:

\[
b = t\hat{w} - \alpha \hat{\hat{w}}.
\]

A change in the educational level in the economy influences benefit \( b \) via \( \hat{w} \) and \( \hat{\hat{w}} \), i.e.,

\[
\frac{b}{h^*} = t \frac{\hat{w}}{h^*} - \frac{\hat{\hat{w}}}{h^*}
\]

(11)
Deriving $\partial \tilde{w} / \partial h^*$ and $\partial \tilde{w} / \partial h^*$ from (9), or rather (10), and using this in (11) yields after some manipulations

$$\frac{b}{h^*} = \frac{n(h^*)}{\bar{n}} \left\{ \frac{1}{w} (1-h^*) - q - w (1-h^*) - (1-q) \right\} - \sigma b,$$

which, as a consequence of our simplifying assumptions from (5), i.e., $h^* q = (1/1+r)(b + \sigma q w)$ and $1 - h^* = q$, reduces to

$$\frac{b}{h^*} = -\frac{n(h^*)}{\bar{n}} \alpha(b + q w),$$

where $n(h^*)$ denotes the number of individuals of ability type $h^*$ and $\bar{n}$ the life expectancy weighted size of a generation. The sign of $\partial b / \partial h^*$ is clearly negative. The intuition here is as follows: the portion of individuals with a higher life expectancy is smaller in a generation with a lower average educational level; due to our assumption of life annuity pension benefits and for given contributions this allows, ceteris paribus, for a higher flat-rate benefit.

The adjustment of the flat-rate benefit $b$ in case of a change of the Bismarckian factor $\alpha$ consists of two effects. Using the reformulated budget constraint (8) and (11) yields

$$\frac{db}{d} = \hat{\tilde{w}} + t \hat{\tilde{w}} - \frac{\hat{\tilde{w}}}{h^*} \frac{dh^*}{d}.$$ (13)

From the budget constraint we can also derive

$$\left. \frac{b}{h^* = \text{const.}} \right| = -w \int_0^{h^*} (1-h) f(h) dh + (1-\sigma) q \int_0^{h^*} f(h) dh = -\hat{\tilde{w}}.$$ (14)

and therefore rewrite (13) as

$$\frac{db}{d} = \left. \frac{b}{h^* = \text{const.}} \right| + \frac{b}{h^*} \frac{dh^*}{d}$$ (15)

where the first term is the (negative) direct budget effect in a fixed contribution rate system and the second term is the indirect effect via the induced change in the educational level. The sign of the latter effect is positive in total with both partial influences being negative. We will show in the following section that the sign of $dh^* / d\alpha$ is in fact negative. The same is true for $\partial b / \partial h^*$, as shown above. If the (positive) indirect effect dominates the (negative) direct budget effect, the flat-rate benefit might even have to increase in case of a higher Bismarckian parameter, despite the assumed fixed contribution rate system. The adjustment in the benefits sphere in case of a pension reform, as represented by (15), illustratesemma 1.
3.2. Level of education

We are interested in the effect of a reform toward a more Bismarckian system (i.e., a marginal increase of \( \alpha \)) on the ability cutoff level \( h^* \) in the education decision. The pension reform is assumed to be a one-time event. Let us analyze \( dh^*/d\alpha \) considering the public budget constraint.

**Proposition 1** A marginal increase of the Bismarckian parameter \( \alpha \) (in a fixed contribution rate system) reduces the educational level in the economy. This is due to a positive life expectancy differential between skilled and unskilled individuals.

Implicit differentiation of indifference condition (4) yields

\[
\frac{dh^*}{d\alpha} = \frac{Z}{D}
\]

where

\[
D := -w(1-t) - g - \frac{1}{1+r} w + \frac{\sigma}{1+r} b \quad \text{< 0}
\]

and

\[
Z := \frac{1}{1+r} \left( 1-h^* \right) w - (1-\sigma)qw + \frac{1}{1+r} \left[ 1 - (1-\sigma) \left( \frac{b}{h^*} \right) \right]_{h^* = \text{const.}}
\]

\[
= \frac{1}{1+r} w \left( 1-h^* \right) - q + \frac{1}{1+r} \sigma qw + \frac{b}{h^* = \text{const.}}.
\]

Considering our simplifying assumption from (5) this reduces to

\[
Z = \frac{1}{1+r} \sigma qw + \frac{b}{h^* = \text{const.}}. \tag{16}
\]

\( Z_{\alpha} \) consists of a positive and a negative component. The first component, \((1/1+r)\sigma qw\), captures the positive effect of an increase in \( \alpha \) on the Bismarckian part of an \( h^* \)-type individual’s pension benefit. Due to the positive life expectancy differential, a skilled individual benefits more from this effect compared to an unskilled individual. On the other hand, the second component captures that skilled individuals lose more from the reduction in the flat-rate benefit \( b \). The reason is, that, due to the higher life expectancy of skilled individuals, their flat-rate pension benefit is higher in present value terms than for unskilled individuals. This effect follows from \((1/1+r)\sigma(\partial b/\partial \alpha)_{h^* = \text{const.}}\). The overall sign of (16) can be shown to
be unambiguously negative, meaning that around the ability cutoff level, fewer individuals prefer to become educated when $\alpha$ increases.

$Z_\alpha$ is negative if

$$qw + \frac{b}{\hat{h}^\text{const.}} < 0 \iff qw < \hat{w}$$

which should always hold if there is at least one (educated) individual earning a higher income in the economy. Appendix B provides a more formal proof. Hence, for $\sigma \neq 0$,

$$\frac{dh^*}{d} < 0$$

unambiguously holds, which implies that the reform discourages human capital investment at the margin and thus reduces the average educational level in the economy. This proves Proposition 1.

Although we do not model other generations explicitly, the effects on those can easily be derived. Note first that any generation following the generation of workers who are affected by the one-time pension reform will perfectly replicate this ‘initial’ generation. Hence, there is only one generation left which may be affected by the reform: the retirees in the period of the reform. We will return to this issue in Section 3.4, where we will deal with welfare effects.

### 3.3. Reform

The previous section took a view from a broader (‘macro’) perspective, arguing that a pension reform toward a higher Bismarckian factor reduces educational effort in the economy. However, a pension reform not only changes the level of education in our model economy but also the redistributive characteristics of the pension system. A priori it is not clear how the reform affects different (skill) groups in society such that, at the end of the day, $h^*$ falls. It may turn out that a pension reform which leads to a lower average educational level and should therefore be undesired by the population as a whole becomes attractive for an increasing subgroup of citizens because they gain from redistribution via the pension system.

In order to analyze the distributional effects, we first look at the individual who represents the transition from net-recipients to net-payers in the system. Here, we are interested in seeing whether individuals who were indifferent in terms of redistribution before the reform will gain or lose through the reform. In the next section, we will then further elaborate on these issues by turning to a general welfare analysis.
In our model, net-recipients are individuals whose pension benefits (in present value terms and considering life expectancy) exceed their contributions. Since all uneducated individuals are equal with respect to labor income and life expectancy, they are all either net-recipients or net-payers. The latter case (from which we will abstain in what follows) can occur if the life expectancy differential is such that the poor-to-rich redistributional effect of the pension system exceeds the rich-to-poor redistributional effect. Within the group of educated individuals there might be both net-recipients and net-payers.

We now analyze the effect of a pension reform on the ability level $\tilde{h}$ characterizing the individual at the transition between net-recipients and net-payers. The implicit definition of $\tilde{h}$ is given by

$$
(1 - \tilde{h})n_w - \frac{1}{1 + r} \ b + \ (1 - \tilde{h})w_r = 0.
$$

The present value of pension benefits for an $\tilde{h}$-type individual exactly equals her pension contributions. Individuals with abilities greater than the threshold ability, i.e., individuals with $h < \tilde{h}$, are net-payers while all other individuals with lower abilities are net-recipients.

Let us first check for the existence and uniqueness of $\tilde{h}$.

**Lemma 2** A contribution rate $t > (1/1 + r)\alpha$ ensures the existence of a unique ability level $\tilde{h} < h^*$.

This result follows from Figure 2 which shows the individual tax bills (or rather pension contributions) and the individual retirement benefits for the different ability types. It illustrates the definition of $\tilde{h}$ for a given pension system with $\alpha, b > 0$. With the distribution function $F(h)$, $\tilde{h}$ determines the size of the group of net-recipients or rather net-payers in the system.

In Figure 2, it is the fact that the tax bill curve is steeper than the retirement benefits curve (for skilled individuals, i.e., to the right of $(1 - h^*)$) that ensures the intersection of tax bill curve and retirement benefits curve. Therefore,

$$
t > \frac{1}{1 + r}
$$

is a sufficient condition for $\tilde{h}$ to exist.

Furthermore, the fact that both the tax bill curve and the retirement benefits curve are linear (and therefore intersect only once) and the governmental budget constraint ensure that $\tilde{h}$ is unique and $\tilde{h} < 0$ (i.e., there are net-payers in the pension system). This proves lemma 2.
After having checked for the existence and uniqueness of $\tilde{h}$ we can now turn to the comparative statics and analyze the effect of a policy change on $\tilde{h} \neq h^*$. Again, we consider a policy change toward a more Bismarckian system, i.e., a marginal increase of $\alpha$.

**P 2** A marginal increase of the Bismarckian factor increases the number of net-recipients relative to net-payers in the pension system if and only if the value of the pension of an $\tilde{h}$-type increases due to the reform.

Hence, whether the number of net-recipients relative to the number of net-payers in the pension system increases or decreases as a consequence of the reform is a priori unclear. Implicit differentiation of (18) yields:

\[
\frac{d\tilde{h}}{d\alpha} = -\frac{A}{B} \tag{20}
\]

where

\[
A := -\frac{1}{1+r} (1-\tilde{h})w + \frac{db}{d} = -\frac{1}{1+r} (1-\tilde{h})w - \hat{w} + \frac{b}{h^*} \frac{dh^*}{d}, \tag{21}
\]

\[
B := -tw + \frac{1}{1+r} w. \tag{22}
\]

Condition (19) ensures that $B < 0$. Therefore, the sign of $d\tilde{h} / d\alpha$ solely depends on the ambiguous sign of $A$. 

As long as the value of the pension increases because the adjustment of the flat-rate benefit due to the increased Bismarckian factor is positive, i.e., $\frac{\partial b}{\partial \alpha} > 0$, or at least not too large in case it is negative, $\hat{h}$ decreases (i.e., the ability threshold $(1 - \hat{h})$ in Figure 2 shifts to the right) implying an increasing number of net-recipients relative to net-payers in the pension system.

The individual whose pension benefits (in present value terms) exactly equal her contributions would now become a net-recipient if the reform were implemented. Emma 1, i.e., the fact that the adjustment of the flat-rate benefit does not only consist of a direct (negative) budget effect but also of an indirect (positive) effect due the change in the average educational level and therefore the average life expectancy of retirees (which might even induce an increase in $b$), explains this result.

3.4. W

Beside an analysis of a pension reform’s effect on the redistributive characteristics of the system as presented above, a further important step in evaluating the Bismarckian pension reform is to ask about welfare implications. Who gains and who loses from the reform and does the economy, or rather society as a whole, gain or lose?

3.4.1. Individual welfare

After making the education decision according to indifference condition (4), the individual optimization problem is to maximize lifetime utility $U$ from consumption in both periods subject to the intertemporal budget constraint $9$. The corresponding agrangians read

$$\Phi^q = U\left(c_1^q, c_2^q\right) + \eta_q \left(1 - t\right) qw + \frac{1}{1 + r} \left(1 - \sigma\right) (b + qw) - c_1^q - \frac{c_2^q}{1 + r}$$

which are identical for all unskilled individuals, and

$$\Phi^h = U\left(c_1^h, c_2^h\right) + \eta_h \alpha \left(1 - t\right)(1 - h) w - hg + \frac{1}{1 + r} b + \left(1 - h\right) w - c_1^h - \frac{c_2^h}{1 + r}$$

for skilled individuals with $h \leq h^*$. $\eta_q$ and $\eta_h$ are the respective agrangian multipliers.
The demand functions are \( c^j_t = c^j_t (w, t, r, \alpha, b, \sigma) \) for unskilled and \( c^h_t = c^h_t (w, t, r, \alpha, b, h, g) \) for skilled individuals, where \( j \in \{1, 2\} \) and \( h \in [0, h^*] \). The corresponding indirect utility functions are \( v^g = v^g (w, t, r, \alpha, b, \sigma) \) and \( v^h = v^h (w, t, r, \alpha, b, h, g) \).

**P 3** The individuals of the post-reform generations only gain from the reform if the value of the pension increases due to the reform.

Using the Envelope Theorem we can determine the effect of a policy change on individual utility.

\[
\frac{\partial v^g}{\partial \theta} = \Phi^g = \eta_q \frac{1}{1 + r} (1 - \sigma)qw + \frac{db}{d}, \tag{23}
\]

\[
\frac{\partial v^h}{\partial \theta} = \Phi^h = \eta_h \frac{1}{1 + r} (1 - h)w + \frac{db}{d}. \tag{24}
\]

Whether an individual gains or loses depends on the direction and the size of the change in the flat-rate benefit as a consequence of the reform.

The value of the pension increases if the flat-rate benefit \( b \) increases, or if at least a potential reduction of the flat-rate benefit does not exceed the increase in the Bismarckian component of an unskilled individual. In that case, both skilled and unskilled individuals unambiguously gain from the reform. However, beyond this threshold, only those (skilled) individuals with a high enough labor income, i.e., those who gain more from the higher Bismarckian benefit than they lose from the reduction of the flat-rate benefit, gain from the reform.

Hence, knowing the size of the adjustment of the flat-rate benefit \( b \) not only allows us to determine the group of individuals who benefit from the reform, but by means of the ability distribution \( F (h) \) we could also analyze the size of ‘interest groups’ in favor or against the reform. Furthermore, (23) then provides additional information on its redistributive consequences. Both aspects would be of special interest in a political economy framework analyzing the political support for the reform.

So far, we ignored the retirees in the period of the reform (generation \(-i\)) whose benefits depend on their own contributions but, in a PAYG system, also on the contributions of the subsequent generation \((i)\) which now has a lower average educational level due to the reform. Let us analyze the change in the pension benefits of these post-reform retirees, assuming that the new Bismarckian factor also applies to their pension scheme.

**P 4** While the unskilled retirees in the period of the reform unambiguously lose from the reform, skilled retirees only gain if they have an above average labor income. Only in case of a right of continuance with respect to the Bismarckian factor, the retirees’ welfare is not affected by the reform.
First, consider the change in the pension benefit of an unskilled retiree.

\[
- \hat{w}_{i,-i} - \hat{w}_{-i} + qw_{-i} = t \frac{\hat{w}_{i,-i}^*}{h_i^*} - \hat{w}_{-i} + qw_{-i} \\
= t \frac{n(h_i^*)}{\hat{n}_{-i}} w (1-h_i^*) - q \frac{h_i^*}{\hat{n}_{-i}} - \hat{w}_{-i} + qw_{-i} \\
= -\hat{w}_{-i} + qw_{-i} < 0,
\]

where \( n(h^*) \) denotes the number of individuals with an ability level \( h^* \) and \( \hat{n} \) the life expectancy weighted size of a generation. The unskilled retirees clearly lose from the reform. Whether a skilled retiree gains or loses depends on whether her labor income exceeds the weighted average income:

\[
- \hat{w}_{i,-i} - \hat{w}_{-i} + (1-h_{-i})w_{-i} = -\hat{w}_{-i} + (1-h_{-i})w_{-i}.
\]

Given our assumption from (5), we find that only if there exists a right of continuance with respect to the Bismarckian parameter for the retirees in the period of the reform, their pension benefits remain unchanged, since

\[
\hat{w}_{i,-i} = t \frac{n(h_i^*)}{\hat{n}_{-i}} w (1-h_i^*) - q \frac{h_i^*}{\hat{n}_{-i}} = 0.
\]

### 3.4.2. Social welfare

Whether the ‘society’ as a whole gains or loses from the reform is not a priori obvious. We now analyze the overall welfare change of skilled and unskilled individuals of a generation \( i \) by means of two exemplary social welfare functions. We assume the before mentioned right of continuance and therefore ignore the generation \(-i\) retirees. First, take a Rawlsian welfare function:

\[
W^R = \min \{ \nu^\theta, \nu^{h=h^*}, \nu^{h=0} = \nu^\theta \}
\]

Therefore

\[
\frac{\partial W^R}{\partial \nu^\theta} \leq 0, \quad (1-\sigma)qw + \frac{db}{d} \leq 0 \\
> 0, \quad (1-\sigma)qw + \frac{db}{d} > 0.
\]


The reform unambiguously reduces social welfare described by a Rawlsian welfare function if the individuals with the lowest utility level, i.e., the unskilled individuals, lose from the reform.

et us now instead assume a Utilitarian social welfare function:

\[
W^U = N \int_0^{h^*} \theta_h v^h_f(h) \, dh + \theta_q v^q_1 f(h) \, dh
\]

(30)

with \( \theta_q \) as the welfare weight of unskilled and \( \theta_h, h \in [0, h^*] \), as the weights for skilled individuals.

\[
\frac{dW^U}{dh} = \theta_h v^h_n(h^*) \frac{dh^*}{dh} + N \int_0^{h^*} \theta_h v^h_f(h) \, dh
\]

\[-\theta_q v^q_n(h^*) \frac{dh^*}{dh} + N \theta_q v^q_1 f(h) \, dh\]

(31)

\[
= \theta_h v^h_n - \theta_q v^q_n(h^*) \frac{dh^*}{dh}
\]

\[+ N \int_0^{h^*} \theta_h v^h_f(h) \, dh + \theta_q v^q_1 f(h) \, dh .\]

Assuming identical welfare weights for all individuals and \( v^{h^*} = v^q \) (which holds for example in a setting with \( U^q = v^q(c^q_1) + (c^q_2/1 + r), U^h = v^h(c^h_1) + (c^h_2/1 + r) \) and \( u^j = u^h(c^j) \) for \( j \in \{q, h\} \) the first term of (32) vanishes and the overall sign of \( dW^U/d\alpha \), i.e., the direction of the welfare change due to the pension reform, depends on the size and direction of the adjustment of the flat-rate benefit relative to the change in the individual Bismarckian benefits, on the educational level in the economy and the distribution of abilities among individuals. A priori, it is by no means clear that the beneficiaries’ gains from the reform would suffice to compensate potential losses. n the other hand, however, with an increase in the flat-rate benefit, i.e., \( db/d\alpha > 0 \), or at least a reduction which is not too large, a marginal reform could also be Pareto-improving.

4. C

In many countries, pension reforms since the 1990s aimed at a reduction of intragenerational redistribution by strengthening the link between individual contributions and benefits. Usually, these reforms are slowly phased in and still need to be fully anticipated by the population. ur results show that it is not clear whether the reforms will be successful in the long run. While we expect that a reduction of work disincentives from (distortive) redistributive taxation will be welfare-enhancing, a negative side-effect of a reform could be that individuals, who are at the brink of taking up, e.g., university studies, will be discouraged to do so. This reduction of educational effort at the margin and the resulting lower average ed-
ucational level of the economy due to a pension reform could dilute economic growth. In an aging society, however, growth is the key to stabilizing social security systems. Policy-makers need to keep these possible effects in mind. The empirically well established life expectancy differential between skilled and unskilled individuals causes this education-disincentive effect, which is driven by a complex combination of (direct) budget effects and (indirect) effects via individuals’ educational decisions.

Especially the possibility of a (higher) education-discouraging effect of a pension reform, which reduces the progressivity of the system, as well as the redistributive consequences of the reform, which change the number of net-payers and net-recipients, has to our best knowledge been ignored in the literature and the policy discussion so far.

With respect to our modelling framework, one should be aware that some underlying assumptions are not entirely innocuous and might have affected our findings. First, we assumed a pension system with a fixed contribution rate. In the real world, there are only few public pension systems which are considered to be of the DC (defined contributions) type. If we assumed the more common DB (defined benefits) or fixed pension benefits system, it would easily be possible to raise \( \alpha \) without lowering the flat-rate benefit \( b \). Obviously, this leads to higher total spending and requires higher contributions to keep the budget balanced. In the past, this additional tax burden has simply been shifted to younger cohorts or yet unborn generations. However, although several countries are still considered to have DB systems, many of them have started to introduce policy measures to keep contribution rates constant in the long run. The reason is that, due to demographic change, contribution rates would increase substantially in the future, inducing strong work disincentives for the young. With increasing factor mobility in a globalized world, hardly any country can afford this development. It is therefore justified to argue based on a fixed contribution rate pension system.

Second, we implicitly assume education to be a zero-one decision in which individuals, when deciding to go for education, earn significantly more than those who remain unskilled. Assuming a continuous education choice would lead to a continuous wage differential between differently skilled persons, which removes the ‘jump’ in Figure 2 that is necessary to have the marginal individual increase her education. However, in most countries we see that educational decisions are discrete and that income differentials between differently skilled persons exist (e.g., when comparing Bachelor’s and Master’s graduates). A similar argument applies when we differentiate individuals according to their (subjective) life expectancy conditional on their educational achievement. In both cases, however, a generalized version of the model which includes educational decisions of the medium and highly educated individuals pension reforms might have positive effects on these groups, too.

Third, in order to keep our analysis simple and to show our main point as clearly as possible, i.e., the possibly education-discouraging effect of seemingly efficiency-enhancing pension reforms, we resorted to a special case of our indifference condition in which both costs and income flows resulting from the educational decisions balance simultaneously. Re-
laxing this assumption requires a rebalancing of cost and income comparisons, which effectively implies that the (critical) ability level of the individual who is indifferent with respect to education changes. This might weaken some of our findings as, e.g., the indirect adjustment effect in the benefit sphere via the induced change in the educational level is no longer unambiguously positive. However, given that this effect is typically more than offset by a negative direct budget effect, the sign of the total effect remains unchanged (only a scenario in which a positive indirect effect dominates would then no longer be possible). Hence, a more general exposition of the model might change some of our findings, but the general intuition about the detrimental effect of pension reforms to educational decisions is still valid. From a policy perspective, any scenario (i.e., real-world parameter specification) in which education is not negatively affected through a pension reform should in fact be welcomed. Our task is to highlight the potential dangers of this policy.

Furthermore, we have argued that various economic parameters determine the consequences of a Bismarckian pension reform with respect to educational attainment, intragenerational redistribution and welfare. The life expectancy differential between skilled and unskilled individuals, the distribution of abilities or rather productivities among individuals and the return to education in terms of labor income were shown to play a role here. Therefore, one and the same reform of a pension system’s benefits scheme in different countries may have completely different consequences, depending on parameter differentials. These insights might serve as a starting point for an empirical testing of our Propositions 1 and 2.

Finally, we want to emphasize the empirical relevance of our results. Interestingly, Krieger and Traub (2011) show that Bismarckian pension reforms aiming at a tighter link between earnings and pension benefits are more often observed in countries with Bismarckian pension systems. The most striking increases of the Bismarckian factor have been observed in countries such as Germany, Italy or Luxembourg; to a lesser degree similar developments can be seen in countries like Austria, Belgium or France. The authors argue that Bismarckian pension reforms are easier to introduce when the implied reform is not ‘too’ fundamental, i.e., when the population is already familiar with the working of this type of pension system. In terms of our model, we not only expect that lifetime pensions of educated individuals decrease in case of a Bismarckian reform, but also that they decrease more than lifetime pensions of less educated individuals. Krieger and Traub (2011) indicate that this is a very likely outcome from past pension reforms as the following two examples show. First, real-world pension system tend to have elements of ‘discretionary’ intragenerational redistribution whose removal automatically increases the Bismarckian factor. In the German pension system, for instance, recognition of times spent on (higher) education was gradually reduced since the 1990s and finally abolished in 2005. This has had a strong negative effect on educated individuals (only) as earnings-free times of education no longer lead to higher pension benefits. Second, several countries (e.g., Finland, Poland, Sweden) have moved from a ‘last/best earnings years’ calculation of pension benefits to a ‘lifetime earnings’ calculation. Given that age-earnings profiles of educated individuals tend to be steeper than those of unskilled individuals, this type of reform has a stronger detrimental effect on the skilled.
APPENDIX

A. T

Equation (6) from the text:

\[ tw \int_0^{h^*_i} (1-h) f(h) \, dh + q \int_0^{h^*_i} f(h) \, dh = \int_0^{h^*_i} P_h f(h) \, dh + P_q \int_0^{h^*_i} f(h) \, dh. \]

The RHS can be written as

\[ b \int_0^{h^*_i} f(h) \, dh + c \alpha \int_0^{h^*_i} (1-h) f(h) \, dh + \left( 1 - \sigma \right) \int_0^{h^*_i} f(h) \, dh \]

or

\[ b \int_0^{h^*_i} f(h) \, dh + \left( 1 - \sigma \right) \int_0^{h^*_i} f(h) \, dh \]

Dividing both sides of the budget constraint (6) by \( z \) then yields

\[ t \hat{w} = b + c \alpha \hat{w}. \]

B. T

\( Z_\alpha \)

The sign of \( Z_\alpha \) is negative if

\[ qw + b \bigg|_{h^* = \text{const.}} < 0. \]

With (14)

\[ qw < \frac{w \int_0^{h^*_i} (1-h) f(h) \, dh + (1-\sigma) q \int_0^{h^*_i} f(h) \, dh}{h^* f(h) \, dh + (1-\sigma) \int_0^{h^*_i} f(h) \, dh} \]

Rearranging and simplifying yields

\[ q \int_0^{h^*_i} f(h) \, dh < \int_0^{h^*_i} (1-h) f(h) dh. \]
This inequality strictly holds if $h^* > 0$, i.e., if there is at least one individual with some ability $h$ who prefers education to remaining unskilled, since $(1 - h^*) = q$.

N

1. Bommier, eroux and ozachmeur (2011) discuss a social planner’s problem who would like to compensate individuals for different life expectancies, given the existence of a pension system.

2. Further topics related to Beveridgean and Bismackian pension systems include migration issues (Krieger, 2003; Rossignol and Taugourdeau, 2006), retirement age (Hougaard Jensen, au and Poutvaara, 2004), unemployment insurance and labor unions (Goerke, 2000).

3. See for example the German Health Report 2006 of the Robert Koch Institute or Schneider and Schneider (2006) who find empirical evidence for Germany that “education is a central determinant of health relevant behavior”.

4. Note that it is nevertheless reasonable to assume that individuals take the effect of changing the pension system into account when deciding about whether or not to go for education. First, the productivity differential always matters; and second, (risk-averse) individuals tend to be well aware of death’s random nature. That is, it appears reasonable to assume that individuals judge the variance of life expectancy higher when it comes to death than when it comes to pensions.

5. This return is equal to the growth rate of the total sum of wages. Without productivity growth this corresponds to Samuelson’s biological interest rate, i.e., the population growth rate.

6. Technically, this may be achieved by setting the level of $g$ appropriately. But note that other levels of $g$ would shift the indifference condition accordingly.

7. Note that due to our assumption of identical generations, $h^*_1 = h^*_2$ holds in every period, except for one: the period in which the reform takes place, since then we expect the educational level to differ between the pre- and the post-reform generation. However, most of the time we can omit the generation superscript and simply write $h^*.

8. Remember that $qw$ is the lowest possible labor income in the economy.

9. The value of consumption in both periods (we assume goods prices to be normalized to one) does not exceed the present value of net income from labor supply and the pension system. Skilled individuals also consider education expenditures in addition to the value of goods consumption.

10. In this very limited partial equilibrium welfare analysis we ignore potential effects of a change in the educational level on wages or on macroeconomic variables such as, e.g., economic growth.

11. ‘DC system’ is the more commonly used term for what technically has to be interpreted as a pension system with fixed contribution rates (cf., e.g., Krieger 2001, 2003, 2008).

12. The seminal contribution in this context is Browning (1975); cf. also Haupt and Peters (1998) or Krieger (2003) for further discussion.

R


En un modelo de dos periodos con heterogeneidad en los agentes se analiza una reforma en las pensiones en la línea de establecer un vínculo más estrecho entre aportaciones y prestaciones (como se observa recientemente en varios países) en un sistema de pensiones con componente Bismarkiano y Beveridgeano. Se muestra cómo un cambio de estas características puede desincentivar la inversión en capital humano en el margen, y ello reduce por tanto el nivel medio educativo en la economía. A diferencia en la esperanza de vida entre individuos formados y no formados proporciona este resultado. Además, se investigan las consecuencias que el sistema de pensiones genera sobre las características de la redistribución intergeneracional en el sentido del número de receptores netos en relación con el de pagadores netos, así como los efectos sobre el bienestar.

Palabras clave: Seguridad Social, educación, esperanza de vida, reforma de pensiones, redistribución.
