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Education Inequality among Different Social Groups

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Abstract

In this paper, we study an education-planning problem by using a mechanism design approach. We consider a model where agents have different abilities in acquiring education and belong to different social groups (for instance, races or genders). Under the information constraint that the abilities of agents are unobservable but group memberships are observable, we derive two sets of education policies derived under *Rawlsian* and *utilitarian* social welfare functions. Our main results show that the utilitarian planner does not discriminate agents by their social group membership, while the Rawlsian planner provides a form of affirmative action policy. We also study second-best optimal education policies in the case of general degrees of inequality aversion. In this case, it is shown that affirmative action is not necessarily supported, and the differences in education levels are determined by the agents' marginal effects of utility to their group's aggregated welfare.

Keywords: Education; Mechanism design; Affirmative action; Different social groups.

JEL Classification: I24, D82.

Introduction

Education inequality is considered as a crucial cause of differences in wage and status, and other social inequalities. Inequalities of education are persistent across different social groups distinguished by, for instance, genders, races or parent's jobs in the forms of gaps in test scores and in school achievements. Some past studies conclude that inequalities in education among social groups arise from differences in their circumstances. For example, Fryer and Levitt (2004) argue that one cause of the observed test score gaps between blacks and whites in the U.S. may be systematically lower quality schools for blacks relative to whites. Filmer (2008)

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insists that, in many countries, inequalities in education persist due to gender stereotyping. In these cases, an individual in a disadvantaged group may achieve a lower level of education than another individual in an advantaged group, even if both individuals exert the same level of effort in acquiring education. On this basis, Roemer (1998), among others, insists that agents should not be held accountable for differences in backgrounds and circumstances, and that social inequalities arising from these differences should be corrected through policy.

Under these situations, in several countries including the U.S., affirmative action is implemented to give children in disadvantaged groups preferential treatments. However, there are active discussions on whether affirmative action should be admitted or not[1]. Supporters of affirmative action argue that it helps to compensate past discrimination against minorities and to achieve more equal opportunity and student diversity. In contrast, its opponents insist that affirmative action leads to forms of reverse discrimination such that less-qualified agents are admitted to enter universities, and results in violation of rights such as the rights to equal consideration.

In this paper, we consider, from a distributional viewpoint, what ethical principle supports affirmative action. Specifically, we show that a form of Rawlsian difference principle supports affirmative action, while a utilitarian principle does not.

We use an extended version of the model introduced by Fleurbaey *et al.* (2002). In our model, agents differ in two respects. First, agents have different abilities relevant to their costs of acquiring education. Second, we assume that each agent belongs to one of two social groups, either A (advantaged) or D (disadvantaged). We consider the situation where *because of socioeconomic and environmental factors*, agents in the socially disadvantaged group tend to pay larger costs of obtaining education than those in the advantaged group[2]. Therefore, we assume that the relative proportion of agents with higher abilities in the disadvantaged group is smaller than in the advantaged group[3].

Moreover, we consider two kinds of benefit from education. First, each agent benefits from his or her own level of education. Examples of this kind of education benefit include the enhancement of human capital and the accumulation of knowledge. Second, each agent also benefits from the average education level of society as a whole as an externality. Examples of this kind of education benefit potentially include a reduction in crime, the development of new technologies and knowledge, and an increase in political awareness. Although there are various forms of education externalities, we introduce education externality in a simple way in which the utility of agents depends on the average education level of society[4]. Note that the impact of education externalities differs across countries. For instance, some researches argue that education externalities are larger in developing countries than in developed countries[5]. Accordingly, in this paper, we analyze how the impact of the education externality affects the distribution of resources on education across different social groups.

We compare two sets of policies derived from utilitarian and Rawlsian social welfare functions. When we employ the utilitarian social welfare function, there is no difference in education policy for the different groups, i.e., the optimal allocation of the level of education and schooling help depends only on the abilities of agents. In contrast, when we adopt the Rawlsian social welfare function, the result is that given the same ability level:

(i) agents in the disadvantaged group achieve a higher education level than those in the advantaged group; and

(ii) agents in the disadvantaged group receive more (less, respectively) schooling help than those in the advantaged group if the impact of the education externality is sufficiently large (small, respectively).

These results imply that a Rawlsian education policy leads to a form of "discrimination" between social groups. In particular, we can interpret result (i) as a form of reverse discrimination in the sense that preferential treatment is given to more disadvantaged agents. This result could be interpreted that the Rawlsian principle supports affirmative action. In the area of political philosophy, there are some arguments on the relationship between affirmative action and Rawls' (1971) theory of justice[6]. Our results provide a new theoretical evidence on the relationship between affirmative action and Rawls' (1971) difference principle. That is, given information asymmetry about the abilities of agents, Rawls' (1971) difference principle would derive an affirmative action policy for education in the sense that agents in the disadvantaged group achieve higher levels of education than those in the advantaged group.

Our main results are based on the two polar social welfare functions, utilitarian and Rawlsian. It would be, however, important to analyze education policies derived from a social welfare function that is a compromise between the two polar functions. Hence, we study a second-best policy derived from a social welfare function with an inequality aversion between utilitarian and Rawlsian[7]. In this case, for simplicity, we focus on education level and income redistribution. It is shown that, for the highest ability agents, the first best allocations are achieved and there is no difference in education levels between the two groups. In contrast, for the lowest ability agents, the difference in education levels between the two groups is determined by the difference in the marginal effects of the lowest welfare in each group: If the influence of most disadvantaged agents (with the lowest ability) in a group on the group's welfare is higher than that in the other group, then it is socially second-best optimal to let those disadvantaged agents in the former group achieve higher education than those in the latter group. This result shows that there may be cases where agents in the advantaged group achieve higher education levels than agents in the disadvantaged group. Hence, affirmative action is not necessarily supported by the social welfare functions with intermediate degrees of inequality aversion.

We introduce the related literature. There are many studies concerning the distribution of public expenditure on education. In a seminal work, Arrow (1971) considers the utilitarian approach to the problem of distributing public spending. Ulph (1977) and Hare and Ulph (1979) analyze situations where both education and income redistribution policies are simultaneously performed, using the model of asymmetric information where agents' talents are unobservable. In a similar setting, Fleurbaey *et al.* (2002) investigate the distribution of public spending on education (schooling help) and income transfers where the planner has a social welfare function with constant elasticity of substitution (CES)-type functions displaying various degrees of inequality aversion.

Several existing studies also consider externalities in education. For example, Green and Sheshinski (1975) extended Arrow's (1971) model to the case where all agents benefit from the average level of education as well as their own level of education. They showed that if an education externality exists, the utilitarian social planner shifts public expenditures to high-ability individuals that can use these resources most efficiently. In other work, De Fraja (2002) considered a similar type of education externality as Green and Sheshinski (1975) in constructing a mechanism for optimal education policies where incomes and the unobservable talents of children differ across households. In addition, De Fraja (2002) employed a utilitarian social welfare function and his model allowed for a private education sector.

Our main contribution to the literature is that we study education differences between different social groups at second-best optimal policy. Especially, affirmative action would be an important issue in many societies in this context. Our results show a normative criterion under which affirmative action could be supported, using a model with different distributions of abilities for different social groups: Among the CES class of social welfare functions, only the Rawlsian social welfare function always supports affirmative action at second-best optimum. Moreover, we show that under the Rawsian criterion, the magnitude of education externality determines which social group receives more schooling help.

The structure of the remainder of the paper is as follows. Section 2 introduces the model. In Section 3, we analyze the case where agents' talents are observable as a benchmark. In Section 4, we examine the distribution problem under incomplete information. In Section 5, we study second-best optimal education policies in the case of general degrees of inequality aversion. Section 6, provides some concluding remarks.

1. Model

We consider an economy where the whole population is divided into two groups, A and D.

For each $j = A, D$, abilities are distributed according to the distribution function $F_j : [\underline{\theta}, \overline{\theta}] \to [0,1]$, with its density function f_j such that $f_j(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$. We assume that, for $j = A, D$, f_j / F_j is decreasing in θ. This is a standard assumption in the mechanism design literature. Let p_j be the proportion of group j and $p_A + p_D = 1$.

In the rest of the paper, we refer to an agent with ability $\theta \in [\underline{\theta}, \overline{\theta}]$ and $j \in \{A, D\}$ as type θj.

We assume that group A is more advantaged than group D in the following way: the proportion of agents with higher abilities in group A is larger than in group D in the sense of reverse hazard rate dominance. That is, for all $\theta \in (\underline{\theta}, \overline{\theta})$:

$$\frac{f_D(\theta)}{F_D(\theta)} > \frac{f_A(\theta)}{F_A(\theta)}$$

As discussed in Introduction, the agent's ability θ is considered as reflecting family background: Lower ability agents are handicapped by their weaker family backgrounds. We also assume that agents in the socially disadvantaged group D tend to pay larger costs of obtaining education (introduced later) than those in the advantaged group A[8]. Each agent chooses an effort level $y \geq 0$ to acquire education. Schooling help, s, is an in-kind transfer provided by the government? We assume that an education production function g gives the educational achievement of an individual as an increasing function of effort and help. Let $e \quad g(y,s)$ be the education level (e.g. years of schooling) when exerting effort y and receiving in-kind help s (e.g. scholarship for tuition fee, or donation of books or stationeries).

Each agent's gain from education is given by $B(e,E)$, where e is the agent's own education level, E is the average education level of society. For simplicity, we assume $B(e,E) \quad e+ aE$, where $a > 0$ represents the magnitude of the education externality:

$$E \equiv \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} e_j(\theta) f_j(\theta) d\theta,$$

where $e_j(\theta)$ is type θj's education level. Note that each agent benefits not only from his or her own education level, but also from the social education level as an externality.

The total effort expenditure of an agent with ability θ is θy. The minimal amount of effort needed to achieve the education level while receiving s is denoted. By definition, $e \equiv g\left[C(e,s),s\right]$. Following Fleurbaey *et al.* (2002), we make assumptions directly on the cost function C and its derivatives.

Assumption. The mapping C is twice continuously differentiable, with partial derivatives satisfying:

(1) $Ce > 0$, $Cs < 0$, $Ces < 0$;
(2) $Ce \to 0$ as $e \to 0$ for all s; $Cs \to 0$ as $s \to +\infty$, and $Cs \to -\infty$ as $s \to 0$ for all e;
(3) C is strictly convex;
(4) $Cee > |Ces|$ and $Css > |Ces|$.

Together, Assumptions (1)–(4) are almost the same as Assumption 1 in Fleurbaey *et al.* (2002, p. 121). Assumption (1) implies that additional education increases cost, additional help decreases cost, and additional help decreases the marginal cost of education. Assumption (2) is a condition to focus on the interior solution to ensure the simplicity of the analysis. Assumption (3) states that the returns to scale in the production of an individual level of education are strictly decreasing. Assumption (4) means that the cross second-order derivative Ces is sufficiently small in absolute terms. This condition holds if education and schooling help are close substitutes.

Each agent receives a monetary transfer $t \in R$ from the public sector. t represents a scheme of income tax and support. The difference between t and s is that t is related to income redistribution when agents work after their education is finished, while s is an in-kind support when agents are under education.

We assume that θ 's utility is given as

$$u_j(\theta) \quad t_j(\theta) + e_j(\theta) + aE - \theta C\left(e_j(\theta), s_j(\theta)\right),$$

where $t_j(\theta)$ and $s_j(\theta)$ are type θj's transfer and schooling help, respectively. We introduce the two social welfare functions (SWF hereafter). The *utilitarian* SWF is given as:

$$\sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} u_j(\theta)\, f_j(\theta)\, d\theta,$$

On the other hand, the *Rawlsian* SWF is defined as:

$$\min_{j \quad A,D} \min_{\theta \in [\underline{\theta}, \bar{\theta}]} \left\{u_j(\theta)\right\}$$

In the following sections, we compare the two kinds of education policies derived from these alternative SWFs.

2. The First-Best Problem

As a benchmark, in this section we consider the case where agents' abilities are observable. We then compare the first-best allocations with respect to the utilitarian and Rawlsian social welfare functions. The problems are:

$$\max_{e,s,t} \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} u_j(\theta)\, f_j(\theta)\, d\theta, \; \min_{j \quad A,D} \min_{\theta \in [\underline{\theta}, \bar{\theta}]} \left\{u_j(\theta)\right\}$$

subject to the balanced budget constraint (*BB*):

$$\sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(t_j(\theta) + s_j(\theta)\right) f_j(\theta)\, d\theta \quad M.$$

where $M > 0$ is an exogenously given amount of money.

Let (e^R , s^R) and (e^U , s^U) be the first-best solutions of the Rawlsian and utilitarian SWFs, respectively. The optimality conditions are:

$$1+a \;\; = \theta Ce(e^k(\theta), s^k(\theta)), \tag{3}$$

$$-1 = \theta Cs(e^k(\theta), s^k(\theta)), \tag{4}$$

$$\sum_{j \;\; A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(t_j(\theta)+s_j(\theta)\right) f_j(\theta)\, d\theta \quad M, \tag{5}$$

where $k = U, R$, and $j \quad A, D$. Notice that in the case of the Rawlsian SWF, all types achieve the same utility level:

$$u_A(\theta) = u_D(\theta) = u_A(\theta') = u_D(\theta') \; \forall\theta, \theta' \in [\underline{\theta}, \; \bar{\theta} \;].$$

Given the optimality conditions (3) and (4) are identical for both SWFs, the first-best solutions are the same. Therefore, we can put (e^C , s^C) ≡ (e^R , s^R) = (e^U , s^U).

By (3) and (4), the solutions are independent of $j \quad A, D$. Let $(e^C(\theta,a),\; s^C(\theta,a))$ be the solution mappings of the above problem. We obtain the following result[9].

Proposition 1. Suppose that Assumptions (1)-(4) hold.

(i) $e^C(\theta,a)$ is decreasing in θ.

(ii) $s^C(\theta,a)$ is increasing (nonincreasing, respectively) in θ if $a < -(Cee+Ces)/Ces$ ($a \geq -(Cee+Ces)/Ces$, respectively).

Proof. Differentiating (3) and (4) by θ and solving the system of the equations, we obtain

$$e^C_\theta(\theta,a) \quad \frac{-1}{d(C'')}\left[(1+ a)C_{ss}+C_{se}\right)\frac{1}{\theta^2}. \tag{6}$$

$$s^C_\theta(\theta,a) \quad \frac{1}{d(C'')}\left[(1+ a)C_{es}+C_{ee}\right)\frac{1}{\theta^2}. \tag{7}$$

where $d(C'') = C_{ee}C_{ss} - (C_{ee})^2 > 0$. From equation (6) and Assumption (4), $e^C_\theta(\theta,a) < 0$. Moreover, from equation (7) and $C_{es} < 0$, $s^C_\theta(\theta,a) \le 0$ if $(1+a)C_{es}+C_{ee} \le 0$.

Proposition 1 states that (i) the higher the ability of an agent, the higher the level of education he/she achieves; and (ii) if the impact of the externality a is sufficiently large (small, respectively), agents with higher abilities receive higher (lower, respectively) levels of help.

An intuition behind the result (ii) is that if the impact of the externality is sufficiently large, then the planner has an incentive to reduce the costs of high-ability agents and so allow them to acquire more education to increase the externality.

Next, note that the left-hand sides of equations (3) and (4) are common for the two social groups. Thus, we obtain the following result.

Proposition 2. Under complete information, the allocation of education and help is the same for the two groups A and D.

Therefore, when the planner can observe the agents' abilities, there is *no discrimination* with respect to education and schooling between the two social groups.

3. The Second-Best Problem

In this section, we analyze the case where the planner cannot observe the agents' abilities. We introduce *Incentive Compatibility* (*IC*). We assume that each agent cannot affect the level of externality, since the population is infinitely large and hence his/her influence is negligible.

(IC) For all θ, θ' $\in$ [$\underline{\theta}$, $\bar{\theta}$] and all $j \quad A, D$,

$$u_j(\theta) \geq t_j(\theta') + e_j(\theta') + aE - \theta C\big(e_j(\theta'), s_j(\theta')\big)$$

This constraint means that any agent has no incentive to misreport his or her ability. Note that agents cannot lie about their group as the planner can observe each agent's group membership. Our problem is then to maximize the two social welfare functions subject to (*BB*) and (*IC*).

We derive the following condition (IC_f) from (IC)[10].

(IC_f) For each $\theta \in$ [$\underline{\theta}$, $\bar{\theta}$] and each j,

$$u_j(\theta) \quad u_j(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} C\big(e_j(x), s_j(x)\big)\, d\theta.$$

(IC) implies the following first order condition: For each $\theta \in$ [$\underline{\theta}$, $\bar{\theta}$] and each j,
(IC ') $t_j'(\theta) + e_j'(\theta) - \theta\Big[C_e\big(e_j(\theta), s_j(\theta)\big)e_j'(\theta) + C_s\big(e_j(\theta), s_j(\theta)\big)s_j'(\theta)\Big] = 0.$

This equation implies

$$t_j{'}(\theta) \quad -e_j{'}(\theta) + \theta\left[C_e\left(e_j(\theta),s_j(\theta)\right)e_j{'}(\theta)+C_s\left(e_j(\theta),s_j(\theta)\right)s_j{'}(\theta)\right].$$

By integrating this equation, we have

$$t_j\left(\overline{\theta}\right)-t_j(\theta) \quad [-e_j{'}(x)]_{\theta}^{\overline{\theta}}+\int_{\theta}^{\overline{\theta}}x\left[C_e\left(e_j(x),s_j(x)\right)e_j{'}(x)+C_s\left(e_j(x),s_j(x)\right)s_j{'}(x)\right]d.$$

Then, given $u_j(\theta) \quad t_j(\theta) + e_j(\theta)+aE-\theta C\left(e_j(\theta),s_j(\theta)\right)$, we obtain:

$$u_j(\theta) \quad u_j\left(\overline{\theta}\right) + \int_{\theta}^{\overline{\theta}}C\left(e_j(x),s_j(x)\right)d\theta.$$

Next, we discuss the second-order condition of (*IC*). Differentiating

$$t_j(\theta')+e_j(\theta')+aE-\theta C\left(e_j(\theta'),s_j(\theta')\right)$$

twice with respect to θ' (with evaluating $\theta'=\theta$), we have the second-order condition as follows:

$$t_j{''}(\theta)+e_j{''}(\theta)-\theta\frac{d}{d\theta}\left[C_e\left(e_j(\theta),s_j(\theta)\right)e_j{'}(\theta)+C_s\left(e_j(\theta),s_j(\theta)\right)s_j{'}(\theta)\right]\leq 0.$$

Note that (*IC*'') holds for all θ. Then, differentiating (*IC*'') by θ,

$$t_j{''}(\theta)+e_j{''}(\theta)-\theta\frac{d}{d\theta}\left[C_e\left(e_j(\theta),s_j(\theta)\right)e_j{'}(\theta)+C_s\left(e_j(\theta),s_j(\theta)\right)s_j{'}(\theta)\right]$$

$$\left[C_e\left(e_j(\theta),s_j(\theta)\right)e_j{'}(\theta) + C_s\left(e_j(\theta),s_j(\theta)\right)s_j{'}(\theta)\right]$$

By the two conditions above, we obtain the following condition.

(IC_s): For all *j* and all $\theta \in [\underline{\theta},\overline{\theta}]$,

$$\left[C_e\left(e_j(\theta),s_j(\theta)\right)e_j{'}(\theta)+C_s\left(e_j(\theta),s_j(\theta)\right)s_j{'}(\theta)\right]\leq 0.$$

We also rewrite (BB). From

$$\sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(t_j(\theta)+s_j(\theta)\right) f_j(\theta)\, d\theta \quad M,$$

and

$$u_j(\theta) \quad t_j(\theta) + \; e_j(\theta) + aE - \theta C\left(e_j(\theta), s_j(\theta)\right)$$

we obtain (*BB'*)

$$\sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(s_j(\theta) - (1+a)e_j(\theta) + \theta C\left(e_j(\theta), s_j(\theta)\right) + u_j(\theta)\right) f_j(\theta)\, d\theta \quad M.$$

In the remainder of our analysis, we solve the utilitarian and the Rawlsian second-best problems under the new constraints (*BB'*), (IC_f) and (*ICs*), instead of (*BB*) and (*IC*). In the next two subsections, we solve the problems by ignoring (*ICs*).

3.1 The Utilitarian Solution

Now we analyze the second-best utilitarian education policies. We solve

$$\max_{e,s,u} \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} u_j(\theta) f_j(\theta)\, d\theta,$$

subject to (*BB'*) and (IC_f).

To solve the problem, we rewrite (*BB'*) and (IC_f) as follows. For each $j \quad A,D$, define

$$p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(s_j(\theta) - (1+a)e_j(\theta) + \theta C\left(e_j(\theta), s_j(\theta)\right) + u_j(\theta)\right) f_j(\theta)\, d\theta \quad M_j,$$

$$m_j'(\theta) \quad -p_j\left(s_j(\theta) - (1+ \; a)e_j(\theta) + \theta C\left(e_j(\theta), s_j(\theta)\right) + u_j(\theta)\right) f_j(\theta),$$

$$m_j(\underline{\theta}) \quad 0, m_j(\bar{\theta}) \quad -M_j,$$

and

$$u_j'(\theta) \quad -C\big(e_j(\theta), s_j(\theta)\big).$$

Then, the utilitarian planner's optimization problem becomes a standard optimal control problem[11]. We introduce four costate variables, denoted λ_A, λ_D, μ_A and μ_D, associated with the state variables m_A, m_D, u_A, and u_D, respectively. The Lagrangian of the problem is defined as:

$$\mathcal{L} \quad \sum_{j \quad A,D} \Big(p_j u_j f_j - p_j \lambda_j \big(s_j - (1+ a) e_j + \theta C\big(e_j, s_j\big)\big) - \mu_j C\big(e_j, s_j\big)\Big).$$

Applying Pontryagin's maximum principle, we obtain the following conditions:

$$\frac{\partial \mathcal{L}}{\partial e_j} \quad -p_j \lambda_j \big(-(1+ a) + \theta C_e\big) f_j - \mu_j C_e \quad 0,$$

$$\frac{\partial \mathcal{L}}{\partial s_j} \quad -p_j \lambda_j \big(1+ \theta C_s\big) f_j - \mu_j C_s \quad 0,$$

$$\frac{\partial \mathcal{L}}{\partial u_j} \quad p_j \ (1-\lambda_j) f_j \quad \mu_j', \ \mu_j\big(\underline{\theta}\big) \quad \mu_j\big(\overline{\theta}\big) \quad 0,$$

$$\frac{\partial \mathcal{L}}{\partial u_j} \quad 0 \quad -\lambda_j'.$$

From these equations, we obtain

$$\frac{1+a}{\theta} \quad C_e\big(e_j^u(\theta), s_j^u(\theta)\big) \tag{8}$$

$$\frac{-1}{\theta} \quad C_s\big(e_j^u(\theta), s_j^u(\theta)\big) \tag{9}$$

where (e^u, s^u) are the second-best utilitarian education policies.

It is obvious that equations (8) and (9) are the same as the optimality conditions under complete information, i.e., equations (3) and (4). Therefore, $(e^u, s^u) \quad (e^C, s^C)$. Then, given that the first-best allocation of education and help (e^C, s^C) does not distinguish agents by their group membership, the same result holds for the allocation of the utilitarian education policies under incomplete information. We summarize the result as follows.

Proposition 3. Suppose that Assumptions (1)–(4) hold. Under incomplete information, the utilitarian education policies do not discriminate between the different social groups in the following sense: For all $\theta \in [\underline{\theta}, \ \overline{\theta}\ ]$,

$$\left(e_A^u(\theta), s_A^u(\theta)\right) \quad \left(e_D^u(\theta), s_D^u(\theta)\right).$$

Thus, when the planner adopts the utilitarian SWF, agents with the same ability level receive the same level of education and help, regardless of their group membership.

As $(e^u, s^u) \quad (e^C, s^C)$, we can see that (IC) does not bind. Hence, any utility levels can be achieved as long as (*IC*) and (*BB*′) are satisfied.

3.2 The Rawlsian Solution

We consider the second-best Rawlsian education policies. The maximization problem is:

$$\max_{e,s,u} \min_{j \quad A,D} \min_{\theta \in [\underline{\theta}, \overline{\theta}]} \left\{u_j(\theta)\right\}$$

subject to (*BB*′) and (IC_f). From (*BB*′) and (IC_f), we obtain:

$$M = \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} \left(s_j(\theta) - (1+a)e_j(\theta) + \theta C\left(e_j(\theta), s_j(\theta)\right) + u_j(\theta)\right) f_j(\theta) d\theta$$

$$= \sum_{j=A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} \left(s_j(\theta) - (1+a)e_j(\theta) + \theta C\left(e_j(\theta), s_j(\theta)\right) + u_j\left(\overline{\theta}\right) + \int_{\theta}^{\overline{\theta}} C\left(e_j(x), s_j(x)\right) dx \right) f_j(\theta) d\theta$$

Then, computing the double integral

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} C\left(e_j(x), s_j(x)\right) dx\, f_j(\theta) d\theta,$$

we have (*BB”*)

$$M \quad \sum_{j \quad A,D} p_j \left[\int_{\underline{\theta}}^{\overline{\theta}} \left(s_j(\theta) - (1+ \ a)e_j(\theta) + \left(\theta + \frac{F_j(\theta)}{f_j(\theta)}\right) C\left(e_j(\theta), s_j(\theta)\right) \right) f_j(\theta) d\theta + u_j\left(\overline{\theta}\right) \right]$$

Since the group memberships are observable, it is obvious that $u_A\left(\overline{\theta}\right) \quad u_D\left(\overline{\theta}\right)$. From this observation, we can put the lowest utility level as $u\left(\overline{\theta}\right)$. Then, (*BB"*) can be rewritten as:

$$u\left(\overline{\theta}\right) \quad M - \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\overline{\theta}} \left(s_j(\theta) - (1 + \ a) e_j(\theta) + \left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C\left(e_j(\theta), s_j(\theta)\right) \right) f_j(\theta) d\theta$$

Then the second-best Rawlsian solution can be obtained by maximizing the right hand side of the above equation. Let be $(e^r, \ s^r)$ he second-best allocation concerning education. The first-order conditions are as follows. For *j=A, D,*

$$\frac{1+a}{\theta + \dfrac{F_j(\theta)}{f_j(\theta)}} \quad C_e\left(e_j^{\ r}(\theta), s_j^{\ r}(\theta)\right) \tag{10}$$

$$\frac{-1}{\theta + \dfrac{F_j(\theta)}{f_j(\theta)}} \quad C_s\left(e_j^{\ r}(\theta), s_j^{\ r}(\theta)\right) \tag{11}$$

Compared with equations (3) and (4), equations (10) and (11) show that the agent with ability θ is treated as if his or her ability were $\theta + \dfrac{F_j(\theta)}{f_j(\theta)} > \theta$. That is, for all *j = A, D*, and all θ,

$$e_j^{\ r}(\theta, a) \quad e_j^{\ C}\left(\theta + \frac{F_j(\theta)}{f_j(\theta)}, a\right), s_j^{\ r}(\theta, a) \quad s_j^{\ C}\left(\theta + \frac{F_j(\theta)}{f_j(\theta)}, a\right).$$

The optimality conditions (10) and (11) imply that the education and help levels of all types except $\overline{\theta} A$ and $\overline{\theta} D$ are distorted. In contrast, as $F_j\left(\underline{\theta}\right) \quad 0$ for each *j*, the education and help levels of agents with the highest ability are at the first-best levels.

From Proposition 1, it is obvious that, for all θ and *j*,

(i) $e_j^{\ r}(\theta, a) > e_j^{\ C}(\theta, a)$; and

(ii) $s_j^{\ r}(\theta, a) > (\leq, \ \text{respectively}) \ s_j^{\ C}(\theta, a)$ if *a* is sufficiently small (large, respectively).

Moreover, if F_j / f_j is increasing in θ, then $e_j^{\ r}$ is decreasing in θ and $s_j^{\ r}$ is increasing (nonincreasing, respectively) in θ when a is sufficiently small (large, respectively).

Now we consider the differences in the levels of education and schooling help for the two groups. Remember the assumption that for all θ ,

$$\frac{f_D(\theta)}{F_D(\theta)} > \frac{f_A(\theta)}{F_A(\theta)}.$$

By this assumption and Proposition 1-(i):

$$e_D{}^r(\theta,a) \quad e^C\left(\theta + \frac{F_D(\theta)}{f_D(\theta)}, a\right) > e^C\left(\theta + \frac{F_A(\theta)}{f_A(\theta)}, a\right) \quad e_A{}^r(\theta,a).$$

Similarly, by Proposition 1-(ii):

$$s_D{}^r(\theta,a) \quad s^C\left(\theta + \frac{F_D(\theta)}{f_D(\theta)}, a\right) < (\geq, respectively)\, s^C\left(\theta + \frac{F_A(\theta)}{f_A(\theta)}, a\right) \quad s_A{}^r(\theta,a),$$

if $a < -(Cee + Ces)/Ces$ ($a \geq -(Cee + Ces)/Ces$, *respectively*).

In sum, we have obtained the following result.

Proposition 4. Suppose that Assumptions (1)–(4) hold. Then, the second-best Rawlsian policies have the following properties:

(i) $e_D^r(\underline{\theta}) \quad e_A^r(\underline{\theta})$, $s_D^r(\underline{\theta}) \quad s_A^r(\underline{\theta})$.

(ii) For all $\theta \in [\underline{\theta}, \overline{\theta}\,]$ $e_D^r(\theta) > e_A^r(\theta)$.

(iii) For all $\theta \in [\underline{\theta}, \overline{\theta}\,]$ $s_D^r(\theta) < (\geq, \text{respectively})\, s_A^r(\theta)$ if $a < -(Cee + Ces)/Ces$ ($a \geq -(Cee + Ces)/Ces$, respectively).

(i) implies that agents with the highest ability across the two groups are not discriminated against in education. That is, they obtain the same levels of education and help.

(ii) means that all agents except the highest-ability agents in the disadvantaged group achieve a higher education level than agents in the advantaged group. This could be interpreted as a form of reverse discrimination in that the more disadvantaged group is given more preferential treatment than the advantaged group.

(iii) suggests that all agents except the highest-ability agents in the disadvantaged group receive higher (lower, respectively) levels of schooling help than agents in the advantaged group whenever the degree of externality is sufficiently small (large, respectively). The magnitude of the externality affects the difference in the level of schooling help across the two groups.

Next, we have the following corollary to Proposition 4.

Corollary. Let $u_j^r(\theta)$ the utility level of type θj under the Rawlsian second-best solution. Then, we have the following properties:

(i) $u_D^r(\bar{\theta})$ $u_A^r(\bar{\theta})$.

(ii) If a $<-(Cee+Ces)/Ces$ $u_D^r(\theta)>u_A^r(\theta)$ for all $\theta\in[\underline{\theta},\bar{\theta})$.

(iii) If a $\geq-(Cee+Ces)/Ces$, $u_D^r(\theta)<u_A^r(\theta)$ for all $\theta\in[\underline{\theta},\bar{\theta})$.

Proof : (i) As shown above, it is obvious that

(ii) If a $<-(Cee+Ces)/Ces$, then by Assumption (1) and Proposition 4, for all $\theta\in[\theta,\bar{\theta})$,

$$
\begin{aligned}
&C\left(e_A{}^r(\theta),s_A{}^r(\theta)\right)-C\left(e_D{}^r(\theta),s_D{}^r(\theta)\right)\\
&<C\left(e_D{}^r(\theta),s_A{}^r(\theta)\right)-C\left(e_D{}^r(\theta),s_D{}^r(\theta)\right)\\
&\qquad\qquad\qquad\qquad <\\
&C\left(e_D{}^r(\theta),s_D{}^r(\theta)\right)-C\left(e_D{}^r(\theta),s_D{}^r(\theta)\right)\quad 0.
\end{aligned}
$$

Then, from (IC_f) and (i), for $\theta\in[\underline{\theta},\bar{\theta})$,

$$
u_A^r(\bar{\theta})-u_D^r(\bar{\theta})\quad \int_{\theta}^{\bar{\theta}}\left[C\left(e_A{}^r(\theta),s_A{}^r(\theta)\right)-C\left(e_D{}^r(\theta),s_D{}^r(\theta)\right)\right]dx<\ 0.
$$

Hence, we obtain $u_D^r(\theta)<u_A^r(\theta)$.

(iii) If a $\geq-(Cee+Ces)/Ces$, then by Assumption (1) and Proposition 4, for all $\theta\in[\underline{\theta},\bar{\theta})$,

$$
\begin{aligned}
&C(e_A{}^r(\theta),s_A{}^r(\theta))-C(e_D{}^r(\theta),s_D{}^r(\theta))>C(e_A{}^r(\theta),s_A{}^r(\theta))-C(e_A{}^r(\theta),s_D{}^r(\theta))\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\geq\\
&C(e_A{}^r(\theta),s_D{}^r(\theta))-C(e_D{}^r(\theta),?\,s_D{}^r(\theta))\quad 0.
\end{aligned}
$$

Then, from (IC_f) and (i), for $\theta \in \left[\theta, \bar{\theta}\right)$,

$$u_A^r\left(\bar{\theta}\right) - u_D^r\left(\bar{\theta}\right) \quad \int_{\theta}^{\bar{\theta}} \left[C\left(e_A{}^r(\theta), s_A{}^r(\theta)\right) - C\left(e_D{}^r(\theta), s_D{}^r(\theta)\right)\right] dx > 0.$$

Hence, we obtain $u_D^r(\theta) > u_A^r(\theta)$.

The corollary means that the differences in utility levels between the social groups depend on the impact of externality. If is sufficiently large (small, respectively), the planner gives larger (smaller, respectively) schooling help to type θD than to type θA .

Then, type θD obtains whigher (lower, respectively) information rent than type θA .

4. A General Case

In the previous section, the second-best policies regarding the two polar cases, the Rawlsian and utilitarian, are studied. In this section, we examine the second-best education policies derived by a more general social welfare function:

$$\sigma^{-1} \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} u_j(\theta)^{\sigma} f_j(\theta) d\theta,$$

were σ represents the degree of inequality aversion. The utilitarian corresponds to $\sigma \quad 1$ while the Rawlsian corresponds to $\sigma \quad -\infty$. Thus, the social welfare function considered in this section is a compromise between the two polar cases.

As shown by Fleurbaey *et al.* (2002), it is quite difficult to solve the problem with three variables in this setting. Therefore, we simplify the model and study the problem without schooling help s or externality.

We slightly modify the constraints (IC_f), (IC_s), and (*BB"*). It is straightforward to check that in the present environment, these constraints are rewritten as follows.

(IC_f): For all *j* and $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$,

$$u_j(\theta) \quad u_j\left(\bar{\theta}\right) + \int_{\theta}^{\bar{\theta}} C\left(e_j(x)\right) d\theta.$$

(IC_s): For all *j* and $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$, $e_j{}'(\theta) \leq 0.$

(BB”):

$$M \quad \sum_{j \quad A,D} p_j \left[\int_{\underline{\theta}}^{\bar{\theta}} \left(\left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C\left(e_j(\theta)\right) - e_j(\theta) \right) f_j(\theta) d\theta + u_j\left(\bar{\theta}\right) \right].$$

Then, our problem is as follows.

$$\max_{e,\, u} \sigma^{-1} \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} u_j(\theta)^{\sigma} f_j(\theta) d\theta,$$

Subject to (*BB”*), (*IC$_f$*) and (*IC$_s$*).

We introduce the following variables and parameters. For each *j*, define

$$B_j \quad u_j\left(\bar{\theta}\right), r_j(\theta) \quad \int_{\theta}^{\bar{\theta}} C\left(e_j(x)\right) d\theta,$$

$$k'(\theta) \quad \sum_{j \quad A,D} p_j \left(e_j(\theta) - \left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C\left(e_j(\theta)\right) \right) f_j(\theta),$$

$$k(\underline{\theta}) \quad 0, k\left(\bar{\theta}\right) \quad \left(p_A B_A + p_D B_D\right) - M,$$

Then, using these notations, our problem can be rewritten as follows.

$$\max_{\left(B_j, y_j\right): j \quad A,D} \sigma^{-1} \sum_{j \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(B_j + r_j(\theta)\right)^{\sigma} f_j(\theta) d\theta,$$

subject to, for each *j*,

$$r_j'(\theta) \quad -C\left(e_j(\theta)\right),\ r_j\left(\bar{\theta}\right) \quad 0,\ r_j(\underline{\theta}) \text{ is free},$$

$$k'(\theta) \quad p_j \left(e_j(\theta) - \left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C\left(e_j(\theta)\right) \right) f_j(\theta),$$

$$y_j(\theta) \quad e_j{}'(\theta) \le 0,$$

$$k(\underline{\theta}) \quad 0, k\left(\bar{\theta}\right) \quad \left(p_A B_A + p_D B_D\right) - M.$$

This problem has five state variables, e_j, r_j, for each j, and k , and two control variables y_j for each j, and two control parameters B_j for each j. We have four costate variables, denoted π_{1j} π_2 π_{3j} for each j associated with the state variables e_j, k, r_j, respectively. π_{4j} is a Lagrange multiplier corresponding to the constraint $y_j \leq 0$.

The Lagrangean of the problem is defined as

$$\Lambda(\theta) \equiv \sum_{j=A,D} \Big[\sigma^{-1} p_j \left(B_j + r_j(\theta)\right)^{\sigma} f_j(\theta) - \pi_{1j}(\theta) C\left(e_j(\theta)\right)$$

$$+\pi_2(\theta) p_j \left(e_j(\theta) - \left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C\left(e_j(\theta)\right) \right) f_j(\theta) + \left(\pi_{3j}(\theta) - \pi_{4j}(\theta)\right) y_j(\theta) \Big].$$

The optimality conditions are obtained as follows (see Leonard and Van Long, 1995, Theorems 6.5.1 and 7.11.1). For each j,

(i) $\frac{\partial \Lambda}{\partial y_j}(\theta) \quad \pi_{3j}(\theta) - \pi_{4j}(\theta) \quad 0,$

(ii) $\frac{\partial \Lambda}{\partial r_j}(\theta) = p_j \left(B_j + r_j(\theta)\right)^{\sigma-1} f_j(\theta) = -\pi'_{1j}(\theta),$

(iii) $\frac{\partial \Lambda}{\partial k_j}(\theta) = 0 = -\pi'_2(\theta),$

(iv) $\frac{\partial \Lambda}{\partial e_j}(\theta) = -\pi_{1j}(\theta) C'\left(e_j(\theta)\right) + \pi_2(\theta) p_j \left(1 - \left(\theta + \frac{F_j(\theta)}{f_j(\theta)} \right) C'\left(e_j(\theta)\right) \right) f_j(\theta) =$

$= \pi'_{3j}(\theta),$

(v) $\pi_{4j}(\theta) y_j(\theta) = 0.$

From (iii), π_2 is constant with respect to θ. (ii) implies $\pi_{1j}(\underline{\theta}) \quad 0$ and

$$\pi_{1j}(\theta) = p_j \int_{\underline{\theta}}^{\theta} \left(B_j + r_j(x)\right)^{\sigma-1} f_j(\theta)\, dx.$$

Moreover, it can be shown (from Leonard and Van Long, 1995, Theorem 7.11.1) that

$$\sum_{j\quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(B_j + y_j(\theta)\right)^{\sigma-1} f_j(\theta)\, d\theta \quad \pi_2,$$

which means that π_2 is described as the total of the marginal social values of increasing B_A and B_D. Note that $\pi_2 = -\left(\pi_{1A}(\bar{\theta}) + \pi_{1D}(\bar{\theta})\right)$. By (i), we have $\pi_{3j}(\theta) \quad \pi_{4j}(\theta)$, which implies $\pi'_{3j}(\theta) \quad \pi'_{4j}(\theta)$. From these observations and (iv), we obtain

$$C'\left(e_j(\theta)\right) \quad \frac{1 + \dfrac{\pi'_{3j}(\theta)}{p_j f_j(\theta)\pi_2}}{\theta + \dfrac{F_j(\theta)}{f_j(\theta)} + \dfrac{\pi_{1j}(\theta)}{p_j f_j(\theta)\pi_2}} \tag{12}$$

In this general problem, it is difficult to find education differences between the two groups A and D. Hence, we here focus on the case of $y_j(\theta) \quad e_j'(\theta) < 0$. In this case, $\pi_{3j}(\theta) \quad \pi'_{3j}(\theta) \quad 0$ from (v), and equation (12) becomes

$$C'\left(e_j(\theta)\right) \quad \left[\theta + \frac{F_j(\theta)}{f_j(\theta)} + \frac{\pi_{1j}(\theta)}{p_j f_j(\theta)\pi_2}\right]^{-1} \tag{13}$$

In this case, $e_A(\theta) < e_D(\theta)$ if and only if

$$\frac{F_A(\theta)}{f_A(\theta)} + \frac{\pi_{1A}(\theta)}{p_A f_A(\theta)\pi_2} > \frac{F_D(\theta)}{f_D(\theta)} + \frac{\pi_{1D}(\theta)}{p_D f_D(\theta)\pi_2}.$$

In this general case at $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$, it is difficult to know which side of the inequality is larger.

We now focus on the two polar abilities, $\underline{\theta}$ and $\bar{\theta}$. At $\underline{\theta}$, equation (13) becomes $C'\left(e_j(\underline{\theta})\right) \quad 1/\underline{\theta}$, since $F_j(\underline{\theta}) \quad 0$ and $\pi_{1j}(\underline{\theta}) \quad 0$. This means that for the highest ability agents, the first-best education level is achieved and there is no education difference between the two groups.

At $\bar{\theta}$ equation (13) becomes

$$C'\left(e_j\left(\bar{\theta}\right)\right) \quad \left[\bar{\theta}+ \frac{1}{f_j\left(\bar{\theta}\right)}+\frac{\pi_{1j}\left(\bar{\theta}\right)}{p_j f_j\left(\bar{\theta}\right)\pi_2}\right]^{-1}$$

$$\left[\bar{\theta}+ \frac{\sum_{i \quad A,D} p_j \int_{\underline{\theta}}^{\bar{\theta}} \left(B_i + r_i\left(\theta\right)\right)^{\sigma-1} f_i\left(\theta\right) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left(B_j + r_j\left(\theta\right)\right)^{\sigma-1} f_j\left(\theta\right) d\theta}{p_j f_j\left(\bar{\theta}\right)}\right]^{-1}$$

$$\left[-_{+} \frac{p_k\left(\int_{\underline{\theta}}^{\bar{\theta}} \left(B_k + r_k\left(\theta\right)\right)^{\quad -1} f_k\left(\theta\right) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left(B_j + r_j\left(\theta\right)\right)^{\quad -1} f_j\left(\theta\right) d\theta\right)}{\pi \quad \left(\bar{\theta}\right)}\right]^{-1}$$

$$\left[\bar{\theta}+ \frac{p_k \Delta_{kj}}{\pi_2 f_j\left(\bar{\theta}\right)}\right]^{-1},$$

where $j \neq k$ and

$$\Delta_{\overline{kj}} \quad \int_{\underline{\theta}}^{\bar{\theta}} \left(B_k + r_k\left(\theta\right)\right)^{\sigma-1} f_k\left(\theta\right) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \left(B_j + r_j\left(\theta\right)\right)^{\sigma-1} f_j\left(\theta\right) d\theta.$$

Note that $\Delta_{\overline{kj}} \quad -\Delta_{jk}$ Interestingly, if $\Delta_{kj} < 0$, then

$$C'\left(e_j\left(\bar{\theta}\right)\right) \quad \left[\bar{\theta}+ \frac{p_k \Delta_{kj}}{\pi_2 f_j\left(\bar{\theta}\right)}\right]^{-1} > \bar{\theta}^{-1} \quad C'\left(e_j^{\,C}\left(\bar{\theta}\right)\right),$$

and thus by convexity of C, $e_j\left(\bar{\theta}\right) > e_j^{\,C}\left(\bar{\theta}\right)$ i.e., the education level is higher than the first-best level.

We here analyze the difference in education level between the two groups. Again, by convexity of C, $e_j(\bar{\theta}) < e_j^{\,C}(\bar{\theta})$ if and only if

$$\frac{p_D \Delta_{DA}}{\pi_2 f_A\left(\bar{\theta}\right)} > \frac{-p_A \Delta_{DA}}{\pi_2 f_D\left(\bar{\theta}\right)} \leftrightarrow \left(p_A f_A\left(\bar{\theta}\right) + p_D f_D\left(\bar{\theta}\right)\right)\Delta_{DA} > 0 \quad \leftrightarrow \Delta_{DA} > 0$$

Thus, for the lowest ability agents, the difference in education levels between groups *A* and *D* is determined by the difference in the marginal effects of B_A and B_D in each group. Note that the marginal effects are not necessarily on the social welfare as a whole, since those marginal effects are not weighted by the probabilities. If the marginal influence of most disadvantaged agents (with the lowest ability) to the their own group's welfare is higher than that in the other group, then it is socially second-best optimal to let those disadvantaged agents in the former group achieve higher education than those in the latter group.

Thus, in the case with a general inequality aversion affirmative action is not necessarily supported even for the lowest ability agents. Rather, for some ability level, agents in the more advantaged group may achieve higher education levels.

5. Concluding Remarks

In this paper, we have studied second-best optimal education policies where agents differ in ability and group membership. We have obtained the following results. First, the Rawlsian education policy leads to a form of "reverse discrimination" in the following sense: agents in the advantaged group achieve a lower level of education than agents in the disadvantaged group, and the former receive less schooling help than the latter if the impact of the externality is sufficiently small (Proposition 4). Second, the education policies derived from the utilitarian social welfare function do not distinguish agents by their group membership (Proposition 3).

The differences in the policies arise through the incentive compatibility constraints. On the one hand, when using the utilitarian social welfare function, the constraints do not bind. Thus, the allocation coincides with the first-best allocation that does not discriminate between agents by their group. On the other hand, when we adopt the Rawlsian social welfare function, the incentive constraints bind. The social planner would then require that higher-ability agents achieve higher levels of education and so transfer income from agents with higher abilities to those with lower abilities. Given asymmetric information, under the incentive compatibility constraints, the planner must provide information rent

$$\int_{\theta}^{\bar{\theta}} C\left(e_j\left(x\right), s_j\left(x\right)\right) dx$$

to higher-ability agents to make them exert a higher level of effort. To reduce the information rent, the planner would lower the cost of the lower-ability agents. To reduce this cost, the planner requires lower-ability agents to exert a lower effort, and thus to achieve a lower education level than the first-best level. Note that in the second-best Rawlsian solution, type is treated as if his or her ability were $\theta+\frac{F_j(\theta)}{f_j(\theta)}$. Then, type θA's virtual ability is lower than type θD's. Hence, the education level of type is lower than θA is lower than that of type θD. An intuition is that, as the relative proportion of agents with higher abilities in the group A is larger than in group D, the planner makes the education level of θA lower than that of θD to decrease the information rent.

Proposition 4 may be interpreted as a justification for an affirmative action policy on education. As discussed in the introduction, the result would show a relationship between affirmative action and Rawls' (1971) difference principle. We have also shown, in section 5, that when the inequality aversion of the social welfare function is between utilitarian and Rawlsian, affirmative action may not be supported as a second-best optimum. In this case, the differences in education levels between the two social groups are determined by the agents' marginal effects of utility to their group's aggregated welfare. Especially for the lowest ability agents, the difference in education levels between the two groups is determined by the difference in the marginal effects of the lowest welfare in each group. As argued in footnote 7, it remains for future research to study education differences derived from Roemer's (1998) social welfare function using a model where agents' preferences may differ and they choose different effort levels.

Appendix

In this appendix, we show that the (ICs) constraint holds for both the utilitarian and Rawlsian second-best solutions. For all $\theta\in\left(\underline{\theta},\overline{\theta}\right)$, and all j,

$$e_j^u(\theta,a) \quad e^C(\theta,a),\ e_j^r(\theta,a) \quad e^C\left(\theta+\frac{F_D(\theta)}{f_D(\theta)},a\right)$$

$$s_j^u(\theta,a) \quad s_j^C(\theta,a),\ s_j^r(\theta,a) \quad s_j^C\left(\theta+\frac{F_D(\theta)}{f_D(\theta)},a\right),$$

and $\theta+\frac{F_D(\theta)}{f_D(\theta)}$ is increasing in θ, it is sufficient to check that the solution (e^C, s^C) satisfies (IC_s).

From equations (8) and (9), we have:

$$C_e\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) + C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) \quad \frac{a}{\theta}.$$

By use of this equation:

$$C_e\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) e_\theta^C(\theta, a) + C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) s_\theta^C(\theta, a)$$

$$\left(\frac{a}{\theta} - C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right)\right) e_\theta^C(\theta, a) + C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) s_\theta^C(\theta, a)$$

$$\frac{a}{\theta} e_\theta^C(\theta, a) + C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) \frac{1}{d\left(C''\right)\theta^2} \Big[\left((1+a) C_{ss}\left(e_j{}^C(\theta), s_j{}^C(\theta)\right)\right. \qquad ,$$

$$\left. + C_{es}\left(e_j{}^C(\theta), s_j{}^C(\theta)\right)\right) +$$

$$+ \left(C_{es}\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) + C_{ee}\left(e_j{}^C(\theta), s_j{}^C(\theta)\right)\right)\Big]$$

where the last equality is from equations (6) and (7) in Section 3. Recall that $d\left(C''\right) > 0$, $C_s < 0$, and $e_\theta^C < 0$. Hence, by Assumption (4), we can conclude that:

$$C_e\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) e_\theta^C(\theta, a) + C_s\left(e_j{}^C(\theta), s_j{}^C(\theta)\right) s_\theta^C(\theta, a) < 0.$$

Thus, we have proved that (IC_s) holds.

Notes

1. Fullinwider (2009) provides a survey on broad topics of affirmative action.
2. See, for instance, Roemer (1998, pp. 5-12) for further discussion on this point.
3. Note that our intention is not to suggest that the asymmetry of the distribution functions reflects variation in inherent ability across different social groups.
4. De Fraja (2002) and Green and Sheshinski (1975), among others, introduce an education externality in this form.
5. See Hanushek (2002) for a discussion. Hanushek (2002) also argues that reducing social inequalities and the presence of education externalities provide major justifications for government intervention in education.

6. For instance, Nagel (2003) discusses a relevance between Rawls' theory of justice and affirmative action. Taylor (2009) argues that most affirmative action policies are incompatible with Rawls' theory of justice, while Valls (2010) responds that Taylor (2009) misinterprets the implications of Rawls's theory for affirmative action policies and that most forms of affirmative action are compatible with Rawls' theory.
7. Another possibility is Roemer's (1998) social welfare function based on Equality of Opportunity (EOp) approach. In this environment, agents with the same type choose the same effort, and thus Roemerian social welfare function coincides with the Rawlsian social welfare function. It remains for future research to consider, in our context, an optimal education policy derived from Roemerian social welfare function in a model where agents' preferences may differ and they choose different effort levels.
8. We can obtain the same results if there are finitely many social groups.
9. The result is almost identical to Proposition 1 in Fleurbaey *et al.* (2002) with the exception that we allow for the education externality.
10. The proof is essentially from Fleurbaey (2002, Section 6).
11. See, for instance, Leonard and Van Long (1992, Theorem 6.3.1).

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Resumen

En este artículo, se estudia un problema de planificación de la educación, utilizando un enfoque de diseño de mecanismos. Consideramos un modelo en donde los agentes tienen capacidades diferentes en la adquisición de la educación y pertenecen a diferentes grupos sociales (por ejemplo, razas o géneros). Bajo la restricción de información de que las capacidades de los agentes son inobservables, pero los miembros de los grupos son observables, derivamos dos conjuntos de políticas de educación derivados en virtud de las funciones de Rawls y utilitarias de bienestar social.
Nuestros principales resultados muestran que el planificador utilitario no discrimina los agentes por su pertenencia a un grupo social, mientras que el planificador de Rawls ofrece una forma de política de acción afirmativa. También estudiamos las segundas políticas óptimas de educación en el caso de grados generales de aversión a la desigualdad. En este caso, se muestra que la acción afirmativa no es necesariamente respaldada, y que las diferencias en los niveles de educación están determinadas por los efectos marginales de utilidad de los agentes para el bienestar agregado de su grupo.

Palabras clave: Educación, diseño de mecanismos, acción afirmativa, diferente grupo social.

Clasificación JEL: I24, D82.