

*Panorama*





## **A literature review on optimal indirect taxation and the uniformity debate\***

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### **Abstract**

A review of the theoretical literature on optimal indirect taxation reveals that analytical arguments in favor of uniform indirect taxation seem rather weak and unrealistic; hence determining the optimal tax structure remains an empirical issue. However, reviewing the empirical-computational studies published so far, shows that most of them operate under rather restrictive and simplistic frameworks. There is little computational support for uniformity, particularly when the models approach real world complexity. It appears that in a many-consumer economy, differentiated and progressive indirect taxation is likely to be the optimal solution.

*Keywords:* Optimal taxation, commodity taxation, indirect taxation, tax evasion

*JEL Classification:* H21, H23, H26, C63.

### **1. Introduction**

A large part of the static optimal taxation literature is concerned with optimal indirect taxation, i.e. taxes on the supply and demand for different goods, focusing on the indirect tax structure and the optimal mix between direct and indirect taxes. Since in a static model any uniform indirect tax structure can be replaced by a proportional direct (income) tax, the tax-mix issue and the issue of optimal indirect taxes are closely related<sup>1</sup>. In this context the question arises: is there a need for differentiated indirect taxation? The purpose of this paper is to provide an up-to-date review of the literature on this topic.

In recent years several policy studies have appeared, arguing that indirect taxes should generally be uniform and distributional concerns should be left solely to direct taxes and welfare benefits (Mirrlees *et al.* (2011); Arnold *et al.* (2011); European Commission (2013); IMF (2014); NOU (2014)). At the same time, there is a tendency to recommend a shift from direct to indirect taxes<sup>2</sup>. In light of this, our review seems relevant to the ongoing tax policy discussion.

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A related survey is Ray (1997), who reviews the literature on optimal commodity taxes and optimal reforms<sup>3</sup>. Like Santoro (2007), who surveys the marginal reform approach (see section 3.6), we are narrower in scope, since we focus mainly on the tax uniformity issue. For that purpose, we put more attention to empirical-computational studies, their results, methods, and modelling assumptions. Our review of analytical results is used mainly to explain computational studies and help in the interpretation of their results. Thus, we do not go into formal mathematical derivations, but focus instead on the intuition and assumptions behind the tax structure results, with special attention to the uniformity issue<sup>4</sup>.

The first attempt to analyze the optimal tax problem was Ramsey (1927). He posed the question: Suppose the government needs to raise a certain amount of tax revenue, how should it collect this revenue in a way that will minimize welfare losses? This started what could be called the ‘Ramsey tradition’, covering models where proportional commodity taxes are combined with zero or linear income tax. Ramsey studied a single-consumer economy in which direct taxation is absent. Later, the model was extended to the many-consumer case (Diamond, 1975; Diamond and Mirrlees, 1971), with the possibility of linear income tax (Deaton, 1979a; Deaton and Stern, 1986; Boadway and Song, 2016), and with externalities (Sandmo, 1975). The model was also interpreted and studied by several other scholars (Baumol and Bradford, 1970; Besley and Jewitt, 1995; Corlett and Hague, 1953; Deaton, 1979b, 1981, 1983; Dixit, 1970, 1975, 1979; Feldstein, 1972; Lerner, 1970; Mirrlees, 1974; Munk, 1977, 1980; Sadka, 1977; Samuelson, 1982; Sandmo, 1976; Stiglitz and Dasgupta, 1971). The only significant contributions in the Ramsey tradition that suggested uniform commodity tax rates in a many-person setting are Deaton (1979a) and Deaton and Stern (1986), for linear Engel curves and linear income tax.

James Mirrlees gave birth to what may be called the ‘Mirrlees tradition’ (Mirrlees, 1971, 1976) covering many-person models with an optimal non-linear income tax. Here the basic problem is asymmetric information, since the tax authority cannot discern each individual’s ability, but can observe only income. Lump-sum taxation of abilities would be the first-best solution, but it is infeasible in practice. Mirrlees (1971) studied originally non-linear income tax, but later studies have focused on a mix, where the optimal or improved non-linear income tax is combined with non-linear or proportional indirect taxes (Atkinson and Stiglitz, 1976; Mirrlees, 1976; Cooter, 1978; Stiglitz, 1982; Laroque, 2005; Kaplow, 2006). As with the Ramsey tradition, several theoretical extensions exist, such as allowing for heterogeneous preferences (Mirrlees, 1976; Saez, 2002), heterogeneous endowments (Mirrlees, 1976; Cremer et al., 2001; Boadway and Pestieau, 2003), relaxing production efficiency (Naito, 1999), incorporating externalities (Pirttilä and Tuomala, 1997) and non-separable utility between commodities and leisure (Christiansen, 1984; Edwards *et al.*, 1994; Nava *et al.*, 1996; Jacobs and Boadway, 2014). A number of studies in the ‘Mirrlees tradition’ suggested uniform commodity taxation in a many-person setting. These include: Atkinson and Stiglitz (1976), Stiglitz (1982), Laroque (2005) and Kaplow (2006).

The basic many-person optimal tax problem for  $H$  individuals, covered in the uniform taxation theorems of Atkinson and Stiglitz (1976) and Deaton (1979a), can be put as follows:

$$\text{Max} \sum_h W(u(t_1, \dots, t_N, T(m_h), \omega_h)) \quad (1.1)$$

$$\text{s.t.} \sum_{h=1}^H \sum_{i=1}^N t_i x_{hi} + \sum_{h=1}^H T(m_h) = G + Hb \quad (1.2)$$

where  $W(\cdot)$  is the welfare function of individual utilities  $u(\cdot)$ ;  $(t_1, \dots, t_N)$  are the  $N$  commodity tax rates;  $T(m_h)$  is the income tax function;  $\omega_h$  is the unobservable ability level (or long-term wage rate) of taxpayer  $h$ ;  $X_{hi}$  represents individuals' demands;  $G$  denotes fixed government expenditure on public goods; and  $b$  is a uniform lump sum grant to  $H$  consumers. Gross income is:  $m_h = \omega_h \ell_h$  where  $\ell$  is labour supply. Labour is the only primary input in these static models. The objective is to maximize the welfare function in (1.1) subject to the tax revenue constraint in (1.2). This is a joint optimization problem of income and commodity taxes. Generally, this problem can be solved by maximizing with respect to indirect tax rates, the income tax function, and the lump-sum grant. In the Atkinson-Stiglitz (briefly A-S) model, the search is for the fully optimal non-linear income tax function. In the Deaton model only linear income taxes are considered. In empirical-computational models, the cardinalist welfare transformation,  $W(\cdot)$ , which determines the social valuation weights, is usually defined by a single parameter, called the inequality aversion rate. Econometric techniques are used to provide estimates for utility-demand parameters in these models.

The mathematical correctness of the A-S and Deaton uniform taxation theorems is not in doubt. The debate is on the empirical validity of these highly abstract models. Briefly, the uniform taxation theorems are based on the following assumptions:

- Identical preferences for all consumers along with different wage rates (abilities).
- Weakly separable utility between commodities and leisure.
- No administrative and compliance costs and no tax evasion.
- The absence of other complicating factors such as externalities, merit goods and imperfect competition.
- Income redistribution is carried out through a uniform lump-sum grant for everyone.
- The A-S model takes it for granted that the optimal non-linear income tax schedule can be implemented in practice.
- The Deaton theorem assumes linear Engel curves for all goods.

Numerous objections have been raised against these assumptions, including:

- Preferences are not identical due to different compositions of households, different needs and endowments apart from different abilities.
- Weakly separable utility between commodities and leisure is probably not valid in respect to many goods.

- Complicating factors, such as administrative costs, tax evasion, externalities and merit goods, all violate the uniform taxation theorems.
- The optimal non-linear income tax schedule a la Mirrlees (1971), which is part of the A-S model, is probably not politically acceptable.
- Linear Engel curves for all goods, which is postulated in Deaton's theorem, is not supported by empirical evidence.
- Income redistribution is much more complicated than the provision of a uniform lump-sum grant for everyone. In actual support systems there may be non-optimal elements.
- A number of empirical-computational studies assume zero or sub-optimal lump-sum grants to represent inadequate redistribution. This modelling approach seems to be more relevant to the situation in developing countries.

The counter-arguments to uniformity appear in both the theoretical literature and in empirical-computational studies. Most theoretical contributions relevant to the uniformity debate deal with models in the Mirrlees tradition, while all empirical-computational studies, apart from one, deal with the extended Ramsey framework, and usually ignore income tax. The advantages of empirical-computational models compared with purely deductive mathematical studies, are in that they can much more easily handle complexities, and can provide some quantitative appreciation of the magnitudes involved. On the other hand, they can only describe optimal outcomes in particular situations, and are not well suited to derive general principles that are applicable under a broad range of circumstances. Moreover, some empirical-computational models deal with a single person economy and are largely irrelevant for distributional considerations. All relevant computational models assume constant producer prices. In principle, supply side could be also brought in, and general equilibrium models could be applied, yet no such studies exist to our knowledge<sup>5</sup>.

The present review indicates that the arguments in favor of a uniform tax structure seem rather weak and unrealistic in respect to analytical results, making empirical-computational studies relevant. Further, we found that there are relatively few empirical-computational studies that are in line with the specifications of the uniform taxation theorems mentioned above, possibly because of the complexity and heavy informational requirements for computing optimal taxes. But it is also possible that some of the prominent theoretical results in favor of uniformity have been given too much weight, thus retarding further research on this subject. Unfortunately, most empirical-computational models operate under rather restrictive and simplistic assumptions. Although the empirical evidence seems thin so far, it appears that in models that approximate real world conditions, usually a differentiated and progressive indirect tax structure will prevail<sup>6</sup>. This is in contrast to the aforementioned policy related studies, which recommend uniform indirect taxation for distributional purposes. The unresolved controversy concerning tax uniformity calls for more empirical-computational research, closer to real world complexity, to find out how the optimal indirect tax structure should look like.

The paper is structured as follows. Section 2 presents the main analytical arguments for and against uniform taxation. Section 3 summarizes relevant results from empirical-computational models. Section 4 examines the treatment of income tax in optimal mixed taxation models. Section 5 takes a critical look at the arguments in favor of uniform taxation that appeared in some recent policy related studies. Some brief conclusions are presented in section 6.

## 2. Theoretical considerations

### 2.1. The Ramsey model and its extensions

The original Ramsey model considers a representative consumer economy, where the consumer can allocate his/her total budget between leisure and a number of goods (Ramsey, 1927). The model assumes constant producer prices and no profits. It should be clear from this setup that this model only focuses on the efficiency aspects of commodity taxation. Income tax is assumed to be zero, which is the same as saying that a proportional income tax is allowed. The specifications of the Ramsey model suggest that in this second-best model the objective is to minimize total dead-weight losses (i.e. Harberger triangles)<sup>7</sup>. The well-known result from the Ramsey model states the following: At the optimum, a small intensification of all indirect taxes should decrease the compensated demand by the same proportion for all goods. Since substitution effects are associated with efficiency losses, it is not surprising that the focus is on compensated demand.

Diamond (1975) extended this model to a many-person model with an endogenously determined redistributive lump-sum grant ( $b$  in eq. 1.2), and showed the need for deviating from the principle of 'equal proportional reduction in compensated demand'. The proportional reduction should be less if the consumption of the good concerned is concentrated among groups, i) having a high welfare valuation of marginal income, or ii) having a high propensity to pay taxes. The first condition implies that given two goods with equal compensated demand elasticities, the one consumed more by higher income earners should be reduced more, implying a higher tax rate. The positive correlation between tax rates and consumer group income implies a progressive tax structure. The second condition is closely linked to the shape of Engel curves. Linear Engel curves mean that marginal budget shares are equal, and the propensity to pay taxes will then be equal.

The Ramsey rule only characterizes properties of the optimum, and does not provide clear guidance about the optimal tax structure. To examine how the actual tax structure will look like, further assumptions are needed. Sadka (1977) proves that in a single-consumer model, a necessary and sufficient condition for uniformity is that compensated elasticities in respect to the wage rate of different commodities are all equal. This means that a decrease in the wage rate following a proportional income tax increase, will reduce compensated demands by the same proportion. Since this is what characterizes marginal changes at the optimum, proportional income tax represents the optimal solution. A preference structure satisfying this condition is when direct utility

can be written as  $U(v(x), L)$ , where  $x$  represents commodities;  $L$  is labour supply and  $v(x)$  is a homogenous function<sup>8</sup>. This type of utility function implies equal Engel elasticities for all goods<sup>9</sup>. It is possible that for some particular groups of goods, a homogeneous sub-utility function will apply. Then these goods should be subject to the same tax rate. But based on empirical evidence, such an assumption cannot be applied to a complete demand system.

Elaborating more on tax structures, Deaton (1981) shows that quasi-separability will lead to a uniform structure in the one-consumer case<sup>10</sup>. Besley and Jewitt (1995) generalize the one-consumer result further and show that it applies to a particular type of utility function. Deaton (1981) also shows that if we move to a many-consumer economy and assume that the planner has preferences in favor of equity, then quasi-separability leads to a progressive tax structure. Weak separability between commodities and leisure leads to a regressive indirect tax structure in the one-consumer case. Introducing an egalitarian planner and many-consumer economy, will move the solution towards progressivity.

Another study focusing on tax structure is Baumoul and Bradford (1970), who derived the so-called inverse elasticity rule for a single-consumer economy, by assuming there are no cross-price effects between commodities, i.e. the demand for all goods depends only on its own price and the price of leisure, namely the wage rate. The rule states that the tax rate should be inversely related to the own price elasticity of a commodity and will be smaller the more complementary is the commodity with labour<sup>11</sup>. Given that necessities typically have low elasticities of demand, this rule calls for a regressive commodity tax structure in a single-consumer economy. A uniform tax structure will prevail if all own price elasticities are the same and utility is weakly separable between commodities and leisure.

In a single-consumer and three-good setting (leisure and two commodities), Corlett and Hague (1953) examine which commodities should be taxed to supplement an existing income tax. Their analysis relies on a marginal reform approach, which considers the welfare change due to the introduction of a differentiated commodity tax structure, when the starting point is a uniform system, alternatively interpreted as an existing proportional income tax. They find that the commodity which is a stronger complement to leisure (in Hicks sense, meaning that the compensated cross derivative of the good with respect to the wage rate is negative) should bear a higher tax. This is a marginal analysis, however, not a global one (see section 3.6). Still, the result also holds true within an optimal design framework (see Sandmo (1976), Jacobs and Boadway (2014)).

## 2.2. The uniform taxation theorems

### 2.2.1. *Optimal non-linear income tax and the Atkinson-Stiglitz theorem*

A prominent result in the literature is the Atkinson-Stiglitz (A-S) theorem (Atkinson and Stiglitz, 1976). Conditional on an optimal non-linear income tax, they study the role of indirect taxation in a many-person redistributive model. The starting point for their analysis is

the Mirrlees (1971) control theoretic optimal non-linear income tax model, where a uniform lump-sum grant is the main redistributive instrument. Incorporating into the Mirrlees (1971) model non-linear commodity taxes, A-S prove, using control theory, that in this situation optimal non-linear commodity taxes will be all zero, provided preferences are weakly separable between commodities and leisure<sup>12</sup>. This implies that optimal redistribution could be achieved by income tax alone. Given that in the absence of evasion and administration, a portion of income tax can be converted into uniform commodity taxes, the A-S result implies the optimality of zero or uniform commodity taxation.

A particular shortcoming of the original A-S theorem is that it assumes non-linear commodity tax functions, in the form  $t_i(x_i)$ , where the tax rate  $t_i$  is a function of the quantity consumed,  $x_i$ . This assumption was adopted for reasons of mathematical convenience, because this way the zero commodity taxation theorem could be proved with control theory in a relatively simple manner. But in practice, quantity dependent taxes can be applied only to a few items. They cannot be applied for the majority of goods, because the government cannot properly observe the quantity purchased and consumed by individuals or households, and much of the tax could be avoided through re-trading between consumers. The non-linear commodity tax problem has been corrected by Christiansen (1984), who extends the A-S theorem to proportional commodity taxes (i.e. constant tax rates), by using conditional Marshallian demand functions within a marginal reform framework, in much the same manner as Corlett and Hague (1953). He shows how leisure complements and substitutes should be taxed or subsidized in the presence of an optimal non-linear income tax. He finds that given weakly separable utility and optimal income tax, optimal commodity tax rates should be zero or uniform.

### **2.2.2. The main objections raised against the Atkinson-Stiglitz theorem**

A number of objections were raised against the A-S theorem, and here we shall examine them one by one.

#### *Identical preferences for all taxpayers*

One of the central assumptions of the Mirrlees (1971) model and the A-S theorem is identical preferences and differing wage rates. However, casual empiricism suggests that taxpayer populations are quite heterogeneous, because of different household compositions and different needs and endowments. The earliest qualification to identical preferences is presented in Mirrlees (1976) section 4, where he shows that when the population is characterized not by a single characteristic (ability) but by multiple characteristics, then the solution is more complicated, and the A-S theorem will not necessarily apply<sup>13</sup>. Mirrlees applied in this analysis advanced mathematical techniques. Later critiques used simpler mathematics to prove the same point. Saez (2002) shows, using Christiansen (1984) conditional Marshallian demand functions approach, that the A-S result is no longer valid when considering the heterogeneity of house-

hold preferences, reflected by different purchases at the same income level. Other heterogeneity conditions that were shown to invalidate the A-S theorem (using Stiglitz's 'self-selection' approach), include different unobservable endowments apart from different abilities (Cremer *et al.*, 2001), and different needs, endowments and multiple forms of labour supply (Boadway and Pestieau, 2003). It should be noted that almost all theoretical studies assume a population with identical preferences. Heterogeneous households, in terms of demographic composition, enter into the picture in a few empirical-computational models.

#### *Weak separability between commodities and leisure*

This is another central assumption in the A-S theorem. Weak separability between commodities and leisure has been rejected in several econometric studies (e.g. Blundell and Walker, 1982; Blundell and Ray, 1984; Browning and Meghir, 1991), thereby invalidating the A-S theorem.

Some recent studies are more ambivalent in relation to commodity/leisure non-separability than those published decades ago. In their contribution to the Mirrlees *et al.* (2011) Review in relation to VAT and excises (chapter 4), Crawford, Keen and Smith empirically reject weak separability between commodities and leisure, but argue that it is far from clear how much differentiation could be justified on these grounds, or which commodities should be taxed more or less heavily. Pirttilä and Suoniemi (2014) note, that despite the large theoretical literature, there has been little empirical work done in trying to establish the relationship between commodity demand and labour supply. Their research indicates that capital income and housing expenses are negatively associated with working hours, whereas child care is positively related. Generally, it appears that more empirical research is needed on this subject.

#### *Perfect competition in labour markets*

In common with many other theoretical models, the A-S and Mirrlees (1971) models assume perfect competition. Yet, in optimal taxation models this assumption has led to rather debatable results. The assumption that wages exactly match productivities has led in the Mirrlees (1971) model to decreasing marginal income tax rates at the higher income range. Evidently, such a tax schedule is not politically acceptable in a world of imperfect competition and information. That means that the Mirrleesian income tax function, which is part of the A-S model, cannot be implemented in practice (Boadway and Pestieau, 2003).

#### **2.2.3. Non-distributional factors**

It is well known from the theoretical literature that differentiated commodity taxation can be justified on grounds other than raising tax revenue for redistribution, such as externalities, (de)merit goods, risks, administrative and compliance costs and tax evasion<sup>14</sup>.

Sandmo (1975) extends the Ramsey framework to include externalities, and shows that additional taxes equal to the marginal external effects should be added to the original solution. Pirttilä and Tuomala (1997) show how externalities should be taken care of in the presence of a non-linear income tax. Studies taking into account the effects of tax evasion on optimal tax rates include: Cremer and Gahvari (1993), Boadway *et al.* (1994), Ray (1997) and Revesz (1997, 2014a, 2014b). Revesz points out that there are different evasion propensities between goods and services produced and marketed by large organizations compared with those produced and marketed by small business. We shall return to this issue in section 5. Cremer and Gahvari (1995) examine the effect of income risks on the optimal taxation of housing and durables. Alm (1996) considers the influence of administrative and compliance costs on optimal tax rates. Besley (1988) deals with the merit goods arguments (i.e. paternalistic concerns). Another possible complicating factor is cross-border shopping. This is examined in Christiansen (1994) and Nygård (2014). Note that all these complicating factors violate the necessary conditions for tax uniformity.

#### **2.2.4. Other possible objections to tax uniformity**

There are also other areas where the A-S model is at odds with what is observed in practice. In developed countries, one may observe in-kind transfers (education, health, child care etc.) accessible at no cost or little cost, and different forms of welfare benefits. There are also all kinds of indirect taxes such as VAT, general sales tax, property taxes and excises on specific goods (e.g. gasoline, tobacco, alcohol and motor vehicles). Moreover, governments correct externalities by implementing Pigovian taxes and subsidies. We shall not examine here the optimality or otherwise of different forms of indirect taxation, but a few comments can be made on welfare benefits and in-kind transfers; because this subject has been taken up in some taxation models that follow the Ramsey tradition (see sections 2.2.7, 3.3.1 and 3.3.2).

In common with Mirrlees (1971), the A-S model assumes a single form of income support carried out through an optimal uniform lump-sum grant (called the demogrant). In practice, redistributive support is much more complicated than that. It is made up of cash support through welfare benefits and various forms of in-kind transfers. Piketty and Saez (2013) section 2, note that in developed countries over 50 percent of social programs represent in-kind transfers, rather than cash payments through pensions, unemployment benefits and the like. The share of in-kind transfers (mainly education and health services) in social programs is much larger in developing countries, where direct support payments are almost non-existent. The heterogeneity of targeted welfare payments can be explained by the heterogeneity of recipient households, a subject that we have already discussed earlier. Given imperfect information by support agencies and possible false reporting by many recipients (particularly in relation to unemployment, disability and means-tested benefits), the perfect optimality of cash payments can be in doubt. The full optimality of in-kind transfers is even more dubious, because here optimality must cover not only scale but also composition.

The implication of sub-optimal lump-sum grants in a heterogeneous population model has been examined theoretically by Deaton and Stern (1986) (see section 2.2.7). In the framework of a model based on linear income tax and linear Engel curves, they reach the conclusion that if differentiated lump-sum grants are not provided in optimal scale and composition, then optimal commodity taxation will be differentiated. Moreover, some empirical-computational studies reviewed in sections 3.3.1 and 3.3.2, suggest that if the demogrant is set at a sub-optimal level, then the optimal solution will involve differentiated and progressive commodity taxation. While these results do not strictly pertain to the A-S model, they do suggest that some of the many missing elements from this highly abstract model can lead to differentiated taxation. This conjecture should be explored further.

### 2.2.5. *The self-selection approach*

The A-S theorem can be proved also using Stiglitz's (1982) 'self-selection' optimization model. This asymmetric information model is applied in a two-person or small group setting. It is a simplified form of the original A-S model. In this type of models, proportional commodity taxes are combined with a non-linear income tax. Marginal income tax rates and corresponding lump-sum taxes or subsidies are determined at separate income tax brackets covering individual taxpayers. Income and commodity tax rates are chosen so as to ensure that high ability persons will not have the incentive to mimic the incomes of lower ability persons. The self-selection approach was used in a number of studies that investigated departures from the A-S solution due to various complications. These include Boadway et al. (1994) on the effect of income tax evasion, Cremer *et al.* (2001) on unobservable endowments in addition to different abilities, Boadway and Pestieau (2003) on different needs, endowments and multiple forms of labour supply, Cremer and Gahvari (1995) on income risks and the purchase of durables, Pirttilä and Tuomala (1997) on externalities and Bastani et al. (2014) on subsidies for child care. Edwards *et al.* (1994) and Nava et al. (1996) examine the effect of leisure substitution and complementarity on optimal tax rates, as well as their effect on the optimal provision of public goods.

The popularity of the self-selection approach for the purpose of extending the A-S model in various ways, is motivated by its assumed better tractability compared with the original control theoretic A-S model<sup>15</sup>. While this may well be true in respect to analytical studies, at least for the purpose of numerical studies there seems to be an easier approach. There is an explicit elasticities based formula for the Mirrlees (1971) income tax problem (see Saez (2001) and Revesz (1989, 2003)). The formula changes slightly following the incorporation of proportional commodity taxes into the income tax model. Denote the marginal commodity tax rate as  $t_m^c = \sum_j t_j \frac{\partial q_j}{\partial m}$  where  $m$  is income and  $q_j$  are commodities. Then the left hand side of the formula for the optimal marginal income tax ( $t_m$ ) will be:  $\frac{t_m + t_m^c}{1 - (t_m + t_m^c)}$  instead of the original  $\frac{t_m}{1 - t_m}$  (for proof see appendix 1). The right hand side remains the same. The end point conditions at the upper and lower boundaries of the wage distribution of the working

population will be then:  $t_m + t_m^c = 0$ . Provided  $t_m^c$  is positive, this implies that at the end points the marginal income tax rate will be negative rather than zero. This result was originally discovered by Cooter (1978). Using the revised optimal income tax formula, it might not be too difficult to investigate various extensions to the A-S model through computational studies.

### ***2.2.6. The Laroque-Kaplow proposition: Sub-optimal income tax***

The Laroque-Kaplow (L-K) proposition extends the A-S theorem to apply to non-optimal income tax functions as well (Laroque, 2005; Kaplow, 2006). It states that with weak separability and identical preferences, it is possible to replace any non-uniform indirect tax structure by zero or uniform commodity taxes, by adjusting the non-optimal income tax function and the demogrant in such a way that all taxpayers will maintain or improve their utility position. Hence the L-K proposition suggests that in a redistributive model, eliminating non-uniform commodity taxes can lead to a Pareto improvement. The L-K proposition shares all the empirical objections to the A-S theorem outlined in sections 2.2.2 to 2.2.4. But it has some other weaknesses as well.

Boadway (2010) discusses this proposition and stresses that if because of some reason (political or administrative), the appropriate income tax adjustment is not carried out, then such a reform may be welfare reducing. Since the L-K proposition does not describe the design of the income tax functions that combined with zero or uniform commodity taxes will yield improved welfare, this critique seems pertinent. Revesz (2014a) claims that the mathematical theorems presented by Laroque (2005) and Kaplow (2006) do not really prove that following the L-K reform a Pareto improvement will necessarily occur<sup>16</sup>. The problem seems to be that Laroque and Kaplow did not recognize properly that the work related disincentive effects of direct and indirect taxes are substantially different.

### ***2.2.7. Optimal linear income tax and the Deaton theorem***

Assuming that income tax is restricted to be linear, Deaton (1979a) proved that in a many-person model with identical preferences, weak separability and linear Engel curves, the optimal commodity tax structure will be uniform. This result is similar to the A-S theorem, but given that income tax is linear, the assumption of linear Engel curves must be added in order to obtain a uniform commodity tax solution. The assumption of linear Engel curves for all goods (with each line starting with positive demand from the lowest disposable income), is dubious and has been rejected in econometric studies; see e.g. Blundell and Ray (1984). Revesz (1997) shows that in a multi-product setting, because of the non-negativity constraint on demand, linear Engel curves for all goods imply nearly homothetic preferences, which is clearly unrealistic<sup>17</sup>. The Deaton (1979a) model shares all the empirical objections to the A-S model presented in sections 2.2.2 to 2.2.4, apart from the infeasibility

of implementing an optimal non-linear income tax schedule. However, an extension of Deaton's theorem takes care of heterogeneous populations, as will be explained below.

Deaton and Stern (1986) assume weakly separable utility, linear Engel curves and linear income tax, and that consumers differ in preferences (and consumption patterns) partly due to differences in observable policy related characteristics (such as age or number of children) and partly due to idiosyncratic preference variation. Differences in preferences are represented in their model by differences in the intercepts of the Engel curves. They show that if i) social valuation weights are correlated with differences in preferences and characteristics, ii) variations in consumption are related to differences in policy related characteristics, and iii) lump-sum grants can be conditioned on policy related characteristics in a linear way, then uniform indirect taxation is optimal. In this case, we only need to design an optimal set of lump-sum grants dependent on policy related characteristics and differentiated indirect taxation is superfluous. Let us just note here that due to imperfect information, the possibility of optimal lump-sum grants is somewhat problematic from a real world perspective, and as indicated earlier, the assumption of linear Engel curves for all goods is not supported by empirical evidence. The Deaton-Stern (1986) theorem has been supported by the numerical results of Ebrahimi and Heady (1988) (see sections 3.3.2 and 3.3.3).

In a recent paper, Boadway and Song (2016) examine analytically certain extensions to the Deaton (1979a) model. They investigate a two-good model where one good is a necessity and the other is a luxury. They find that if income tax (linear or non-linear) is less progressive than optimal, then the necessity should be taxed at a lower rate than the luxury. Given that a commodity tax model where only a lump-sum grant is present but no income tax, is effectively a model with a particular form of linear income tax (see section 3.1), the conclusion of Boadway and Song applies to these models as well. In section 3.3.2 we shall examine empirical-computational models of this type, where the lump-sum grant is sub-optimal. The numerical results from these models accord with the Boadway-Song theorem.

Another conclusion presented in their paper is that if a linear income tax function is optimal, but low-income households are unable to afford luxury goods, it may be optimal to tax necessity goods at lower rates than luxuries. The computational studies of Revesz (1997, 2014a, 2014b) examine the case of non-linear Engel curves, where luxuries are consumed only by high income earners. The numerical results from these models (discussed in section 3.3.3) lead to the same conclusion as Boadway and Song (2016).

### **3. Empirical-computational studies**

#### **3.1. Methodological issues**

To apply the theory in empirical-computational models raises several questions: What kind of information is needed? Which type of utility function should be used? What kind of methods and specifications should be applied? The first order conditions and the budget

constraint will make our need for information about the individuals' demand and its derivatives immediately apparent. We will also need information on individuals' supply functions, i.e. labour supply. Without imposing further restrictions, such as commodity/leisure separability, the wage rate and leisure consumption will influence the demand for commodities through substitution and complementarity and not only income effects. Given that separability between commodities and leisure has been rejected in several econometric demand studies (see section 2.2.2), it does not appear to be a realistic restriction. On the other hand, there are few reliable estimates on leisure substitution or complementarity parameters (Jacobs and Boadway, 2014).

In line with earlier discussion, one could argue that the functional form of demand should be flexible enough to allow for non-linear Engel curves and non-separable utility. Yet we should be aware that undertaking an optimal design analysis requires the demand and labour supply functions to be consistent with consumer theory globally and not only locally, since the optimal price structure could be far from the point at which the functions are consistent with theory. So-called flexible functions do not automatically exhibit these properties in a global sense<sup>18</sup>. The property of a flexible functional form is its ability to take on any set of price and income elasticities at a particular data point, unrestricted by a priori assumptions. This seems very desirable, but it comes at a cost. As Caves and Christensen (1980, p. 423) make it clear, outside the initial data points the estimated flexible form indirect utility function may not be monotonic or strictly quasi-convex (implying quasi-concavity of direct utility). An example of a popular and widely used flexible functional form is the almost ideal demand system (AIDS) introduced by Deaton and Muellbauer (1980)<sup>19</sup>. This function may provide an approximation to any arbitrary utility function and may exhibit many desirable properties locally, such as the quasi-convexity of indirect utility. However, since it is an approximation, it can only show consistency with demand theory locally, and there is no guarantee for the same applying globally.

In addition, in a redistributive model the social welfare function must be specified in detail for a many-consumer economy. This raises issues in regard to the cardinalisation of utility and political value judgments, which are usually captured by the inequality aversion rate. Household composition also raises inter-personal comparability issues. One way to resolve the problem is to assign politically determined utility weights according to household composition. A more objective approach is to use demographic equivalence scales, derived from household expenditure surveys (Ray, 1989). Determining the equivalence scales requires the decomposition of average household expenditure between household members, divided into categories such as parents, non-parents, children by age group, etc. Whether demographic equivalence scales can be determined objectively, free of value judgments, is an open question.

Most of the numerical models on commodity taxation published so far exclude income tax, or at least the variable part of income tax. Given a linear income tax function commonly defined as:  $\alpha + \beta m_h$ , then  $\beta m_h$  represents the variable part. Models where only  $\alpha$  is present are in effect also linear income tax models. For example, assume an initial situation where income tax is  $\alpha + \beta m_h$  and all commodity taxes are zero. Then a transformation reduces the variable

income tax rate ( $\beta$ ) to zero, while at the same time it increases commodity taxes (relative to producer prices) and the lump-sum grant by the factor  $1/(1 - \beta)$ . This will effectively not change anything, by virtue of zero homogeneous utility and demand. But in the new situation, only the lump-sum grant,  $\alpha/(1 - \beta)$ , is left in the model, which will then represent a particular form of linear income tax.

After a utility function has been selected and all the information needed is at hand, we can calculate optimal taxes. This is not a trivial task since taxes, prices, quantities and elasticities are interdependent in a non-linear way. Some kind of iterative numerical method must be employed to yield a solution. If we relax the assumption of constant producer prices, the computation of optimal taxes will also require information about the producer sector, which would render the task even harder.

Having reviewed some of the difficulties with empirical-computational studies, we can now turn to examine a number of contributions. To our knowledge, apart from the self-selection model of Bastani et al. (2014), no attempt has been made to test numerically the theory of non-linear income tax along with commodity taxation according to the “Mirrlees tradition”. We shall therefore review mainly studies under the extended Ramsey model, i.e., linear or proportional taxation. The studies reviewed consider only the demand side, assuming constant producer prices. With the exception of four studies, the demographic composition of households is excluded from these models.

### 3.2. Representative consumer models

We start the review of empirical-computational models with a number of studies that are considering a representative consumer economy. To what extent these one-person models are relevant to distributional models is an open question. Whatever the case may be, they illustrate various statistical and mathematical approaches to the commodity tax optimization problem<sup>20</sup>.

Atkinson and Stiglitz’s (1972) paper is the first, to our knowledge, to compute optimal tax rates from empirical data. In computing optimal taxes for five commodity groups they consider two demand systems: The linear expenditure system (LES) based on estimates by Stone (1954) and the direct addilog demand system based on estimates by Houthakker (1960)<sup>21</sup>. In both cases there is separability between commodities and leisure and they assume elastic labour supply. According to theory they should get a solution which is regressive, and so they do<sup>22</sup>.

Fukushima and Hatta (1989), using the same data set and the same model, find that reducing the (compensated) labour supply elasticities works in favor of a uniform system. With what they consider as more reasonable values, they find the structure to be fairly uniform.

Harris and McKinnon (1979) also calculated optimal tax rates for five product groups using a Stone-Geary (LES) function with leisure and commodities. The optimal structure,

they conclude, varies with the assumed compensated labour supply elasticities. Fukushima (1991) uses the same data but with lower labour supply elasticity, which yields a result somewhat closer to a uniform solution.

Asano and Fukushima (2006) estimate the joint decision of leisure and commodity demand without imposing any separability restriction. They use Deaton's AIDS and compute optimal tax rates for ten commodity groups in Japan. Their conclusion is that the optimal structure is reasonably close to a uniform one, which suggests that the welfare losses associated with tax uniformity are small. Although the Asano and Fukushima model provides important improvements, their results are weakened by uncertainty to what extent the model complies with properties required according to demand theory (see section 3.5).

Another study within the one-person economy framework is Nygård (2014). Using a LES system, he includes cross-border shopping and focuses on cross-border exposed goods for Norway. The goods purchased across the border are assumed to be non-taxable and externality-generating. He optimizes commodity taxes conditional on a pre-existing income tax. As expected, he shows that goods purchased at home should be taxed more leniently, because of the distortions caused by cross-border shopping. In particular, he shows how the effects get stronger because these goods are externality-generating. In general, the tax structure obtained is highly differentiated. When neglecting cross-border shopping and external effects, he gets a more uniform solution, though still more differentiated than that of Asano and Fukushima (2006).

### 3.3. Many-consumer economy

#### 3.3.1. *Models without a lump-sum grant*

In almost all theoretical models, redistributive support is represented by a single uniform lump-sum grant for everyone, called the demogrant. In a number of empirical-computational models the demogrant is missing. This is supposed to represent the situation in many developing countries where direct support payments are absent. Looking back at eq. (1.2), the budget constraint in these models appears as:  $\sum_i^N \sum_h^H t_i x_{ih} = 0$ .  $H_b$  is missing because  $b$  is set to zero.  $T(m_p)$  and  $G$  are missing because in the models described below, there is no income tax or fixed public goods expenditure requirements. Given the above constraint, commodity taxes and subsidies must add up to the same totals. The presence of egalitarian objectives and the absence of a demogrant, imply progressive indirect taxation/subsidization at the optimum, because there is no other way to provide support to the needy, as limited as it may be. Here we shall review some of these models.

As far as we are aware, the first calculation of optimal taxes within a many-person framework is presented in Deaton (1977). Deaton's model relies heavily on simplifying assumptions. By employing what he calls strategic aggregation, he ends up having to consider

the behavior of only two consumers, the marginal utilities weighted social representative consumer and the average consumer. He calculates optimal taxes for eight goods. His study is based on inelastic labour supply and linear Engel curves. His specification of the welfare function is based on Atkinson (1970), and is similar to the welfare functions used in studies that will be reviewed later. In the absence of distributional goals, his results indicate a uniform tax structure. He finds that when the concern for equity increases, the structure becomes more differentiated and luxuries are taxed more heavily than necessities.

Heady and Mitra (1980) use a Stone-Geary LES utility function, implying both separability and linear Engel curves for nine goods, including leisure. Basically they find that the tax structure is progressive, depending on what assumption is adopted about equity.

Sensitivity of optimal tax rates to different demand systems is considered by Ray (1986) in a model that excludes leisure. He calculates optimal tax rates for nine goods from Indian data, conditional on the prices, incomes and elasticities observed at a particular point of time. That means that his optimal tax rates are not optimal in a strict sense, but only reflect what the tax rates would have been had the initial situation constituted the optimum<sup>23</sup>. He compares the linear expenditure system (LES) with the restricted non-linear preference system (RNLPS), which is a specialization of the non-linear preference system (NLPS) introduced by Blundell and Ray (1984)<sup>24</sup>. The RNLPS allows for non-linear Engel curves. He finds that results from the two demand systems agree at low level of concern for equity, but diverge when the inequality aversion rate increases. At low levels of inequality aversion they approach a uniform solution. Ray (1986) also finds that the scale of redistribution achieved through the taxation/subsidization of nine broad product groups is fairly limited.

A study that follows up Ray (1986) is Majumder (1988). Using the same conditional method, he tests other non-linear Engel curves specifications on the same data and discovers that the results are sensitive to the exact specifications of Engel curves non-linearity.

Murty and Ray (1987) use the general functional form of Blundell and Ray (1984), the non-linear preference system (NLPS), to investigate sensitivity to the assumption of weakly separable utility between goods and leisure. Their results indicate that the tax rates for the nine goods considered are highly sensitive to deviations from weak separability.

Murty and Ray (1989) develop an iterative algorithm, based on the marginal tax reform approach of Ahmad and Stern (1984), to calculate optimal tax rates. Because of the iterative calculations, their approximation to optimal tax rates is probably better than in some of the studies mentioned earlier. Their results from LES and RNLPS are similar when inequality aversion is low, but diverge as the inequality aversion rate becomes higher.

Ray and Blacklow (2002) extend this model and incorporate demographic effects when they use RNLPS and LES to study optimal taxes in Australia for nine goods. The optimal tax rates, they conclude, move away from uniformity when demographics are introduced and affect the social welfare weights. The effect is more significant when considering the RNLPS

than LES. In line with Murty and Ray's (1989) findings, their results also indicate that LES and RNLPS agree at low inequality aversion, but diverge at higher inequality aversion rates. Furthermore, optimal tax rates appear to be more sensitive to the choice of functional form than to the inclusion of demographic effects.

It should be noted that a redistributive model without a lump-sum grant is hardly realistic in any country. Even in developing countries where direct support payments are absent, there is some kind of redistributive support provided through public education, health care and other in-kind transfers, which can be represented for modelling purposes by a demogrant. Alternatively, the set of subsidized goods in a zero lump-sum grant model should include education, health, child care and public transport, provided there is some information on their consumption by income groups and on their demand elasticities.

### 3.3.2. *Models with sub-optimal lump-sum grants*

Another class of empirical-computational models deals with positive but sub-optimal lump-sum grants. The budget constraint of such models with identical preferences appears as:  $\sum_i^N \sum_h^H t_i x_{ih} = Hb$ . But in this case the demogrant ( $b$ ) is not the solution from full welfare function optimization. In this section we shall only deal with cases where ' $b$ ' is below the optimal value, and the variable part of income tax ( $\beta m_h$ ) is missing.

The demogrant can be sub-optimal for two reasons. The first is because it is given exogenously below the optimal level. When the lump-sum grant is sub-optimal, it is reasonable to assume that progressive indirect taxation is needed to compensate for sub-optimal support provided to the needy. Sub-optimal demogrants can represent the situation in countries where because of the proliferation of the "shadow economy", and/or because of serious wastage in the public sector, expenditure on redistribution does not reach the politically preferred level (Revesz, 2014b)<sup>25</sup>. The theoretical study by Boadway and Song (2016) (discussed in section 2.2.7) found that given a sub-optimal linear income tax, optimal commodity tax rates should be progressive. Because of the absence of the variable part of income tax in the models described in this section, the sub-optimal demogrant alone represents in effect a form of sub-optimal linear income tax (see section 3.1), and indeed, the numerical results from these models accord with the Boadway-Song theorem.

Another reason why the demogrant can be sub-optimal arises in populations with heterogeneous characteristics. Under these conditions, varying amount of support should be provided, depending on policy related characteristics of the household. In this case the budget constraint will be:  $\sum_i^N \sum_h^H t_i x_{ih} = \sum_h b_h$ . When the optimization does not take into account all the information available in the model about household characteristics, then the resulting lump-sum grants will be sub-optimal. For example, in the model of Ebrahimi and Heady (1988), there are two rates of child benefits, depending on the age of the child. When the two rates are reduced to one (the sub-optimal case) then optimal commodity tax rates

become differentiated. This result confirms the theoretical contribution of Deaton and Stern (1986), discussed in section 2.2.7.

Ray (1989) and Ebrahimi and Heady (1988) investigate the impact on optimal tax rates of child benefits. Both of them use data from a UK database covering four composite goods. Ray (1989) allows for non-linear Engel curves but does not include leisure, while Ebrahimi and Heady (1988) include leisure and linear Engel curves that are not always parallel across households. Since Ebrahimi and Heady combine a many-person heterogeneous population model with variable labour supply and differentiated lump-sum grants, their study is probably one of the most sophisticated and comprehensive empirical-computational models published so far. We shall return to their study in the next section. Ray (1989) finds support for progressivity when lump-sum grants (made up only of child subsidies) are sub-optimal, especially when inequality aversion is high. Ebrahimi and Heady (1988) conclude that commodity tax rates will not be uniform when the demogrants (lump sum grants conditioned on demographic characteristics of the household plus a uniform component) are not set optimally, or if preferences are not weakly separable between commodities and leisure.

Asano et al. (2004) use an AIDS system without variable labour supply, which has much in common with Ray (1989). If the lump-sum grant is zero or is set at a sub-optimal level, they get a progressive structure, in line with Ray (1989). When the uniform lump sum grant is free to change in the course of tax optimization, they get a regressive tax structure. Given that in their model labour supply is fixed, it is not clear how they obtained finite tax rates from full optimization. In a redistributive model where the only primary input (labour) is fixed, one would expect the optimal solution to tax away and redistribute all incomes equally through the variable demogrant (implying infinite commodity tax rates), since output would be unaffected. In such a model, the optimal demogrant will be average income.

### 3.3.3. *Models with endogenously determined optimal lump-sum grants*

In these models the lump-sum grant is free to change in the course of optimization, in line with the specifications in eq. (1). In most of the scenarios described below, the budget constraint is similar to eq. (1.2), although in some of these models the variable part of income tax ( $\beta m_h$ ) is missing<sup>26</sup>. As explained in section 3.1, a model where  $\alpha$  is present but  $\beta m_h$  is absent, implies a particular form of linear income tax.

Ebrahimi and Heady (1988) present some scenarios where lump-sum grants are determined through full tax optimization, taking into account all demographic characteristics and the tax revenue constraint. Under these conditions, they find that optimal commodity tax rates will be uniform, provided utility is weakly separable between commodities and leisure and provided Engel curves are linear and parallel across households, in line with the analysis of Deaton and Stern (1986). If either of these conditions is violated, optimal tax rates will not be uniform. In their model, non-parallel Engel curves represent a mild case of non-lin-

earity. But even that slight non-linearity of Engel curves has led to perceptible differentiation in tax rates.

Revesz (1997) uses LES first with nine goods for everyone, then with 9 goods for the poor and 18 goods for higher income earners. In both cases leisure is separable from other goods and a uniform demogrant is calculated from the tax revenue constraint. In the 9 goods model he gets results in line with Deaton (1979a), namely uniformity. When applying the 18 goods setup, he gets a progressive structure, because of the non-linearity of Engel curves. This model illustrates the commonly observed situation, where the consumption of luxuries starts at intermediate or high income levels. The 9 goods parameter estimates used in this study are taken from Deaton and Muellbauer (1980).

Revesz (1997) has been developed further, to become a fairly comprehensive numerical model of optimal indirect taxation in Revesz (2014a, 2014b). Unlike other numerical studies mentioned earlier, Revesz (2014a, 2014b) does not calibrate his calculations to empirical data. Instead, he uses in his numerical examples arbitrary but plausible numbers. He justifies this approach by presenting approximate formulas for optimal commodity tax rates. The numerical results are used to illustrate and substantiate the analytical approximations. Revesz (2014a, 2014b) finds that under non-linear Engel curves and logarithmic utility specifications and without income tax, optimal commodity tax rates are highly differentiated and progressive. The dispersion of tax rates is further increased in the presence of “real life” complexities, such as evasion, administrative costs, externalities and non-separable utility between commodities and leisure. The dispersion of tax rates is reduced if an exogenously given non-linear income tax function is incorporated into the model, or if the inequality aversion rate is low, or the average compensated elasticity of labour supply is low.

A computational study by Bastani, Blomquist and Pirttila (2014) examines the effect of strong leisure substitutes, such as child-care and aged-care services on optimal commodity tax rates, using a Stiglitz (1982) type self-selection model. In the self-selection model the zero intercept of the optimal non-linear income tax function represents an endogenously determined lump-sum grant. The model assumes that an hour child care will generate an additional hour of work. Bastani et al. (2014) examine optimal taxes-subsidies and changes in labour supply in a model involving two composite goods plus child care and leisure, and four population groups - low and high wage earners, parents and non-parents. They find that provided child care is not fully paid by the government, progressive taxation of commodities is justified.

### **3.4. Relevance to the uniform taxation theorems**

Having reviewed the empirical-computational models published so far, the question arises to what extent are these models relevant to the uniform taxation theorems? The models covered in section 3.2 (single-consumer economy) and section 3.3.1 (the absence of income tax and a lump-

sum grant) appear far removed from the specifications in eq. (1) of the A-S and Deaton models. The framework of the sub-optimal demogrant models, discussed in section 3.3.2, is already closer to those of the uniform taxation theorems. In fact, there are two theoretical studies – Deaton and Stern (1986) and Boadway and Song (2016) – that are directly relevant to these numerical models. The models in section 3.3.3, with endogenously determined optimal lump-sum grants, are perhaps the closest to the framework of the uniform taxation theorems. But even here, some departures from the original specifications can be found. All the models in section 3.3.3 are based on populations with non-identical preferences. With the demographic models of Ebrahimi and Heady (1988) and Bastani *et al.* (2014), preferences are heterogeneous because of different household characteristics. In the models of Revesz (1997, 2014a, 2014b), two different sets of preferences are specified for low and high-income taxpayers, in order to obtain non-linear Engel curves with LES. Such a segmented utility framework is not really needed for non-linear Engel curves, but it is necessary in these particular models<sup>27</sup>. Despite some departures from the specifications in eq. (1), the models discussed in section 3.3.3 appear to be quite relevant to the tax uniformity debate.

### 3.5. Some critical remarks on flexible demand functions

The demand systems used in early empirical-computational work (such as LES) relied on very strict assumptions, such as separability and linear Engel curves. This does have the advantage of being perfectly consistent with consumer theory. More recent studies have avoided putting a priori assumptions on behavior, by using flexible functional forms. The results risk inconsistency with consumer theory. While this issue is neglected in the literature, several examples can be given.

The study of Asano and Fukushima (2006) shows important improvements (such as increasing the number of goods and not imposing separability together with non-linear Engel curves), but their results are weakened by the fact that nothing is done in order to check global characteristics at the optimum. They discuss theory consistency for estimated elasticities at the sample mean, but make no attempt to clarify whether the same conditions will be fulfilled at the optimum.

The same criticism applies to other studies, such as Murty and Ray (1989) and Ray and Blacklow (2002). The flexible form demand system, NLPS, is not globally quasi-concave (Blundell and Ray, 1984, p. 802). How the restricted versions (such as RNLPS) perform in this regard has not been explained.

### 3.6. An alternative approach: Optimal marginal reforms

Another approach to the tax optimization problem is to consider marginal reforms instead of globally optimal tax designs (Ahmad and Stern, 1984). From a practical point of

view, its advantage is considerably lower information requirements. Whereas the globally optimal tax approach demands knowledge about individuals' complete demand system, the marginal reform approach only needs information about individuals' consumption expenditures, aggregate demand derivatives and tax rates in the initial situation (see Santoro (2007) for a survey). Moreover, the need for global consistency with demand theory is not present, so flexible functional forms could be used with more ease. The marginal reform approach also seems to be more robust to the choice of specifications than the globally optimal tax approach (Madden, 1995; Decoster and Schokkaert, 1990). All this makes marginal reform analysis attractive, but it remains somewhat limited in scope. It only indicates the direction for welfare improvement, without ensuring that the solution is the best possible outcome.

#### **4. The treatment of income tax**

A particular problem, that diminishes the practical relevance of both the theoretical and the empirical-computational literature, is the treatment of income tax. In many computational models income tax is missing entirely. Almost all relevant theoretical studies deal with A-S model or its simplified version, the Stiglitz self-selection model. As noted in section 2.2.2, the Mirrlees (1971) type income tax schedule, which represents the optimal non-linear income tax function in the A-S model, suggests decreasing marginal income tax rates at the higher income range. This does not appear to be politically acceptable in a world of imperfect information and competition. The income tax schedules emerging from the Stiglitz self-selection model are even less realistic. For example, in a two-person model, the marginal income tax rate on the higher ability person will be zero and on the lower ability person it will be positive, with a lump-sum tax separating the two.

A linear income tax appears more realistic. It is used in a number of empirical-computational models, apart from the single-consumer and the zero lump-sum grant models. Nonetheless, it still departs from the piecewise linear income tax schedules encountered in practice. Given that income tax has a strong influence on the structure of optimal indirect taxation, the question arises, what would be the structure of optimal indirect taxes in the presence of actual direct tax schedules (broadly defined, including social security contributions)? There are very few papers that touch on this subject. Revesz (2014a, 2014b) incorporates hypothetical piecewise-linear income tax schedules into some of his scenarios. He finds that the presence of a non-linear and non-optimal income tax schedule reduces the dispersion and progressivity of optimal commodity tax rates. Given the worldwide trend to reduce and flatten income taxes, this finding suggests that following the reduction of income tax, the progressivity of indirect taxation should be increased. This conjecture ought to be explored further. There is certainly a need to construct optimal indirect taxation models that incorporate actual or proposed income tax schedules.

Apart from greater realism, there is another advantage in using indirect tax optimization models with actual income tax schedules, plus a lump-sum grant that represents actual social

spending per person (including welfare payments and in-kind transfers), and an actual figure for expenditure on public goods<sup>28</sup>. This way, the optimal solution for indirect taxes will be, to some extent, free of value judgments. With  $T(m_h)$  and  $b$  fixed in equations (1.1) and (1.2), the problem is no longer joint optimization of income and commodity taxation, but the optimization of indirect taxes alone. In this case another constraint is added. Not only must the solution satisfy eq. (1.2), but it must also satisfy the condition that the  $b$  obtained from the solution must equal to the actually observed lump-sum grant per person (denoted  $\hat{b}$ ). Assuming that the social welfare function ( $W$ ) is characterized by a single inequality aversion rate, the way to ensure that both eq. (1.2) and  $b = \hat{b}$  are satisfied, is to run optimizations with different values of the inequality aversion rate, and then select the solution that yields a lump-sum grant equal to  $\hat{b}$ . This will be the required optimal solution. This solution simply represents optimal indirect tax rates under given empirical conditions. Distributional value judgments are reflected in  $\hat{b}$  and  $T(m_h)$ .

## 5. Uniformity and policy related studies

In recent years some important policy related studies have supported uniform indirect taxation. These include Mirrlees et al. (2011), Arnold et al. (2011), European Commission (2013), IMF (2014) and NOU (2014). The common rationale is that income taxation and welfare payments are more suitable tools for redistributive purposes than progressive indirect taxation. This reasoning is in line with the A-S, L-K and Deaton theorems mentioned earlier<sup>29</sup>. Yet, these analytical results are based on empirically incorrect assumptions, as explained in section 2. The well-known labour and saving disincentives and evasion-avoidance problems associated with income tax, and false reporting with welfare payments, cast doubts about their suitability to address fully all distributional objectives. It is paradoxical that while these policy studies (with the possible exception of Mirrlees *et al.*, 2011) call for a reduction in income tax and its replacement by consumption taxes, at the same time, they expect income tax to be the sole tax instrument to mitigate distributional problems. Some of these policy studies also mention other factors in favor of uniform taxation. These supposed benefits include:

- More effective support to the needy through lower income tax rates or higher welfare payments than through reduced taxes on necessities.
- Lower administrative costs with uniform indirect taxation.
- Narrowing the scope for tax evasion with uniform tax rates.
- In non-VAT systems, due to cascading taxes on inputs, efficiency losses may arise as a result of non-uniform taxes on the inputs to production<sup>30</sup>.

Revesz (2014a, 2014b) suggests that computational models can be made as complex as required for realistic representation<sup>31</sup>. In elaborate models, the abovementioned issues could be examined quantitatively through appropriate enhancements. There is no need to relegate these subjects to intuitive discussions separately from other issues. Arguably, policy analysis

should rely more on empirical estimates combined with comprehensive computational modelling.

Some policy related studies note that even if sometimes there are reasons to adopt non-uniform commodity tax rates, the gains from such a policy (apart from rectifying externalities), are unlikely to outweigh the cost of a more cumbersome tax administration due to non-uniform taxes. This opinion is echoed in a number of studies, including Mirrlees et al (2011), Piketty and Saez (2013), Keen (2013) and IMF (2014). But here the question arises, is this view well-founded on empirical observations? Revesz (2014a, 2014b) points out that there is evidence to suggest that goods and services produced and marketed by large organizations, tend to be less evasion prone than those produced and marketed by small business. Other goods with lower evasion propensities include products where a large percentage passes through border checkpoints, and highly visible goods, such as real estate and motor vehicles. In view of widely differing evasion propensities, the assumption that uniform tax rates will lead to a less costly and more effective tax administration, does not seem to be always tenable. In fact, in countries that face severe compliance problems, the optimal solution is likely to involve markedly differentiated tax rates on administrative ground alone<sup>32</sup>. This is an important issue and a fertile area for future empirical research.

## 6. Conclusions

This review indicates that the theoretical arguments in favor of uniform indirect taxation for distributional purposes seem rather weak and unrealistic. Almost all empirical-computational studies published so far yield non-uniform optimal tax rates; however, the frameworks of many of these models are significantly different from those underlying the uniform taxation theorems. Many-person computational models consistently yield progressive tax structures. The unresolved controversy concerning commodity tax uniformity calls for more empirical and computational research. Care must be taken when using flexible functional forms in future computational studies, to ensure consistency with theoretical requirements, such as global quasi-concavity of direct utility. The uncritical acceptance of the tax uniformity proposition in some recent policy related studies is a cause for concern. Hopefully, in the future policy advice will rely more on comprehensive and realistic empirical-computational modelling.

## Appendix 1

In order to prove the change in the left hand side of the optimal marginal income tax formula (discussed in section 2.2.5), we shall follow here the steps outlined in appendix A

of Revesz (1989). When proportional commodity taxes are present in the model, the Lagrangian defined in (A.2) will be:

$$L = \int_{w_0}^{\bar{w}} u \{t[x, m(x)], t_m[x, m(x)]\} f' dw + \lambda \int_{w_0}^{\bar{w}} (t[x, m(x)] + \sum_j t_j^c q_j[x, m(x)]) f' dw - \lambda R$$

Here an indirect revenue term  $\sum_j t_j^c q_j$  has been added to the direct revenue term (t). Given fixed commodity tax rates ( $t_j^c$ ), the demand for commodities (q) does not depend directly on t and  $t_m$ , but only depends on them indirectly, through changes in income (m). By applying the  $\delta$  operator variations, a new term is added to (A.12):

$$\delta t^c = \sum_j \sum_i t_j^c \frac{\partial q_j}{\partial x_i} \delta x_i = \sum_j \sum_i t_j^c \frac{\partial q_j}{\partial m} \frac{\partial m}{\partial x_i} \delta x_i = \sum_j t_j^c \frac{\partial q_j}{\partial m} \delta m = t_m^c \delta m$$

Making the  $t_m^c$  term compatible with other derivatives:  $t_m^c = \sum_i t_j^c \frac{\partial q_j}{\partial m} = \sum_i t_j^c \frac{\partial q_j}{\partial w} \frac{\partial w}{\partial m} = \frac{t(c)'}{m'}$ , hence  $\delta t^c = \frac{t(c)'}{m'} \delta m$  Using the definition of  $\delta m$  from (A.10) and combining the variations in  $\delta u$  (from A.9),  $\delta t$  (from A.11) and  $\delta t^c$  from above, we obtain the following replacement for (A.12).

$$\delta L = \int_{w_0}^{\bar{w}} \left[ -\frac{\partial u}{\partial y} \delta t + \lambda \delta t - \frac{\lambda (t' + t(c)') w^2}{(m')^2} \frac{\partial \ell}{\partial \dot{w}_{w_0}} \frac{d(\delta t)}{dw} - \lambda \frac{(t' + t(c)') w}{m'} \frac{\partial \ell}{\partial y} \delta t \right] f' w$$
. Notice that here the pair  $(t' + t(c)')$  replaces  $t'$  in (A.12). The term  $T' = t' + t(c)'$  will replace  $t'$  everywhere in the following steps, leading to the extended version of the optimal marginal income tax formula (eq. (10) in Revesz (1989) and eq. (5) in Revesz (2003)), where  $\frac{t_m}{1 - t_m}$  is replaced by  $\frac{t_m + t_m^c}{1 - (t_m + t_m^c)}$  on the left hand side. The fact that there is no explicit change on the right hand side of the marginal income tax formula, suggests that the combined direct plus indirect tax rates in the presence of proportional commodity taxes will be similar to the optimal solution obtained with income tax alone. Due to possible changes in endogenous variables (incomes, commodities and elasticities), a perfect equivalence in outcomes is unlikely to occur.

### Notes

1. Such interchangeability between taxes is only valid in the absence of heterogeneous evasion and administrative costs.
2. The principal arguments are that taxes on consumption affect less saving and investment decisions compared to income taxes, cause less work disincentives, are less vulnerable to tax evasion, hence are more favorable to economic growth.
3. Ray (1997) places a strong emphasis in this survey on his own studies (see sections 3.3.1 and 3.3.2).
4. Several surveys deal with the theory of optimal indirect taxation, for example Auerbarch (1985).

5. General equilibrium models could be useful when examining distortions on the production side in non-VAT indirect taxation systems.
6. Progressive indirect taxation means higher taxes on luxuries and lower taxes or subsidies on necessities.
7. The first-best solution would be to impose a lump-sum tax.
8. Note, the definition of weakly separable utility is also  $U = U(v(x), L)$ , but without a homogeneity condition on  $v$ .
9. Atkinson and Stiglitz (1972) also showed that with weakly separable utility and homogeneous preferences, optimal commodity tax rates will be uniform in a single-person model. In fact, this theorem applies also in a many-person setting, because given homothetic preferences for goods, differentiated indirect taxation cannot be used as a screening device for ability-to-pay.
10. Quasi-separable utility defines weak separability in respect to the minimum cost function. In this case the cost function is defined as:  $c(u, w, \mathbf{p}) = c(u, w, \Omega(u, \mathbf{p}))$  where  $\mathbf{p}$  represents prices,  $w$  is the wage rate,  $u$  is utility and  $\Omega(u, \mathbf{p})$  is a homogeneous function of degree one with respect to prices, representing a perfect consumer price index.
11. An earlier version of the inverse elasticity rule was discussed in Ramsey (1927).
12. A rigorous mathematical treatment of the A-S model is presented by Ruiz del Portal (2012) for the non-linear commodity tax case. He introduces into the literature second order effects, including bunching of consumers and kinks in the tax schedule, and a sub-utility function defined as  $v(x_1 \dots x_n, z)$ , which includes income ( $z$ ), in addition to commodities. He finds that the A-S theorem remains valid subject to these complications. Hellwig (2010) presents a similar mathematical analysis for linear taxes.
13. Apart from the heterogeneity issue, in section 5 dealing with mixed taxation involving proportional and non-linear commodity taxes, Mirrlees (1976) concluded that if at least one good is subject to a non-linear tax, then proportional taxes should bear more heavily on goods that high ability individuals have relatively stronger taste for. This objection to uniformity is derived from a slightly different model than the A-S theorem.
14. Revesz (2014a, 2014b) presents approximate formulas for the effect of these factors on optimal tax rates.
15. The income tax functions obtained from the self-selection models are piecewise-linear, however, they differ from actual income tax schedules that are also piecewise-linear. Actual income tax schedules are continuous functions of income, while those obtained from the self-selection model may be discontinuous.
16. Looking from a different perspective, Revesz (1997) demonstrates analytically and with numerical results that a revenue-neutral replacement of progressive indirect taxation by progressive direct taxation, which leads to zero commodity taxes, will reduce labour supply and social welfare. This finding also appears to contradict the L-K proposition.
17. In the Deaton (1979a) model, in order to have the same propensity to pay taxes at all income levels (i.e. have a constant marginal indirect tax rate), all linear Engel curves must start with positive values from the lowest disposable income.
18. Utility functions with few parameters, such as LES, CES and Cobb-Douglas, satisfy global compatibility with demand theory (including global quasi-concavity) with ordinary parameter estimates.
19. AIDS utility is defined in terms of the minimum cost function as:  

$$\log m = \alpha_0 + \sum_i \alpha_i \log p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \log p_i \log p_j + U \beta_0 \prod_i p_i^{\beta_i} \quad \text{s.t.} \quad \sum_i \beta_i = 0; \quad \sum_i \gamma_{ij} = 0; \quad \gamma_{ij} = \gamma_{ji}$$
with corresponding demand:  $w_i = \alpha_i + \sum_j \gamma_{ij} \log p_j + \beta_i \log(\frac{m}{P})$   
where  $m$  is total expenditure,  $w$  is budget share,  $U$  is utility,  $p_i$  are prices and  $P$  is a general price index.
20. In terms of analytical structure, these models are close to Ramsey (1927).

21. The Stone-Geary linear expenditure system (LES) is defined as:

$$\begin{aligned} \text{utility } u &= \sum \beta_i \log(x_i - \gamma_i) & \text{s. t. } \sum_i \beta_i &= 1 \\ \text{demand } p_i x_i &= p_i \gamma_i + \beta_i (m - \sum p_i \gamma_i) \end{aligned}$$

where  $m$  is income,  $x_i$  is the quantity demanded,  $p_i$  is price,  $\beta_i$  and  $\gamma_i$  are parameters.

Houthakker's direct addilog utility-demand is defined as:

$$\text{utility } u = \left(\frac{\alpha_1}{\beta_1}\right) x_1^{\beta_1} + \left(\frac{\alpha_2}{\beta_2}\right) x_2^{\beta_2} + \dots + \left(\frac{\alpha_n}{\beta_n}\right) x_n^{\beta_n}$$

with demand relationship  $(\beta_i - 1) \log x_i - (\beta_j - 1) \log x_j = \log p_i - \log p_j - \log\left(\frac{\alpha_i}{\alpha_j}\right)$

22. For these theoretical results refer to Deaton (1981) and Baumol and Bradford (1970).
23. It appears that Ray (1986) performed only the first step of what should be a multi-step iterative computational process.
24. The cost function underlying the NLPS is written as  $c(u, p) = [a(p, \alpha) + ub(p, \alpha)]^{\frac{1}{\alpha}}$ , where  $a(\cdot)$  and  $b(\cdot)$  are homogeneous of degree  $\alpha$  in prices,  $p$ . When  $\alpha = 1$  this reduces to the Gorman Polar Form family of systems with LES being the most well-known. We obtain the NLPS by specifying  $a(\cdot) = \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} p_i^{\frac{\alpha}{2}} p_j^{\frac{\alpha}{2}}$ ,  $b(\cdot) = \prod_{k=1}^n p_k^{\alpha \beta_k}$  and  $\sum_{k=1}^n \beta_k = 1$ . We can then derive the NLPS demand function in budget share terms,  $w_i = \sum_{j=1}^n \gamma_{ij} Z_i^{\alpha/2} Z_j^{\alpha/2} + \beta_i (1 - \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} Z_i^{\alpha/2} Z_j^{\alpha/2})$ , s.t.  $\sum_{i=1}^n \beta_i = 1$ ,  $\gamma_{ij} = \gamma_{ji}$ , where  $Z_i = p_i/x$  and  $x$  is aggregate expenditure. The RNLPS follows from this when setting  $\gamma_{ij} = 0$ , for  $i \neq j$ . RNLPS still allows for both non-linearity and non-separability, however, price flexibility is more restricted than for NLPS.
25. Revesz (2014b) also presents some illustrative calculations on the dispersion of commodity tax rates in the presence of sub-optimal lump-sum grants, starting from zero and up to the optimum.
26. These include Ebrahimi and Heady (1988) and Revesz (1997).
27. Had these models applied identical LES preferences, some luxury goods would show up with negative demand for low income consumers. Setting these negative values to zero would upset the budget constraint. Thus, two separate LES utility functions were needed in order to resolve this problem.
28. For a more realistic representation,  $b$  can be made a function of income, i.e.  $b(m_h)$ . With this setup,  $\bar{b}$  will equal average  $b(m_h)$ .
29. Generally, most policy related studies accept that there is a role for Pigovian taxes to correct negative externalities, particularly on the environmental front. However, there is a less supportive view on differentiated indirect taxation, when it comes to distributional or paternalistic concerns, or to leisure complements and substitutes.
30. Examination of this issue requires either general equilibrium modelling or a comparative analysis of these production efficiency losses with the gains to consumers from a simpler non-VAT sales tax system.
31. But it should be noted that most empirical-computational models published so far display a fairly low level of complexity. Possible exceptions are: Ebrahimi and Heady (1988), Revesz (1997, 2014a, 2014b), Ray and Blacklow (2002), Bastani *et al.* (2014) and Nygard (2014).
32. In other words, a representative consumer model that includes specifications in relation to evasion and administrative/compliance costs, is likely to yield differentiated tax rates depending on these specifications.

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## Resumen

La revisión de la literatura teórica sobre imposición indirecta óptima pone de relieve que los argumentos en favor de una imposición indirecta uniforme parecen bastante débiles y escasamente realistas, por lo que determinar la estructura fiscal óptima permanece como un interrogante empírico. Adicional-

mente, de la revisión de los estudios empírico-computacionales publicados hasta la fecha cabe derivar que los mismos operan bajo marcos bastante restrictivos y simplistas. Existe escasa evidencia computacional para apoyar la uniformidad en la imposición indirecta, sobre todo cuando los modelos tratan de reflejar la complejidad del mundo real. A tenor de lo expuesto, parece que en una economía formada por muchos consumidores, una imposición indirecta no uniforme y progresiva puede ser la solución óptima.

*Palabras clave:* Imposición óptima, imposición sobre productos, imposición indirecta, evasión fiscal.

*Clasificación JEL:* H21, H23, H26, C63.